Cycles of Conflict: An Economic Model*

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Abstract

We propose a model of cycles of conflict and distrust. Overlapping generations of agents from two groups sequentially play coordination games under incomplete information about whether the other side consists of "bad types" who always take bad actions. Good actions may be misperceived as bad and information about past actions is limited. Conflict spirals start as a result of misperceptions but also contain the seeds of their own dissolution: Bayesian agents eventually conclude that the spiral likely started by mistake, and is thus uninformative of the opposing group's type. The agents then experiment with a good action, restarting the cycle.

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Mutual benefits from trust and cooperation not withstanding, inter-group conflict is pervasive. In his study of the Peloponnesian War, Thucydides (2000) traces the origins of conflict as much to fear and distrust as to other factors such as greed and honor. He argues that the Peloponnesian War became inevitable precisely because each side saw war as inevitable and did not want to relinquish the first mover advantage to the other (see also Kagan, 2004).[1] This view of conflict, sometimes referred as the Hobbesian view or the spiral model, has a clear dynamic implication: if Group A's actions look aggressive, Group B infers that Group A is likely to be aggressive and acts aggressively itself (e.g., Jervis, 1976, Kydd, 1997). Moreover, unless Group A can fully understand that Group B is acting aggressively purely in response to its own actions, it will take this response as evidence that Group B is aggressive. As a result, conflict spirals.

The ubiquity of "conflict spirals" throughout history provides *prima facie* support for this view. A leading example is ethnic conflict: Donald L. Horowitz argues "The fear of ethnic domination and suppression is a motivating force for the acquisition of power as an end" (Horowitz, 2000, p. 187), and suggests that such fear of ethnic domination was the primary cause of the rise in ethnic violence following the withdrawal of colonial powers. Horowitz also suggests (p. 189, italics in the original) "The imminence of independence in Uganda aroused 'fears of future ill-treatment' along ethnic lines. In Kenya, it was 'Kikuyu domination' that was feared; in Zambia, 'Bemba domination'; and in Mauritius, ... ['Hindu domination']... Everywhere the word *domination* was heard. Everywhere it was equated with political control."

More recent examples of such spirals are provided by conflicts in Northern Ireland, the Balkans, Lebanon, Iraq, Gaza and the West Bank, and Turkey. For instance, many accounts of the dynamics of the Serbian-Croatian war emphasize Croatian fears from the aggressive posturing of Milosevic, which were instrumental in triggering more aggressive Croatian actions—including the adoption as the national symbol of the sahovnica, associated with the fascist pre-Yugoslavia Ustasha regime, and a variety of discriminatory policies towards the Serbian minority (e.g., Posen, 1993).[2] Another example comes from the recurrent Colombian civil wars fought between the Liberal and Conservative parties and their supporters starting in the early 1850s. After civil wars in 1851, 1854, 1859-63, 1876, 1884-85, 1895, and 1899-1902, conflict resumed again in the 1940s in part because of Conservative fears in the

[1] The fear motive for conflict is also referred to as the "Hobbesian trap" or the "security dilemma" (following Schelling, 1960). It is modeled by, among others, Baliga and Sjostrom (2004) and Chassang and Padro i Miquel (2010). On models of conflict spirals, different from the ones we discuss below, see also Fearon and Laitin (1996) and the references therein.

[2] DellaVigna et al. (2012) provide further evidence highly suggestive of a conflict spiral in this context. They show that Croatians who received nationalistic radio broadcasts from the Serbian side were more nationalistic and more supportive of anti-Serbian actions. Kaplan et al. (2005) provide evidence consistent with a "cycle of violence" from the Israeli-Palestinian conflict (but see also Jaeger and Paserman, 2008).

face of growing Liberal popularity that they would be permanently excluded from power. Triggered by the murder of Liberal leader Jorge Eliécer Gaitán, the most notorious episode of civil conflict, *La Violencia*, erupted in 1948. The subsequent widespread agitation led by street mobs in Bogotá was in turn interpreted by the Conservatives as a move against them by the Liberals, leading to all-out civil war (Hartlyn, 1988; Safford and Palacios, 2002).[3]

This classical view of conflict and distrust is incomplete, however, because it only explains how conflict *starts* and not how it *stops*—even though most conflict spirals come to an end sooner or later. For example, sectarian conflict in Northern Ireland has ended starting with a cease-fire in 1994 ultimately leading to the Good Friday agreement in 1998; war and conflict between different ethnic and national groups in the Balkans have mostly ended; historical Franco-German distrust and animosity has made way for vibrant trade and economic and diplomatic cooperation; and many bloody ethnic wars that seemed intractable after the end of World War II have abated dramatically over the past two decades. Even in Colombia, the cycle of wars made way to durable peace brokered by a power-sharing agreement in 1957, which interestingly was led by some of the most hard-line leaders such as the Conservative Laureano Gómez. So rather than *infinite* conflict spirals—where conflict once initiated never subsides—history for the most part looks like a series of *conflict cycles*, where even long periods of conflict eventually end.

This paper proposes a simple model of conflict spirals, and then shows that such spirals contain the seeds of their own dissolution—thus accounting for not only the onset but also the end of conflict. The basic idea of our approach is simple: once Groups A and B get into a phase in which they are both acting aggressively, the likelihood that a conflict spiral has been triggered by mistake at some point increases over time. This implies that aggressive actions—which are at first informative about how aggressive the other side truly is—eventually become uninformative. Once this happens, one group will find it beneficial to experiment with cooperation and, unless the other group is truly aggressive, cooperation will resume—until the next conflict spiral begins.

Formally, our model features a coordination game between overlapping generations of (representatives of) two groups. The "bad" action in the coordination game may correspond to fighting or initiating other types of conflict, and is a best response to bad actions from the other party, while the "good" action is optimal when good actions are expected. Each side is uncertain about the type

[3] Spiral effects might account not only for violent conflict between nations and ethnic groups, but also for distrust between groups and within organizations. Guiso, Sapienza and Zingales (2009) document deep-rooted distrust among some nations, and show that it is associated with lower international trade and foreign direct investment, and Bottazzi, Da Rin, and Hellmann (2011) show a similar pattern for international business ventures. Kramer (1999) surveys a large social psychology literature on distrust in organizations.

of their opponents, who may be—with small probability—"committed" to the bad action. The two distinguishing features of our approach are: (1) noisy Bayesian updating, so that individuals (groups) understand that conflict may be initiated because of a misperception or unintended action; and (2) "limited memory," so that there is limited information about the exact sequence of events in the past, and when and for what reason conflict started is unknown. Both features are plausible in practice. Indeed, a widespread view in international relations is that misperceptions are both very common and of central importance for conflict spirals (Jervis, 1976). Limited memory also plays an important role in our theory. Without it, one party would understand that the spiral was started by its own action being misperceived by the other side and would attempt to rectify this misperception. Such detailed understanding of the origins of conflict are often not possible; instead, participants have only limited understanding of exactly when and how conflict started. This feature is captured by our limited memory assumption (relaxations of which are discussed below).

These features together generate a distinctive pattern where, in the unique sequential equilibrium of this dynamic game, a spiral of distrust and conflict is sometimes initiated and persists, but must also endogenously come to an end. The first contribution of our model is to show that because of limited information about the past, when the current generation sees conflict (but not how it came about), it often responds by choosing a bad action, perpetuating the spiral.[4] The main contribution of our model is to show that such spirals of conflict eventually terminate: when an individual or group reasons that there have been "enough" chances for a conflict spiral to have gotten started (call this number T), they conclude that the likelihood that it started by mistake—rather than being started intentionally by a truly aggressive adversary—is sufficiently high, and they therefore experiment with the good action. In our baseline model, these two forces lead to a unique equilibrium which features a mixture of deterministic and stochastic cycles. In particular, a single misperceived action stochastically initiates a conflict spiral, which then concludes deterministically at the next time t that is a multiple of T.[5]

Our model can be best described as a reputation model with limited records and overlapping generations, and it is, to our knowledge, the first such model in the literature. Liu and Skrzypacz (2013), Liu (2011), and Monte (2011) also study reputation models with limited records, but their

[4] This necessity of some form of limited memory or information about past signals and actions for generating conflict sprials is quite general, but is not discussed in standard informal accounts of conflict spirals (e.g., Posen, 1993).

[5] We certainly do not claim that every possible model of conflict spirals leads to cycles. For example, Rohner, Thoenig and Zilibotti (2013) develop a dynamic "Hobbesian" model of conflict where information about a group's type accumulates over time, leading asymptotically to either permanent war or permanent peace. The key difference is that, because of limited memory, information does not accumulate in our model, so this "asymptotic learning" does not occur.

models do not generate deterministic cycles, and the mechanism for cycles is quite different from ours (for example, players in our model have no incentives to manipulate their reputations, since they are all short-lived). Studies of overlapping generations games include, among others, Lagunoff and Matsui (1997) and Bhaskar (1998), which present anti-folk theorems, and Anderlini and Lagunoff (2001), Lagunoff and Matsui (2004), and Kobayashi (2007), which provide folk theorems with limited memory and altruism across generations (a possibility we consider in the online appendix). Anderlini, Gerardi, and Lagunoff (2010) present a model of equilibrium conflict cycles where each group is held below what its minmax payoff would be if it were a single decision-maker. Acemoglu and Jackson (2012) study a coordination game with overlapping generations and imperfect monitoring, and show how social norms change over time when "prominent" players can try to improve the social norm. Their model does not feature incomplete information about player types or deterministic cycles.

A central and distinguishing feature of our model is the uniqueness of equilibrium and the associated cycles, which are driven by the endogenously changing information content of players' actions. Pesendorfer (1995) also generates cycles with changing information content, but the logic of his model, which is based on signaling, is completely different. Finally, as compared to reputation models where players' types follow a Markov process (Mailath and Samuelson, 2001; Phelan, 2006; Wiseman, 2009; Ekmecki, Gossner, and Wilson, 2012), it is noteworthy that our model predicts cycles in an environment that is stationary and that is also a natural dynamic version of canonical conflict spiral models.[6]

The rest of the paper proceeds as follows. The next section presents our baseline model and main results, including a range of comparative statics. Section II extends our results to the case in which calendar time is unobserved, while Section III considers the case in which more signals about the past are available. Section IV concludes. Appendix A provides the proofs omitted from the text, and Appendix B—which is available online—relaxes several additional simplifying assumptions adopted in the baseline model.

[6] Also somewhat related are repeated games with imperfect public monitoring in which behavior fluctuates with the public history (see Green and Porter (1984) and Abreu, Pearce, and Stacchetti (1988, 1990) for canonical analyses, and Yared (2010) for an application to cycles of war and peace), and stochastic games with perfect information in which behavior changes with the state (e.g., Alesina (1988), Baron (1996), Dixit, Grossman, and Gul (2000), Battaglini and Coate (2008), Acemoglu, Golosov, and Tsyvinski (2010)). These models differ from ours in that they do not have incomplete information about types, deterministic cycles or equilibrium uniqueness. Tirole (1996) studies a simplified model of imperfect public monitoring generating collective reputation, though again with no cycles or endogenously changing information content of actions.

I Baseline Model

In this section, we present our baseline model, which formalizes in the simplest possible way how conflict spirals can form but cannot last forever when individuals are Bayesian and have limited information about the history of conflict.

I.A Model and Equilibrium Characterization

Two groups, Group A and Group B, interact over time $t = 0, 1, 2, \ldots$. At every time t, there is one active player ("player t") who takes a pair of actions $(x_t, y_t) \in \{0,1\} \times \{0,1\}$, where $x_t = 1$ and $y_t = 1$ are "good" ("honest," "peaceful") actions and $x_t = 0$ and $y_t = 0$ are "bad" ("cheating," "aggressive") actions; as will become clear, x_t is player t's action toward player $t-1$, and y_t is player t's action toward player $t+1$. In even periods the active player is a member of Group A, and in odd periods the active player is a member of Group B. A different player is active every period. A key assumption is that players observe very little about what happened in past periods: in particular, before player t takes her action, she observes only a signal $\tilde{y}_{t-1} \in \{0,1\}$ of player $t-1$'s action toward her. This assumption captures the feature that agents may know that there is currently conflict without fully knowing when and how this was initiated in the past. We assume that $\tilde{y}_{t-1}$ is determined as:

$$\begin{aligned}
\Pr\left(\tilde{y}_{t-1} = 1 | y_{t-1} = 1\right) &= 1 - \pi \\
\Pr\left(\tilde{y}_{t-1} = 1 | y_{t-1} = 0\right) &= 0,
\end{aligned}$$

where $\pi \in (0,1)$ is the probability of a "misperception."[7] Thus, a good action sometimes leads to a bad signal, but a bad action never leads to a good signal (both this and the assumption that nothing from the past history beyond the last period is observed are later relaxed).

Each group consists either entirely of normal types or entirely of bad types. The probability that a group is bad (i.e., consists of bad types) is $\mu_0 > 0$. Playing $(x_t = 0, y_t = 0)$ is a dominant strategy for the bad type of player t.[8] For $t > 0$, the normal type of player t has utility function

$$u\left(x_t, \tilde{y}_{t-1}\right) + u\left(y_t, x_{t+1}\right),$$

[7] There are several ways of interpreting the misperception probability π. Most simply, one group may literally misperceive the other's action, or a group's leaders may try to do one thing but mistakenly do another. An alternative assumption, which is mathematically identical, is that, even when a group's type is normal, a fraction π of its members are extremists or "bad types" who always play 0; for example, they may be "provacateurs" who benefit from sending the groups into conflict (cf. Rabushka and Shepsle, 1972, Glaeser, 2005, Baliga and Sjostrom, 2011).

[8] This is equivalent to bad types perceiving the the game as a prisoner's dilemma rather than a coordination game (and is thus much weaker than a long-run player in a standard reputation-formation model having a dominant strategy, which requires that she would rather face the worst possible action of her opponent every period than play a different action herself once).

so her overall payoff is the sum of her payoff against player $t-1$ and her payoff against player $t+1$.[9] By writing payoffs as a function of the realized signal $\tilde{y}_{t-1}$ (rather than the action y_{t-1}), we are following the literature on dynamic games with imperfect monitoring in ensuring that no additional information is obtained from realized payoffs.[10] The normal type of player 0 has utility function $u(\tilde{y}_0, x_1)$.[11]

We assume that each "subgame" between neighboring players is a (sequential-move) coordination game, and that $(1,1)$ is the Pareto-dominant equilibrium as formally stated next.

Assumption 1 (Coordination Game with $(1,1)$ Pareto-Dominant)

1. $u(1,1) > u(0,1)$.
2. $u(0,0) > u(1,0)$.
3. $u(1,1) > u(0,0)$.

We also assume that the probability that a group is bad is below a certain threshold $\mu^* \in (0,1)$:

Assumption 2 (Favorable Prior Beliefs)

$$\mu_0 < \mu^* \equiv 1 - \frac{1}{1-\pi} \frac{u(0,0) - u(1,0)}{u(1,1) - u(1,0)}.$$

Assumption 2 is equivalent to assuming that normal player 0, with belief μ_0, plays $y_0 = 1$ when she believes that player 1 plays $x_1 = 1$ if and only if he is normal and sees signal $\tilde{y}_0 = 1$.

We can now explain the logic of the model. Assumption 1 ensures that in any sequential equilibrium player t does indeed play $x_t = 1$ if and only if he is normal and sees signal $\tilde{y}_{t-1} = 1$. In view of this, Assumption 2 implies that normal player 0's prior about the other group is sufficiently favorable that she plays $y_0 = 1$.

Next, consider normal player 1. If he sees signal $\tilde{y}_0 = 1$, then he knows the other group is normal—since bad types take the bad action, which never generates the good signal. In this case, his belief about the other group is even better than player 0's, so he plays $y_1 = 1$ (in addition to playing $x_1 = 1$).

But what if player 1 sees signal $\tilde{y}_0 = 0$? In this case, he clearly plays $x_1 = 0$, and moreover, by Bayes rule, his posterior belief that the other group is bad rises to

$$\mu_1 = \frac{\mu_0}{\mu_0 + (1-\mu_0)\pi} > \mu_0,$$

[9] Changing this utility function to $(1-\lambda) u(x_t, \tilde{y}_{t-1}) + \lambda u(y_t, x_{t+1})$ for $\lambda \in (0,1)$ would have no effect on the results or in fact on the expressions that follow.

[10] Little would change if payoffs depended directly on y_{t-1} rather than $\tilde{y}_{t-1}$ (and were observed after choosing y_t), or conversely, if payoffs depended on $\tilde{y}_t$ rather than y_t. In the former case if, in addition to changing the utility function, we also dropped the assumption that $\Pr(\tilde{y}_{t-1} = 1 | y_{t-1} = 0) = 0$ (as we do in one of the extensions in the online appendix), we would lose equilibrium uniqueness as in Bagwell (1995). We thank an anonymous referee for pointing this out.

[11] Note that this makes action x_0 irrelevant, so we ignore it (equivalently, assume that player 0 only chooses $y_0 \in \{0,1\}$).

which follows in view of the fact that $\tilde{y}_0 = 0$ may have resulted from the other side being bad (probability μ_0), or from the bad signal following the good action when the other side is normal (probability $(1 - \mu_0)\,\pi$). Now if μ_1 is sufficiently high—in particular, if it is above the cutoff belief μ^*—then player 1 plays $y_1 = 0$ after seeing signal 0.[12]

Now suppose that up until time t normal players play $y = 0$ after seeing signal 0, and consider the problem of normal player t. Again, if she sees signal 1, she knows the other group is normal and plays $(x_t = 1, y_1 = 1)$. But if she sees signal 0, she knows that this could be due to a bad signal arriving at *any* time before t, because a single bad signal starts a spiral of bad actions. Thus, her posterior is

$$\mu_t = \frac{\mu_0}{\mu_0 + (1 - \mu_0)\left(1 - (1 - \pi)^t\right)},$$

which follows since the probability of no bad signal arriving at any time before t, conditional on the other side being normal, is $(1 - \pi)^t$, and thus the total probability of player t seeing $\tilde{y}_{t-1} = 0$ is $\mu_0 + (1 - \mu_0)\left(1 - (1 - \pi)^t\right)$.

If μ_t is above the cutoff belief μ^*, then player t again plays $y_t = 0$ after seeing signal 0. Crucially, note that μ_t is decreasing in t, and furthermore that $\mu_t \to \mu_0$ as $t \to \infty$. Recall that $\mu_0 < \mu^*$. Thus, there is some first time T—given by equation (A1) in the appendix—at which $\mu_T \leq \mu^*$. And at this time, player T plays $y_T = 1$ *even if she sees signal* 0. Thus, any spiral of bad actions that started before time T ends at T.

Finally, consider the problem of normal player $T + 1$. He knows that player T plays $y_T = 1$ if and only if she is normal. Thus, player $T + 1$ is in exactly the same situation as player 1, and play from period $T + 1$ on is exactly like play from period 1 on. Hence, play is characterized by cycles of length T, in which a single bad signal at some time t starts a spiral of bad actions that lasts until the next multiple of T.

A central feature of the above argument is that it holds regardless of beliefs about future play. Consequently, equilibrium is unique up to one technicality: if μ_T exactly equals μ^*, then cycles can be of length either T or $T + 1$, and this can eventually lead to "restarts" of cooperation occurring at a wide range of times. To avoid this possibility, we make the following genericity assumption on the parameters:

Assumption 3 (Genericity) $\mu_t \neq \mu^*$ for all $t \in \mathbb{N}$.

[12] We do not assume that $\mu_1 > \mu^*$. But if $\mu_1 < \mu^*$, then the conflict "cycle" that emerges is the trivial cycle where cooperation always restarts immediately after a misperception.

We now state our main result for the baseline model, establishing that there is a unique equilibrium and that it displays cycles. A similar cyclic equilibrium structure will arise in all of the extensions studied later in the paper and in the online appendix.

Proposition 1 *Under Assumptions 1-3, the baseline model has a unique sequential equilibrium. It has the following properties:*

1. *At every time $t \neq 0 \bmod T$, normal player t plays good actions $(x_t = 1, y_t = 1)$ if she gets the good signal $\tilde{y}_{t-1} = 1$, and plays bad actions $(x_t = 0, y_t = 0)$ if she gets the bad signal $\tilde{y}_{t-1} = 0$.*

2. *At every time $t = 0 \bmod T$, normal player t plays the good action $x_t = 1$ toward player $t-1$ if and only if she gets the good signal $\tilde{y}_{t-1} = 1$, but plays the good action $y_t = 1$ toward player $t+1$* regardless of her signal*.*

3. *Bad players always play bad actions $(x_t = 0, y_t = 0)$.*

It is straightforward to turn the above discussion into a proof of Proposition 1, and we omit this formal proof (which may be found in an earlier version of the paper).

The unique equilibrium described in Proposition 1 has several features that we believe are both interesting from a modeling perspective and suggestive of real-world cycles of conflict and distrust. First, the proposition implies that conflict cycles have both stochastic and deterministic elements: conflict spirals start with random misperceptions, but end at pre-determined dates where conflict becomes uninformative. Second, the probability of the groups experiencing conflict increases over time, as they get more and more chances to accidentally trigger a conflict spiral. Third, a group's posterior belief that the other group is bad conditional on observing conflict decreases over time, as conflict becomes relatively more likely to have been triggered by mistake. Fourth, "restarts" of cooperation occur when this posterior drops below the threshold belief μ^* needed to sustain cooperation—this happens when a group becomes sufficiently optimistic about the other group's type that it is worth experimenting with the good action. Fifth, as noted in the Introduction, limited memory plays an important role in our model: when they are in the midst of a conflict spiral, the groups do not know when the spiral began or who started it. In particular, if a group could somehow learn that they had started the conflict (i.e., that they acted at the first period t for which $\tilde{y}_t = 0$), then they would realize that the conflict is completely uninformative about the other group's type, and would therefore restart cooperation. Though uncertainty about who started a conflict is a necessary factor for spirals in our model, we show in Section III that such uncertainty arises even if each group observes signals about a series of actions from the past (but not the entire past history).[13]

I.B Interpretation and Discussion

A central application of our model is to civil and international wars. Consider, for example, two groups (or two countries) that repeatedly face the potential for conflict. For each potential conflict, the groups sequentially choose between two actions, one of which corresponds to aggression or war. The "security dilemma," or the "Hobbesian trap," suggests a coordination game form in which a group or country likes taking the aggressive action if and only if the other side is aggressive. In our overlapping-generations setup, this exactly corresponds to parts 1-2 of Assumption 1, implying that aggression is a best response to the belief that the other side has been aggressive so far or is expected to be aggressive in the future.[14] Part 3 of Assumption 1 then implies that both sides are better off without such aggression.[15]

It is also useful to return to the two central features of our approach emphasized in the Introduction, misperceptions and limited memory, in this context. It is certainly plausible that non-aggressive acts are sometimes viewed as aggressive by the other party or that, as pointed out in footnote 7, some aggressive elements within normal groups can instigate conflict even when the group itself is normal or non-aggressive. Our limited memory assumption, positing that the past history of signals is not fully observed (especially in the less extreme form used in Section III), is also reasonable in this context. Even though we all have access to history books, it is difficult to ascertain and agree on how and exactly when a given conflict started, and indeed disagreement over "who started it" appears to be a pervasive feature of conflict. In Section III, we also point out that even if certain key, symbolic events are always remembered, lack of full memory of all past events leads to similar dynamics.

[13] More generally, in our model a player who assesses that the other group is more likely to be bad after observing conflict also assesses that the other group is more likely to have started the current conflict spiral. Thus, leaders who are more confident that the other group started the conflict tend to fight, while leaders who unsure who started the conflict are peaceful.

[14] Jervis (1976) and Baliga and Sjostrom (2004) also model conflict as a coordination game rather than, in particular, a prisoner's dilemma. We do not think there is a "right" answer as to whether conflict should be modeled as a coordination game or a prisoner's dilemma, and believe that both approaches can be useful (for example, Chassang and Padro i Miquel (2010) consider a prisoner's dilemma-like game), although we do think the standard Schelling/Hobbes story describes a coordination game. If the stage game in our model were a prisoner's dilemma rather than a coordination game, the only equilibrium would be "Always Defect."

[15] One might argue that our baseline model would better capture the "first-mover advantage" aspect of war or conflict if we allowed player t's payoff from choosing war after getting the peaceful signal from player $t-1$ to differ from her payoff from choosing war prior to player $t+1$'s playing peace; that is, if we allowed a player's payoff to depend on whether she moves first or second in a given conflict. Our results would not be affected by this generalization so long as each potential conflict remains a coordination game (i.e., a player always wants to match her opponent's action or signal, regardless of whether she moves first or second).

I.C Comparative Statics and Welfare

We next provide comparative statics on the average duration of conflict and results on social welfare (when the probability of a misperception π is small), which are both of interest in and of themselves and useful for building intuition about the mechanics of the model.

By a *conflict spiral*, we mean a sequence of consecutive periods t with outcome $(\tilde{y}_t = 0, x_{t+1} = 0)$.[16] The *average duration of conflict* is then the expected average length of a conflict spiral, conditional on both groups being normal. Formally, let l_m be the length of the m^{th} conflict spiral for a given sample path. The average duration of conflict is

$$E\left[\lim_{M\to\infty}\frac{1}{M}\sum_{m=1}^{M} l_m\right],$$

where the expectation is taken over sample paths, assuming both groups are normal. The focus on both groups being normal is natural: when at least one group is bad there is conflict in almost every period (i.e., in every period except possibly for multiples of T), so in this case the average duration of conflict is not very interesting.

Our comparative static results are as follows: First, the average duration of conflict is greater when $u(0,0)$ is higher, $u(1,0)$ is lower, or $u(1,1)$ is lower, as all of these changes make experimenting with the good action less appealing (i.e., they decrease μ^*) and thus increase T (the time when cooperation is restarted) without affecting the probability of the onset of conflict. Second, the average duration of conflict is greater when the prior probability of the bad type is higher, as this makes players more pessimistic about the other group (i.e., increases μ_t for all t) and thus increases T—again without affecting the probability of the onset of conflict. Finally, increasing the misperception probability π has an ambiguous affect on the average duration of conflict. On the one hand, when π is higher, a bad signal is less informative about the opposing group's type, making players more optimistic (decreasing μ_t), and thus decreasing T and the average duration of conflict. This effect is quite intuitive: when the intent behind aggressive actions is less ambiguous, it takes longer to rebuild trust after an apparently aggressive action. On the other hand, increasing π also mechanically makes miscoordination more likely when the other group is good, which decreases μ^* and thus increases T and the average duration of conflict, and also makes it more likely that conflict begins earlier within each cycle, which also increases the average duration of conflict.[17] Summarizing, we have the following result:

[16] The term "conflict spiral" may imply an explicit escalation of conflict over time. Though there is a limited type of escalation in the baseline model — since in every conflict spiral between normal groups, the first period of conflict consists of a misperceived good action followed by a genuine bad action — there is no literal increase in the severity of conflict. We show how this can be incorporated into our model in the online appendix.

[17] Either effect can dominate. For example, an implication of Proposition 3 is that the average duration of conflict

Proposition 2 *The average duration of conflict is increasing in* $u(0,0)$*, decreasing in* $u(1,0)$*, decreasing in* $u(1,1)$*, increasing in the prior probability of the bad type* μ_0*, and ambiguous in the misperception probability* π*.*

Another object of interest is expected social welfare, averaged across all players (which roughly corresponds to the long-run fraction of periods spent in conflict). An interesting observation here is that expected social welfare when both groups are normal is bounded away from the efficient level $2u(1,1)$, even as the probability of a misperception π goes to 0. Thus, not only do some players receive payoff less than $2u(1,1)$ for all $\pi > 0$ (which is immediate), but the fraction of players who get less than this amount does not vanish as $\pi \to 0$. The intuition is that while, as $\pi \to 0$, the probability of a conflict spiral starting each period goes to 0, the expected length of a conflict spiral conditional on its starting goes to infinity. This is because when π is small conflict is very informative and it therefore takes a long time for cooperation to restart after a misperception. This result is in stark contrast to what would happen in a static setting, where, as $\pi \to 0$, the players could coordinate on the good outcome with probability approaching 1.[18]

In contrast, expected social welfare when both groups are normal does converge to the efficient level $2u(1,1)$ when *both* the probability of a misperception π and the prior probability that a group is bad μ_0 go to 0 (regardless of the relative sizes of these probabilities). Thus, both the probability of accidental conflict and the fear of the other group's true intentions must be small for efficiency to prevail. The intuition here can be seen from examining the formula for μ_t: if μ_0 is vanishingly small, then any positive probability of conflict $1-(1-\pi)^t$ is large enough that a player who observes conflict will restart cooperation. Hence, the probability that conflict ever actually occurs in a given T-period cycle goes to 0 when both π and μ_0 go to 0.

Formally, we have the following result, where social welfare is evaluated according to the limit-of-means criterion (proof in the appendix).[19]

Proposition 3 *Suppose that both groups are normal. Then the following hold:*

1. *The limit of expected social welfare as* $\pi \to 0$ *is less than the efficient level* $2u(1,1)$*.*

goes to infinity as $\pi \to 0$. On the other hand, decreasing π by a amount that is small enough that T remains constant necessarily decreases the average duration of conflict.

[18] More precisely, in the "static" (i.e., two-period) version of our model, when both groups are normal, the probability that both players play 1 converges to 1 and payoffs converge to the full information payoffs as $\pi \to 0$.

[19] That is, if player t's payoff is u_t, social welfare is defined to be $\lim_{N\to\infty} \frac{1}{N}\sum_{t=0}^{N} u_t$.

2. For any sequence $(\pi_n, \mu_{0,n})$ converging to $(0,0)$ as $n \to \infty$ (such that Assumptions 1-3 hold for all n), the limit of expected social welfare as $n \to \infty$ equals the efficient level $2u(1,1)$.

II Unobserved Calendar Time

One highly stylized aspect of the baseline model is the strict dependence of behavior on calendar time. This section shows that the key conclusion that limited memory leads to conflict cycles rather than infinite conflict spirals—along with the underlying reason why this is so—does not depend on this feature.

The most direct way of eliminating dependence on calendar time is assuming that players do not know calendar time: each player observes a signal $\tilde{y}_{t-1}$ of her predecessor's action and then chooses actions (x_t, y_t), without knowing t. This approach is intuitively appealing, but it introduces the somewhat delicate issue of what players believe about calendar time when they enter the game. Here, we simply assume that players have an "improper uniform prior" about calendar time, in that they take the probability of observing signal $\tilde{y}_{t-1}$ to equal the long-run fraction of periods in which the signal equals $\tilde{y}_{t-1}$.[20] Player 0, however, is assumed to know calendar time (e.g., she can infer this from the fact that she does not observe a signal).

In such a model, normal players play $y_t = 1$ in response to $\tilde{y}_{t-1} = 1$, as they know the other group is normal after observing $\tilde{y}_{t-1} = 1$. There can be no equilibrium in which normal players play $y_t = 0$ in response to $\tilde{y}_{t-1} = 0$ with probability 1. To see why, suppose that there were such an equilibrium. Then $\tilde{y}_{t-1} = 0$ would be observed almost surely in the long run. But then, by Assumption 2, a normal player would believe that the opposing group is bad with probability $\mu_0 < \mu^*$ after observing $\tilde{y}_{t-1} = 0$ and would therefore play $y_t = 1$. Notably, this is precisely the reason why conflict spirals must eventually come to an end in the baseline model.

To characterize the equilibrium, suppose that in response to $\tilde{y}_{t-1} = 0$ normal players play $y_t = 0$ with some probability $p \in [0,1]$ and play $y_t = 1$ with probability $1-p$. Then, when both groups are good, the long-run fraction of periods in which $\tilde{y}_{t-1} = 0$—denoted by q—is given by $q = \pi + (1-\pi) qp$, or

$$q = \frac{\pi}{1-(1-\pi)p}.$$

This expression follows because if there is a misperception then $\tilde{y}_{t-1} = 0$ with probability 1, while if there is not a misperception then $\tilde{y}_{t-1} = 0$ with probability qp.

[20] See Liu and Skrzypacz (2013) for a rigorous foundation of this approach.

Now a normal player's assessment of the probability that the other group is bad after observing $\tilde{y}_{t-1} = 0$ is given by

$$\mu = \frac{\mu_0}{\mu_0 + (1 - \mu_0)\, q}.$$

For her to be indifferent between playing $y_t = 0$ and $y_t = 1$, it must be that $\mu = \mu^*$, or

$$q = q^* \equiv \frac{\mu_0}{1 - \mu_0} \frac{1 - \mu^*}{\mu^*}.$$

This holds if and only if

$$p = p^* \equiv \frac{q^* - \pi}{q^* (1 - \pi)}.$$

Summarizing, we have the following result:[21]

Proposition 4 *Under Assumptions 1 and 2, the model with an improper uniform prior over calendar time has a unique symmetric sequential equilibrium. It has the following properties:*

1. *Normal player* 0 *plays the good action* $y_0 = 1$ *toward player* 1*.*
2. *At every time* $t > 0$*, normal player* t *plays good actions* $(x_t = 1, y_t = 1)$ *if she gets the good signal* $\tilde{y}_{t-1} = 1$*. If she gets the bad signal* $\tilde{y}_{t-1} = 0$*, she plays the bad action* $x_t = 0$ *toward player* $t - 1$ *and plays the good action* $y_t = 1$ *toward player* $t + 1$ *with probability* $1 - p^*$*.*
3. *Bad players always play bad actions* $(x_t = 0, y_t = 0)$*.*

Thus, even when calendar time is not observed, our model generates cycles. Instead of the deterministic restarts of cooperation of the baseline model (which obviously cannot occur when calendar time is not observed), there is now a constant probability of restarting cooperation every period. But the fundamental reason why cycles must occur is the same: If conflict spirals lasted forever, they would be uninformative in the long run, just as in the baseline model. Consequently, cooperation must restart at some point, and because there is no possibility of conditioning on calendar time in the present model, cooperation restarts stochastically as a result of mixed strategies.

III More Information About the Past

Another stylized aspect of the baseline model is the assumption that players observe a signal of only the most recent action y_{t-1} and get no information about any earlier actions. Though this

[21] Here, a *symmetric* equilibrium is one in which all normal players (except for player 0) use the same strategy. Restring to symmetric equilibrium is without loss of generality if, in addition to being unable to condition on calendar time, players are unable to condition on their own names.

simple information structure allows us to explicitly characterize equilibrium and show that it features "restarts" of cooperation every T periods, it is not necessary for our main intuition for cycling. This section shows that when players observe the previous K signals, for any integer K, there are still restarts of cooperation (though not necessarily at regular intervals); in particular, we show that there are still infinitely many times t at which a normal player plays the good action $x_t = 1$ even if she observes K bad signals, and that this occurs for essentially the same reason as in the baseline model. Also, the event that player t observes K bad signals always occurs with positive probability, in particular with probability at least π^K.

Formally, let us modify the baseline model by supposing that players observe the previous K signals, for some fixed integer K. That is, before choosing her action, player t observes $(\tilde{y}_{t-K}, \tilde{y}_{t-(K-1)}, \ldots, \tilde{y}_{t-1})$, where this vector is truncated at 0 if $t < K$. Player t's utility function is still given by $u(x_t, \tilde{y}_{t-1}) + u(y_t, x_{t+1})$, exactly as in the baseline model.

Proposition 5 *Under Assumptions 1 and 2, in any sequential equilibrium of the model where players observe the last K signals, there are infinitely many times t at which normal player t plays the good action $y_t = 1$ toward player $t+1$ with positive probability when she observes all bad signals (i.e., when $\tilde{y}_{t-k} = 0$ for all $k \in \{1, \ldots, K\}$).*

Proposition 5 and its proof show that our main intuition for cycling goes through when players observe any number of past signals, not just one. However, when $K > 1$ cycling is no longer regular (i.e., there is no longer a restart of cooperation every T periods), and explicitly characterizing equilibrium seems very challenging.[22]

A final remark on observing more information about the past: In the context of war or ethnic conflict, it is sometimes argued that grievances from the distant past can be a salient source of conflict and distrust (e.g., massacres or desecration of holy sites). Proposition 5 can be modified to allow for this possibility. Suppose that every instance of conflict (i.e., every time that $\tilde{y}_t = 0$ or $x_t = 0$) leads to a lasting grievance for the opposing group with some probability. If players forget earlier grievances but always remember the exact timing of their group's last $\tilde{K}$ grievances for some (potentially arbitrarily large but finite) $\tilde{K}$, no matter how long ago these originated, then the same argument leading to Proposition 5 (in the appendix) implies that there are again infinitely many times at which normal players restart cooperation even if the last K signals are all bad and they remember $\tilde{K}$ grievances.

[22] The main difficulty is that when a player observes all bad signals, she has to update her beliefs about the last time a player restarted cooperation, which can be an intractable updating problem.

Consequently, conflict cycles emerge even when players can remember unboundedly distant grievances with positive probability.

IV Conclusion

This paper has proposed a model of cycles of inter-group conflict and distrust based on the classical idea that conflict is often caused by distrust and misperceived aggression. In a dynamic context, a real or perceived aggression from one group makes it appear as innately aggressive to the other side, which in response acts more aggressively itself. When the first group cannot be sure whether this new aggression is a response to its own action or is due to the other side's actually being aggressive, a spiral of aggression and conflict forms. But—as our model shows—such a spiral cannot last forever, because it eventually becomes almost certain that a conflict spiral will have gotten started accidentally, at which point aggressive actions become completely uninformative of the other side's type. At such a time, a group experiments with cooperation, and trust is restored.

In the text, we showed that cycles of conflicts also arise when players cannot condition on calendar time, when players observe multiple past outcomes of the groups' relationship rather than just one, and when players can recall a finite number of "grievances" against the other group from arbitrarily far in the past. We consider three additional extensions of our baseline model in the online appendix. Specifically, we show that the equilibrium of our model remains essentially unchanged when bad actions can also be misperceived as good, and that cycles persist (at least in some equilibrium) when players care about the welfare of future members of their own group. Finally, we also show that enriching the space of actions yields a model with both cycles of recurrent conflict (as in the baseline model) and cycles of escalating actions within each conflict episode.

Though our basic mechanism is simple, it is both different from existing explanations for cyclic behavior in dynamic games and, we believe, potentially relevant for understanding why seemingly intractable conflicts ultimately end, and why cooperation and conciliation often follow periods of distrust. For example, in the context of the *La Violencia* episode in Colombia already discussed in the Introduction, Hartlyn (1988) emphasizes that "learning" by both parties was crucial to the ending of the conflict and the onset of peace. Hartlyn puts special stress on learning that the two parties could cooperate, writing, "The national political leadership "learned" the value of conciliation and compromise" (p. 71) and "The response by party leaders built upon historical antecedents of compromise and the "political learning" that stemmed from the combination of their earlier failed negotiations and their mutual horror and fear in the face of *La Violencia*" (p. 54).

Of course, serious empirical analysis is needed to determine whether the mechanism we highlight—agents concluding that long-lasting conflicts are no longer informative about the true intentions of the other party—can indeed account for cycles of distrust and conflict in practice. There are also several possible areas for future research on the theoretical side. For example, it would be interesting to study the more complex reputational incentives, as well as the possibility of experimentation, that would emerge if players lived for more than one period, and also to consider different ways in which players might learn about the history of conflict and cooperation between groups.

Appendix A: Proofs

Proof of Proposition 3. Rearranging the definition of T, one can check that T is the least integer greater than $\log\left(\frac{\mu^*-\mu_0}{\mu^*(1-\mu_0)}\right)/\log(1-\pi)$, i.e.,

$$T = \left\lceil \frac{\log\left(\frac{\mu^*-\mu_0}{\mu^*(1-\mu_0)}\right)}{\log(1-\pi)} \right\rceil . \tag{A1}$$

This of course implies that $(1-\pi)^T \leq \frac{\mu^*-\mu_0}{\mu^*(1-\mu_0)} \leq (1-\pi)^{T-1}$, and therefore that $\lim_{\pi\to 0}(1-\pi)^T = \frac{\mu^*-\mu_0}{\mu^*(1-\mu_0)}$. Now, by Proposition 1, expected (limit-of-means) social welfare equals expected average social welfare within each T-period block. Consider for example the first block, consisting of periods 0 to $T-1$. Continuing to let u_t be player t's payoff, and assuming that both groups are normal, this equals

$$\frac{1}{T}\left[\begin{array}{c} E\left[u_0+u_{T-1}\right] \\ +\sum_{t=1}^{T-2}\left[\left(1-(1-\pi)^t\right)2u(0,0)+(1-\pi)^t\pi\left(u(1,1)+u(1,0)\right)+(1-\pi)^{t+1}2u(1,1)\right] \end{array}\right].$$

We are interested in evaluating this expression as $\pi\to 0$, which also implies (from (A1)) $T\to\infty$. Thus, the expression of interest is

$$\begin{aligned}
& \lim_{\pi\to 0}\frac{1}{T}\left[\begin{array}{c} E\left[u_0+u_{T-1}\right]+\left(T-1-\frac{1-(1-\pi)^{T-1}}{\pi}\right)2u(0,0) \\ +\left(1-(1-\pi)^{T-1}-\pi\right)\left(u(1,1)+u(1,0)\right)+\left(\frac{1-(1-\pi)^T}{\pi}-2+\pi\right)2u(1,1) \end{array}\right] \\
= \; & \lim_{\pi\to 0}\frac{1}{T}\left[\begin{array}{c} E\left[u_0+u_{T-1}\right]+\left(T-1+(1-\pi)^{T-1}\right)2u(0,0) \\ +\left(1-(1-\pi)^{T-1}-\pi\right)\left(u(1,1)+u(1,0)\right)+2\left(\frac{1-(1-\pi)^T}{\pi}-2+\pi\right)\left(u(1,1)-u(0,0)\right) \end{array}\right] \\
= \; & 2u(0,0)+2\lim_{\pi\to 0}\frac{1}{T}\left(\frac{1-(1-\pi)^T}{\pi}\right)\left(u(1,1)-u(0,0)\right),
\end{aligned}$$

where the second equality follows by state rearrangement, and the third one follows simply from canceling the terms that go to zero and noting that $(T-1)/T\to 1$ as $T\to\infty$. The first part of the proposition then follows by observing that

$$\begin{aligned}\lim_{\pi\to 0}\frac{1}{T}\left(\frac{1-(1-\pi)^T}{\pi}\right) &= \lim_{\pi\to 0}\left(\frac{1-\frac{\mu^*-\mu_0}{\mu^*(1-\mu_0)}}{T\pi}\right)=\lim_{\pi\to 0}\left(\frac{1-\frac{\mu^*-\mu_0}{\mu^*(1-\mu_0)}}{-T\log(1-\pi)}\right)\\ &= \frac{1-\frac{\mu^*-\mu_0}{\mu^*(1-\mu_0)}}{-\log\left(\frac{\mu^*-\mu_0}{\mu^*(1-\mu_0)}\right)}<1,\end{aligned}$$

where the inequality holds for all $\mu_0>0$. Finally, the second part of the proposition follows by observing that

$$\begin{aligned}\lim_{n\to\infty}\frac{1}{T}\left(\frac{1-(1-\pi_n)^T}{\pi_n}\right) &= \lim_{n\to\infty}\left(\frac{1-\frac{\mu^*-\mu_{0,n}}{\mu^*(1-\mu_{0,n})}}{T\pi_n}\right)=\lim_{n\to\infty}\left(\frac{1-\frac{\mu^*-\mu_{0,n}}{\mu^*(1-\mu_{0,n})}}{-T\log(1-\pi_n)}\right)\\ &= \lim_{n\to\infty}\left(\frac{1-\frac{\mu^*-\mu_{0,n}}{\mu^*(1-\mu_{0,n})}}{-\log\left(\frac{\mu^*-\mu_{0,n}}{\mu^*(1-\mu_{0,n})}\right)}\right)=1,\end{aligned}$$

where the final equality uses $\mu_{0,n}\to 0$. ■

Proof of Proposition 5. Suppose not. Then there exists a time $\bar{T}$ such that at all times $t\geq\bar{T}$ normal player t plays $y_t=0$ after observing all bad signals. Suppose that both groups are normal, and observe that the probability that player $\bar{T}+K$ observes all bad signals is at least π^K. In this event, all subsequent players play $y_t=0$ and thus observe all bad signals. In the alternative event that player $\bar{T}+K$ observes at least one good signal, the probability that player $\bar{T}+2K$ observes all bad signals is still at least π^K. Hence, the overall probability that player $\bar{T}+2K$ observes all bad signals is at least $1-\left(1-\pi^K\right)^2$. Now it is easy to see by induction on m that player $\bar{T}+mK$ observes all bad signals with probability at least $1-\left(1-\pi^K\right)^m$. Hence, normal player $\bar{T}+mK$'s belief that the other group is bad when she observes all bad signals is at most

$$\tilde{\mu}_{\bar{T}+mK}=\frac{\mu_0}{\mu_0+(1-\mu_0)\left(1-\left(1-\pi^K\right)^m\right)}.$$

This belief converges to μ_0 as $m\to\infty$, so it follows from Assumption 2 that $\tilde{\mu}_{\bar{T}+MK}<\mu^*$ for some integer M. Therefore, normal player $\bar{T}+MK$ would deviate to playing $y_{\bar{T}+MK}=1$ after observing all bad signals, which yields a contradiction and establishes the desired result. ■

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Online Appendix B of "Cycles of Conflict: An Economic Model" by Daron Acemoglu and Alexander Wolitzky

Additional Extensions

Two-Sided Errors

The analysis of the baseline model was simplified by the assumption that only the good action can generate the good signal. This section shows that our main conclusions still apply when either action can generate either signal.

In particular, assume now that the signal $\tilde{y}_{t-1}$ is distributed as follows:

$$\begin{aligned} \Pr\left(\tilde{y}_{t-1} = 1 | y_{t-1} = 1\right) &= 1 - \pi \\ \Pr\left(\tilde{y}_{t-1} = 1 | y_{t-1} = 0\right) &= \pi', \end{aligned}$$

where $\pi, \pi' \in (0,1)$ and $\pi + \pi' < 1$. The assumption that $\pi + \pi' < 1$ means that the good action is more likely to generate the good signal than is the bad action, and is thus essentially a normalization.

As in the baseline model, Assumption 1 guarantees that normal player t plays $x_t = 1$ if and only if $\tilde{y}_{t-1} = 1$. It is straightforward to see that the appropriate analog of Assumption 2, which guarantees that normal player t plays $y_t = 1$ if and only if her assessment of the probability that the other group is bad after observing $\tilde{y}_{t-1}$ is below a threshold $\mu^*_{2-SIDED}$, is the following.

Assumption 2'

$$\mu_0 < \mu^*_{2-SIDED} \equiv 1 - \frac{u(0,0) - u(1,0)}{(1-\pi)\left(u(1,1) - u(1,0)\right) + \pi'\left(u(0,0) - u(0,1)\right)}.$$

The analog of Assumption 3 is then:

Assumption 3' $\mu_t \neq \mu^*_{2-SIDED}$ for all $t \in \mathbb{N}$.

Denote normal player t's assessment of the probability that the other group is bad after observing $\tilde{y}_{t-1} = 0$ by μ_t (as usual), and denote her assessment of this probability after observing $\tilde{y}_{t-1} = 1$ (which equals 0 in the baseline model, due to one-sided errors) by μ'_t. To compute these probabilities, let

$$M = \begin{pmatrix} 1-\pi & \pi' \\ \pi & 1-\pi' \end{pmatrix}$$

be the Markov transition matrix governing the evolution of $\tilde{y}_t$ in the event that both groups are normal, under the hypothesis that normal players play $y_t = 1$ if and only if $\tilde{y}_{t-1} = 1$. That is, if both groups are normal and $\tilde{y}_t = 1$, then $\tilde{y}_{t+1} = 1$ with probability $1 - \pi$; if, on the other hand, $\tilde{y}_t = 0$, then $\tilde{y}_{t+1} = 1$ with probability π'. Then, by Bayes rule,

$$\mu_t = \frac{\mu_0 (1 - \pi')}{\mu_0 (1 - \pi') + (1 - \mu_0) \left(1 - M^t_{(1,1)}\right)},$$

where $M^t_{(1,1)}$ is the $(1,1)$ coordinate of the t^{th} power of M. This is simply because the probability of observing $\tilde{y}_{t-1} = 0$ conditional on the other group being bad equals $1 - \pi'$, while the probability of observing $\tilde{y}_{t-1} = 0$ conditional on the other group being good equals $1 - M^t_{(1,1)}$. Similarly,

$$\mu'_t = \frac{\mu_0 \pi'}{\mu_0 \pi' + (1 - \mu_0) M^t_{(1,1)}}.$$

In the baseline model, it was the case that $\mu_t \to \mu_0$ as $t \to \infty$, so Assumption 2 guaranteed the existence of a time T such that $\mu_T < \mu^*_{2-SIDED}$. With two-sided errors, $M^t_{(1,1)} \to \frac{\pi'}{\pi + \pi'}$ as $t \to \infty$, so $\mu_t \to \mu_\infty$ as $t \to \infty$, where

$$\mu_\infty = \frac{\mu_0 (1 - \pi')}{\mu_0 (1 - \pi') + (1 - \mu_0) \frac{\pi}{\pi + \pi'}}.$$

If $\mu_\infty < \mu^*_{2-SIDED}$, then Assumption 2 guarantees the existence of a smallest time $T_{2-SIDED}$ such that $\mu_{T_{2-SIDED}} < \mu^*_{2-SIDED}$, and there is a deterministic cycle with period $T_{2-SIDED}$, as in the baseline model. If on the other hand $\mu_\infty \geq \mu^*_{2-SIDED}$, then there is no deterministic cycle, and in particular a bad signal always leads to a spiral of bad actions that lasts until the next accidental good signal.

Summarizing, we have the following result.

Proposition 6 *Under Assumptions 1, 2', and 3', the model with two-sided errors has a unique sequential equilibrium. If $\mu_\infty < \mu^*_{2-SIDED}$, then the equilibrium has the following properties:*

1. *At every time $t \neq 0 \bmod T_{2-SIDED}$, normal player t plays good actions $(x_t = 1, y_t = 1)$ if she gets the good signal $\tilde{y}_{t-1} = 1$, and plays bad actions $(x_t = 0, y_t = 0)$ if she gets the bad signal $\tilde{y}_{t-1} = 0$.*

2. *At every time $t = 0 \bmod T_{2-SIDED}$, normal player t plays the good action $x_t = 1$ toward player $t - 1$ if and only if she gets the good signal $\tilde{y}_{t-1} = 1$, but plays the good action $y_t = 1$ toward player $t + 1$ regardless of her signal.*

3. Bad players always play bad actions ($x_t = 0, y_t = 0$).

If instead $\mu_\infty \geq \mu^*_{2-SIDED}$, *then the equilibrium has the following properties:*

1. *At every time* $t > 0$, *normal player* t *plays good actions* ($x_t = 1, y_t = 1$) *if she gets the good signal* $\tilde{y}_{t-1} = 1$, *and plays bad actions* ($x_t = 0, y_t = 0$) *if she gets the bad signal* $\tilde{y}_{t-1} = 0$.

2. *Normal player* 0 *plays the good action* $y_0 = 1$ *toward player* 1.

3. *Bad players always play bad actions* ($x_t = 0, y_t = 0$).

Proof. Since player $t+1$ plays $x_{t+1} = 1$ if and only if he is normal and $\tilde{y}_t = 1$, it follows that (normal) player t plays $y_t = 1$ if and only if his belief that the other group is bad is below the cutoff $\mu^*_{2-SIDED}$. Now one can compute that $M^t_{(1,1)} = \frac{\pi' + \pi(1-\pi-\pi')^t}{\pi+\pi'}$. In particular, $M^t_{(1,1)} > \pi'$ for all t, and hence $\mu'_t < \mu_0$ for all t. Therefore, Assumption 2' implies that player t always plays $y_t = 1$ after seeing signal $\tilde{y}_{t-1} = 1$. Finally, $\mu_t > \mu^*_{2-SIDED}$ for all $t < T_{2-SIDED}$ (with the convention that $T_{2-SIDED} = \infty$ if $\mu_\infty \geq \mu^*_{2-SIDED}$), by definition of $T_{2-SIDED}$, so player t plays $y_t = 0$ after seeing $\tilde{y}_{t-1} = 0$, for all $t < T_{2-SIDED}$. The remainder of the argument is as in the baseline model. ■

Forward-Looking Behavior

Another stark assumption in the baseline model is that players do not care at all about future periods. We now relax this by retaining the assumption that agents are short-lived but assuming that they care about their group's future utility.[23] Even though, not surprisingly, forward-looking behavior can introduce multiple equilibria, we can obtain a clean characterization of the subset of equilibria that have the same cyclic structure as the unique equilibrium in the baseline model. In particular, we show that in every such equilibrium cooperation restarts at least as frequently as in the baseline model. Hence, the average duration of conflict is reduced.

Formally, modify the baseline model by supposing that normal player t's payoff is now

$$\sum_{\tau=0}^{\infty} \delta^{2\tau} u_{t+2\tau}$$

for some $\delta \in (0,1)$, where u_τ is player τ's payoff in the baseline model. Everything else is exactly as in the baseline model. Our result is the following.

[23] An alternative interpretation is that each group consists of a single long-lived agent that can only remember the most recent signal.

Proposition 7 *Let μ_t be defined as in the baseline model. For odd integers k let*

$$\mu^*_{T'-k} = \frac{\frac{1-\delta^k(1-\pi)^k}{1-\delta^2(1-\pi)^2}\left[(2-\pi)\,u(1,1)+\pi u(1,0)-2u(0,0)\right]-\left(u(1,1)-u(0,0)\right)}{\frac{1-\delta^k(1-\pi)^k}{1-\delta^2(1-\pi)^2}\left[(2-\pi)\,u(1,1)+\pi u(1,0)-2u(0,0)\right]+2u(0,0)-u(1,1)-u(1,0)},$$

and for even integers k let

$$\mu^*_{T'-k} = \frac{\frac{1-\delta^{k-1}(1-\pi)^{k-1}}{1-\delta^2(1-\pi)^2}\left[(2-\pi)\,u(1,1)+\pi u(1,0)-2u(0,0)\right]-\left(1-\delta^k\right)\left(u(1,1)-u(0,0)\right)}{\frac{1-\delta^{k-1}(1-\pi)^{k-1}}{1-\delta^2(1-\pi)^2}\left[(2-\pi)\,u(1,1)+\pi u(1,0)-2u(0,0)\right]+\left(2-\delta^k\right)u(0,0)-\left(1-\delta^k\right)u(1,1)-u(1,0)}.$$

Then the model with forward-looking behavior has a sequential equilibrium that coincides with the sequential equilibrium of the baseline model, but with restart time T' rather than T, if and only if

1. *$\mu^*_{T'-k} \leq \mu_{T'-k}$ for all $k \in \{1, \ldots, T'-1\}$, and*
2. *$\mu^*_0 \geq \mu_{T'}$.*

In particular, in every such equilibrium the restart time T' is no greater than T.

The intuition is the following: Consider a candidate equilibrium with restart time T'. Since player T' restarts cooperation whatever player $T'-1$ does, the incentives of player $T'-1$ are exactly in the baseline model, so she will play just as in the baseline model. But player $T'-2$ now has an additional reason to take a good action toward player $T'-1$ after a bad signal: provided that the other group is normal, this will help player T' obtain payoff $u(1,1)$ rather than $u(0,0)$ against player $T'-1$. Similarly, player $T'-4$ has yet stronger incentives to restart cooperation because of the additional payoffs that this might generate for $T'-2$ against $T'-3$ and $T'-1$. One can now compute the cutoff belief for player $T'-k$ to restart cooperation as $\mu^*_{T'-k}$, which then implies that no player will restart cooperation prior to time T' if and only if $\mu^*_{T'-k} \leq \mu_{T'-k}$ for all $k \in \{1, \ldots, T'-1\}$ (the first condition in Proposition 7).[24] If, on the other hand, player T' does not restart cooperation, then in this candidate equilibrium cooperation will not restart until time $2T'$. Hence, cooperation restarts at time T' if and only if $\mu_{T'} \leq \mu^*_{T'-T'}$, i.e., if and only if $\mu^*_0 \geq \mu_{T'}$ (the second condition in Proposition 7). Finally, the restart time T' cannot exceed T, as player T's incentive to restart cooperation in the model with forward-looking behavior is never less than her incentive to restart cooperation in the

[24] The need to distinguish between odd and even k comes because the only "extra incentive" from helping player T' comes only from her interaction with player $T'-1$ (as she always takes the good action toward player $T'+1$), while for earlier players the extra incentive comes from their interactions with both their predecessors and their successors.

baseline model, and her posterior would be the same as in the baseline model if (counterfactually) the restart time did exceed T.[25]

Proof. For the first part of the proposition, it is clear that the only potentially profitable deviations are deviations by player $T'-k$ to $y=1$ after the bad signal for $k \in \{1, \ldots, T'-1\}$ and deviations by player T' to $y=0$ after the bad signal.

Consider first deviations by player $T'-k$. Since player T' always restarts cooperation, player $T'-k$'s action is inconsequential for the expected payoff of players $t \geq T'+1$, so player $T'-k$ needs only take into account the effect of her action of the payoff on players $T'-k+2, T'-k+4, \ldots, T'-1$ (for k odd) or $T'-k+2, T'-k+4, \ldots, T'$ (for k even). Now if the opposing group is bad, then player $T'-k+2\tau$ gets payoff $u(0,0)$ against each of her opponents, regardless of player $T'-k$'s action, for $\tau \in \{1, \ldots, \lceil (k-1)/2 \rceil\}$. If instead the opposing group is normal, then by taking action $y=1$ rather than $y=0$, player $T'-k$ increases player $T'-k+2\tau$'s probability of getting $u(1,1)$ rather than $u(0,0)$ against his predecessor and getting $(1-\pi)u(1,1)+\pi u(1,0)$ rather than $u(0,0)$ against his successor from 0 to $(1-\pi)^{2\tau}$. This increases player $T'-k+2\tau$'s expected payoff by a total of

$$(1-\pi)^{2\tau}\left[(2-\pi)u(1,1)+\pi u(1,0)-2u(0,0)\right].$$

Thus, for k odd, playing $y=1$ is optimal for player $T'-k$ with belief μ if and only if

$$\begin{aligned} &-\mu\left(u(0,0)-u(1,0)\right)-(1-\mu)\left(u(1,1)-u(0,0)\right) \\ &\qquad +(1-\mu)\left[(2-\pi)u(1,1)+\pi u(1,0)-2u(0,0)\right]\left[\begin{array}{c} 1+\delta^2(1-\pi)^2+\delta^4(1-\pi)^4 \\ +\ldots+\delta^{k-1}(1-\pi)^{k-1} \end{array}\right] \geq 0, \end{aligned}$$

or

$$\mu \leq \mu^*_{T'-k}.$$

The expression for k even is similar, except that since player T' always plays $y_{T'}=1$ his benefit from player $T'-k$'s taking action $y=1$ rather than $y=0$ when the opposing group is normal is only

$$(1-\pi)^k\left(u(1,1)-u(0,0)\right).$$

So, for k even, playing $y=1$ is optimal for player $T'-k$ with belief μ if and only if

$$\begin{aligned} &-\mu\left(u(0,0)-u(1,0)\right)-(1-\mu)\left(1-\delta^k\right)\left(u(1,1)-u(0,0)\right) \\ &\qquad +(1-\mu)\left[(2-\pi)u(1,1)+\pi u(1,0)-2u(0,0)\right]\left[\begin{array}{c} 1+\delta^2(1-\pi)^2+\delta^4(1-\pi)^4 \\ +\ldots+\delta^{k-2}(1-\pi)^{k-2} \end{array}\right] \geq 0, \end{aligned}$$

[25] Note that Proposition 7 allows for multiple equilibria, because the expectation that player T' will restart cooperation reduces earlier players' incentive to restart cooperation (as they can count on player T' to restart) and increases player T''s own incentive to restart cooperation (as she knows that if she does not restart then no one will restart until time $2T'$).

or

$$\mu \leq \mu^*_{T'-k}.$$

Next, consider deviations by player T'. The argument here is nearly identical, noting that if player T' does not restart cooperation then the next restart occurs in T' periods.

Finally, to show that the cycle length T' in any such equilibrium is at most T, consider the strategy profile with cycle length $T' > T$. Then player T will deviate by restarting cooperation after the bad signal, as his posterior is μ_T and his benefit from playing $y = 1$ rather than $y = 0$ is at least as large as in the baseline model. ■

Recurrent Conflict Versus Escalation

As noted in footnote 16, the cycles of conflict captured in our baseline model resemble recurrent episodes of conflicts alternating with episodes of peace, rather than escalation of the intensity of conflict within a given conflict episode. In this subsection, we show that our mechanism can also generate this type of "escalation spiral".[26]

Consider the unobserved calendar time model of Section II, modified to have three possible actions, 0, $\frac{1}{2}$, and 1, and three possible signals, also called 0, $\frac{1}{2}$, and 1 (there are still two possible types, and action 0 is still dominant for bad types). Here, 0 and 1 are the bad and good actions/signals as usual, while $\frac{1}{2}$ is a new, intermediate action/signal, corresponding to "limited conflict," so that the switch from $\frac{1}{2}$ to 0 is an escalation of conflict. We assume that the game remains a coordination game (in particular, $\left(\frac{1}{2}, \frac{1}{2}\right)$ is a Nash equilibrium), that "more cooperative" equilibria are Pareto-preferred, that a group's payoff has increasing differences in its own action and the other group's action, and that a group is always better off when the other group is more cooperative. Formally, the following conditions are sufficient to ensure this:

1. $1 \in \arg\max_{x \in \left\{0, \frac{1}{2}, 1\right\}} u\left(x, 1\right), \frac{1}{2} \in \arg\max_{x \in \left\{0, \frac{1}{2}, 1\right\}} u\left(x, \frac{1}{2}\right), 0 \in \arg\max_{x \in \left\{0, \frac{1}{2}, 1\right\}} u\left(x, 0\right).$

2. $u\left(x, y\right)$ has increasing differences in $\left(x, y\right)$: if $x \geq x'$ and $y \geq y'$, then $u\left(x, y\right) - u\left(x', y\right) \geq u\left(x, y'\right) - u\left(x', y'\right)$.

3. $u\left(x, y\right)$ is non-decreasing in y.

4. $u\left(1, 1\right) > u\left(\frac{1}{2}, \frac{1}{2}\right) > u\left(0, 0\right).$

[26] See, for example, Jervis (1976). We thank a referee for drawing our attention to this issue.

We also assume that the conditional distribution of signals is given by

$$\begin{array}{ccc} \Pr(\tilde{y}_t = 1|y_t = 1) = 1-\pi & \Pr\left(\tilde{y}_t = \frac{1}{2}|y_t = 1\right) = \pi & \Pr(\tilde{y}_t = 0|y_t = 1) = 0 \\ \Pr\left(\tilde{y}_t = 1|y_t = \frac{1}{2}\right) = \rho & \Pr\left(\tilde{y}_t = \frac{1}{2}|y_t = \frac{1}{2}\right) = 1-\rho-\rho' & \Pr\left(\tilde{y}_t = 0|y_t = \frac{1}{2}\right) = \rho' \\ \Pr(\tilde{y}_t = 1|y_t = 0) = 0 & \Pr\left(\tilde{y}_t = \frac{1}{2}|y_t = 0\right) = \pi' & \Pr(\tilde{y}_t = 0|y_t = 0) = 1-\pi' \end{array}$$

Thus, a good action can generate a good signal or an intermediate signal; a bad action can generate a bad signal or an intermediate signal; and an intermediate action can generate any signal. Finally, assume that $\pi + \rho < 1$ and $\pi' + \rho' < 1$, so that a good action is more likely to generate a good signal than is an intermediate action, and a bad action is more likely to generate a bad signal than is an intermediate action.

Below, we derive conditions under which a sequential equilibrium of the following form exists.[27]

1. Normal player 0 plays $y_0 = 1$.

2. At every time $t > 0$, normal player t plays good actions $(x_t = 1, y_t = 1)$ if she gets the good signal $\tilde{y}_{t-1} = 1$, and plays intermediate actions $\left(x_t = \frac{1}{2}, y_t = \frac{1}{2}\right)$ if she gets the intermediate signal $\tilde{y}_{t-1} = \frac{1}{2}$. If she gets the bad signal $\tilde{y}_{t-1} = 0$, she plays the bad action $x_t = 0$ toward player $t-1$, and mixes between playing the bad action $y_t = 0$ and the intermediate action $y_t = \frac{1}{2}$ toward player $t+1$.

3. Bad players always play bad actions $(x_t = 0, y_t = 0)$.

Note that such an equilibrium displays recurrent conflict exactly as in our baseline model or our model with unobserved calendar time but also displays escalation within each conflict spiral: each conflict spiral starts with the misperception of a good action as an intermediate action (which leads to genuine intermediate actions), and then may involve the misperception of an intermediate action as a bad action (which leads to genuine bad actions). Thus, our framework can fairly naturally accommodate escalation of conflict as well as periodic conflict.

To understand when an equilibrium of the conjectured form exists, first observe that in any equilibrium players take more aggressive actions when they believe the opposing group is more likely to be bad.

Lemma 1 *In any sequential equilibrium, normal player t's optimal action toward her successor y_t is non-increasing in her belief.*

[27] Unlike in the baseline model, we do not claim that equilibrium is unique here.

Proof. In any sequential equilibrium, normal player $t+1$ plays $x_{t+1}=\tilde{y}_t$ and bad player $t+1$ plays $x_{t+1}=0$. Hence, letting $U(y_t)$ be normal player t's expected payoff from taking action y_t against player $t+1$ given belief μ, we have

$$\begin{aligned}
U(0) &= (1-\pi')\,u(0,0)+\pi'\left((1-\mu)\,u\left(0,\frac{1}{2}\right)+\mu u(0,0)\right)\\
U\left(\frac{1}{2}\right) &= (1-\rho-\rho')\left[(1-\mu)\,u\left(\frac{1}{2},\frac{1}{2}\right)+\mu u\left(\frac{1}{2},0\right)\right]+\rho\left[(1-\mu)\,u\left(\frac{1}{2},1\right)+\mu u\left(\frac{1}{2},0\right)\right]+\rho' u\left(\frac{1}{2},0\right)\\
U(1) &= (1-\pi)\left[(1-\mu)\,u(1,1)+\mu u(1,0)\right]+\pi\left[(1-\mu)\,u\left(1,\frac{1}{2}\right)+\mu u(1,0)\right].
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{\partial U(0)}{\partial \mu} &= -\pi'\left[u\left(0,\frac{1}{2}\right)-u(0,0)\right]\\
\frac{\partial U\left(\frac{1}{2}\right)}{\partial \mu} &= -(1-\rho-\rho')\left[u\left(\frac{1}{2},\frac{1}{2}\right)-u\left(\frac{1}{2},0\right)\right]-\rho\left[u\left(\frac{1}{2},1\right)-u\left(\frac{1}{2},0\right)\right]\\
\frac{\partial U(1)}{\partial \mu} &= -(1-\pi)\left[u(1,1)-u(1,0)\right]-\pi\left[u\left(1,\frac{1}{2}\right)-u(1,0)\right].
\end{aligned}$$

Proof. Increasing differences implies that $u\left(\frac{1}{2},\frac{1}{2}\right)-u\left(\frac{1}{2},0\right)\geq u\left(0,\frac{1}{2}\right)-u(0,0)$, and $u(x,y)$ non-decreasing in y then implies that $u\left(\frac{1}{2},1\right)-u\left(\frac{1}{2},0\right)\geq u\left(0,\frac{1}{2}\right)-u(0,0)$. The assumption that $\pi'+\rho'<1$ now implies that $\frac{\partial U(0)}{\partial\mu}\geq\frac{\partial U\left(\frac{1}{2}\right)}{\partial\mu}$. Increasing differences also implies that $u(1,1)-u(1,0)\geq u\left(\frac{1}{2},1\right)-u\left(\frac{1}{2},0\right)$ and $u\left(1,\frac{1}{2}\right)-u(1,0)\geq u\left(\frac{1}{2},\frac{1}{2}\right)-u\left(\frac{1}{2},0\right)$, and therefore $\frac{\partial U\left(\frac{1}{2}\right)}{\partial\mu}\geq\frac{\partial U(1)}{\partial\mu}$. It follows that normal player t's optimal action y_t is non-increasing in her belief. ■ ■

Now let $q_{\tilde{y}}$ be the long-run fraction of periods t in which $\tilde{y}_t=\tilde{y}$ when both groups are normal. In an equilibrium of the form conjectured above, letting p be the probability that normal player t plays the bad action $y_t=0$ after the bad signal $\tilde{y}_{t-1}=0$, we have

$$\begin{aligned}
q_0 &= q_{\frac{1}{2}}\rho'+q_0\left[(1-p)\,\rho'+p\,(1-\pi')\right]\\
q_{\frac{1}{2}} &= q_1\pi+q_{\frac{1}{2}}(1-\rho-\rho')+q_0\left[(1-p)\,(1-\rho-\rho')+p\pi'\right]\\
q_1 &= q_1(1-\pi)+q_{\frac{1}{2}}\rho+q_0(1-p)\,\rho
\end{aligned}$$

This system of equations may easily be solved for q_0, $q_{\frac{1}{2}}$, and q_1; we omit the details.

Finally, let $\mu^*_{\frac{1}{2}}$ be the cutoff belief that makes normal player t indifferent between actions $y_t=0$ and $y_t=\frac{1}{2}$, and let μ^*_1 be the cutoff belief that makes her indifferent between actions $y_t=\frac{1}{2}$ and $y_t=1$, which may be easily computed from the above formulas for $U(0)$, $U\left(\frac{1}{2}\right)$, and $U(1)$. Letting

$\mu^{\tilde{y}_{t-1}}$ be normal player t's posterior belief after observing signal $\tilde{y}_{t-1}$, we have

$$\begin{aligned}
\mu^0 &= \frac{\mu_0 (1-\pi')}{\mu_0 (1-\pi') + (1-\mu_0) q_0} \\
\mu^{\frac{1}{2}} &= \frac{\mu_0 \pi'}{\mu_0 \pi' + (1-\mu_0) q_{\frac{1}{2}}} \\
\mu^1 &= 0.
\end{aligned}$$

So an equilibrium of the desired form exists only if $\mu^0 = \mu^*_{\frac{1}{2}}$, or equivalently if

$$q_0 = \left(1-\pi'\right) \frac{\mu_0}{1-\mu_0} \frac{1-\mu^*_{\frac{1}{2}}}{\mu^*_{\frac{1}{2}}}.$$

This equation implicitly fixes the mixing probability p at some $p^* \in [0,1]$. Finally, by Lemma 1, an equilibrium of the desired form exists if and only if Assumption 2 holds and it is optimal for normal player t to play action $y_t = \frac{1}{2}$ after observing signal $\tilde{y}_{t-1} = \frac{1}{2}$ when $p = p^*$; that is, if and only if Assumption 2 holds and $\mu^{\frac{1}{2}} \in \left[\mu^*_1, \mu^*_{\frac{1}{2}}\right]$ when $p = p^*$.[28]

While this characterization is not very explicit, it does show that an equilibrium of the conjectured form should exist for a wide range of parameters.

[28] Assumption 2 implies that it is optimal for normal player 0 to play $y_0 = 1$. It also implies that it is optimal for normal player t to play action $y_t = 1$ after observing signal $\tilde{y}_{t-1} = 1$, as $\mu^1 = 0 < \mu_0$.