Pass-Through as an Economic Tool*

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Abstract

Pass-through rates (at which a monopolist passes on increases in her cost to consumers) play the same role in the comparative statics of monopoly that elasticities do in competitive markets. This makes simple assumptions about them (e.g. that they stay on the same side of 1 over a range of prices) useful for the identification and testing of a wide range of industrial organization models, if data on exogenous cost variations are available. I review the notion of pass-through, establishing a number of new results that show why it is a crucial parameter of monopoly optimization. I then use it to provide a complete (novel) characterization of the relationship between firm and industry mark-ups and profits within and across industrial organization in the classic Cournot (1838)-Spengler (1950) double marginalization problem. I discuss a variety of other applications, some novel to this paper (Cournot competition and the effects of increases in competition on prices) and some from my work elsewhere (two-sided markets) and the work of others (conjectural variations models, mergers in differentiated Bertrand markets and international macroeconomics). I briefly discuss the implications of these results for empirical work, emphasizing the weakness of common functional forms. Finally, I highlight a parametric class of demand functions first proposed by Bulow and Pfleider (1983), based on the stronger assumption of constant pass-through, that has a number of useful properties.

[F]or many questions of policy analysis, it is not necessary to identify fully specified economic models that are invariant to classes of policy modifications. All that may be required for any policy analysis are combinations of subsets of the structural parameters, corresponding to the parameters required to forecast particular policy modifications, which are often much easier to identify (i.e. require fewer and weaker assumptions).

–James J. Heckman and Edward J. Vytlacil[1] describing Marschak's Maxim[2]

I. Introduction

Elasticities of supply and demand play fundamental roles in the economic analysis of competitive markets. Will raising a tax increase or decrease revenue? It depends on the elasticities of supply and demand. However, a monopolist optimizes her prices by taking into account the elasticities of both the demand curve she faces and her own marginal costs. Elasticities therefore determine the level, rather than the comparative statics, of monopoly pricing. What takes its place is the pass-through rate at which a monopolist with linear cost finds it optimal to pass-through increases in that cost to consumers. Like elasticities in competitive problems, a wide variety of comparative statics in problems with market power depend on this unit-less measure of the responsiveness of the firm to shocks to the economic environment. This article develops and elaborates these claims.

Understanding the fundamental role of pass-through is like any tool in a theorist's work kit: it is useful for analyzing the working of economic systems and, on occasion, allowing the deduction from reasonable premises of non-obvious and even policy-relevant conclusions. But perhaps a more important application of the framework developed here, and the one I will emphasize, is empirical and related to Marschak's Maxim, articulated in the quote above by Heckman and Vytlacil. Marschak argued that the approach (now common in empirical

[1]Heckman and Vytlacil (2007).

[2]Marschak (1953).

industrial organization) of making assumptions and gathering data necessary to fully identify a demand system is often unnecessary to determine a particular counterfactual. Often only one or a few parameters are needed for a particular prediction, as my example about taxation suggests. By determining analytically which parameters need be identified, we can dramatically increase both the plausibility and interpretability of identifying assumptions. In the industrial organization problems I consider, if exogenous cost variations are available, the only ancillary assumptions[3] needed to identify a number of policy-relevant effects is that pass-through stays on the same side of some pre-specified boundary such as 1 over a range of prices or that it is monotonic in cost over that range. Just as elasticities serve as the appropriate Marschakian device in the taxation, I will show here that in a wide variety of industrial organizations problems, pass-through serves a similar role.

To show how and why this is the case, I begin in Section II by developing the idea of pass-through in the simplest monopoly pricing problem. I discuss how the commonly used second-order condition of log-concavity of demand implies that pass-through is less than one-for-one (cost-absorbing) and offer an equally tractable, much weaker second-order condition which allows for arbitrary (positive) pass-through rates. I then show that pass-through is exactly the inverse second-order elasticity of profits with respect to mark-up, implying that it provides a unit-less measure of the sharpness of definition of the monopoly problem and therefore applies equally well to quantity and pricing decisions. Finally, I discuss a joint result with Michal Fabinger (Fabinger and Weyl, 2008) that pass-through determines the split between consumer and producer surplus under monopoly.

To demonstrate the logic of pass-through, I turn in Section III to a particular simple application, the classic Cournot (1838)-Spengler (1950) double marginalization problem. I prove that whether demand is cost-absorbing or cost-amplifying determines whether the mark-ups of the two firms are strategic substitutes or complements, as each firm's mark-up acts essentially as a tax on the other firm. Therefore knowledge of whether demand is

[3]On top of the basic model set-up and classical assumptions about optimization and common knowledge.

globally cost-absorbing or cost-amplifying, assuming that it is either one or the other, along with knowledge of whether pass-through increases or decreases in cost implies a complete ranking of firm mark-ups and profits within and across as well as final prices across industrial organizations (Nash, Stackelberg and integrated). Both because of the direct importance of the double marginalization problem and its relationship to others, these results have a variety of applications. These include testing the model, the empirical recovery of pass-through rates from market structure, antitrust regulation of vertical relationships, vertical relationships among tax authorities, endogenous determination of market structure, monopoly pricing with inter-temporal complementarity in consumption (habit-forming goods, goods with switching costs, tying, etc.), strategic regulatory interactions and common agency problems.

In Section IV I turn from depth to breadth, overviewing a variety of applications of pass-through to models in industrial organization, some novel, some from my previous work and some from the work of others. I show that the same identification is possible in the quantity (Cournot) competition problem using pass-through as in the double marginalization problem. I also use recent work by Gabaix et al. (2005) to show that the asymptotic effect of competition on prices in the classic Perloff and Salop (1985) model of differentiated products Bertrand competition is determined by whether the distribution of consumer utility is cost-abosrbing (log-concave) or cost-amplifying (log-convex). I then discuss some applications from my previous work. The crucial policy implications of the Rochet and Tirole (2003)(RT2003) model of two-sided markets turn on pass-through and in fact the model can be non-parametrically over-identified using exogenous cost variations (Weyl, 2008d). Pass-through allows the extension of the double marginalization problem to the RT2003 model of two-sided markets (Weyl, 2008b); the greater complexity of this model in fact leads to more rather than less over-identification. Similar over identification is possible (Weyl, 2008c) in the other leading model of two-sided markets, that of Armstrong (2006). I also review applications from others' work. Seade (1986) shows that pass-through is useful in the broad conjectural variations model, offering the hope that some of the results here on the Cournot problems can be extended beyond the two-firm, symmetric linear cost models considered here. I review a recent literature (Froeb et al., 2005; Farrell and Shapiro, 2008) showing that pass-through is the crucial determinant of the magnitude of unilateral effects in differentiated Bertrand industries, as well as previewing Weyl (2008a) which shows that the same applies to differentiated Cournot industries.

Of course precisely because it does not propose to identify a full structural model, my Marschakian approach requires exogenous variation to identify its (small number) of parameters of interest. Section V discusses these issues related to empirical applications. Pass-through can be measured by a regression using quantitatively observable exogenous (uncorrelated with demand shocks) variations in costs or by structural estimation of the relevant properties of demand. There are certainly cases when neither of these approaches are feasible. Nonetheless, the results here suggest that in some applications it is crucial that (computational) structural empirical analysis, in addition to recovering elasticities and cross-elasticities, also measure pass-through rates from data rather than imposing them through restrictive functional form assumptions. In fact I show that (the homogeneous consumer, single-product form of) nearly all functional forms for demand commonly used in empirical industrial organization (at least those globally obeying monopolist second-order conditions) have pass-through less than one and monotonically increasing.

While the "weak" identifying assumptions I employ allow for only qualitative (sign) identification of counter-factuals, Section VII shows how quantitatively precise identification can be achieved by a strengthening of my identifying assumptions from pass-through being above or below 1 over a range of prices to pass-through being constant. This gives rise to a parametric class of demand functions first discussed by Bulow and Pfleider (1983) which have a number of useful properties. It is extremely tractable in a wide range of problems. It has the same number of degrees of freedom as typical statistical distributions used to generate demand functions. It is to my knowledge the only monopoly demand function allowing for a full range of pass-through rates independent of elasticities and demand levels given prices.

It provides a natural notion of extrapolation from marginal properties of demand, such as elasticities and prices, to the useful infra-marginal property of average consumer surplus, exploiting the relationship between pass-through and average surplus. I conclude in Section VII by discussing directions for future research. Longer proofs are collected into a series of appendices following the main text.

II. Monopoly Pricing and Pass-Through

A. Basics

Consider the problem of a monopolist facing consumer demand $D(\cdot)$ (assumed decreasing and thrice continuously differentiable) and constant marginal cost of production c. The familiar first-order condition is given by:

$$m \equiv p - c = \mu(p) \equiv -\frac{D(p)}{D'(p)} = \frac{p}{\epsilon(p)} \tag{1}$$

where $\epsilon(p)$ is the elasticity of demand. Thus μ is the ratio of price to elasticity of demand or, in mathematical terms, the inverse hazard rate of demand. I refer to this as the firm's *market power* because of its connection monopoly pricing: an upward shift in the market power function allows the monopolist to profitably charge a higher mark-up.

B. Second-order conditions and pass-through

Equation (1) is merely first-order condition. A common condition ensuring its sufficiency for optimization is that demand is log-concave, which is equivalent to market power being decreasing. However this condition is grossly sufficient[4] for this purpose and is substantively

[4]For an extensive discussion of the properties of log-concave functions (and particularly probability distributions), see Bagnoli and Bergstrom (2005). The authors also discuss a wide variety of economic applications where log-concavity is assumed.

restrictive. In particular it restricts the pass-through rate[5], the amount a monopolist finds it optimal to raise prices in response to a small increase in cost. Simple implicit differentiation shows that a monopolist's optimal pass-through of linear cost is given by

$$\rho \equiv \frac{dp}{dc} = \frac{1}{1-\mu'} \tag{2}$$

Therefore as first noted by Amir et al. (2004) log-concavity (convexity) is equivalent to pass-through being less (greater) than 1-for-1. I will therefore generally refer to log-concave demand as "cost-absorbing" and log-convex demand as "cost-amplifying", using the terminology of Rochet and Tirole (2007). A much weaker condition than cost absorbtion that makes equation (1) sufficient for the monopolist's optimization is that $\mu' < 1$ which is the same as marginal revenue declining in quantity[6]. This condition is equivalent to $\frac{1}{D}$ being concave and is therefore known in the mathematical literature[7] on the subject as -1-, $\alpha = -1$- or $p = -1$-concavity as first introduced by Brascamp and Lieb (1976). It is a regularity condition common used in auction theory (Myerson, 1981). It assumes that as mark-up increases, the marginal incentive to increase mark-up declines. I therefore refer to this condition as "mark-up contraction" (MUC). I call $\mu' \leq 1$ weak MUC. The main testable implication of this assumption is that a firm facing a binding price control will choose to charge at the controlled price.

Proposition 1. *If demand exhibits MUC then any solution to equation (1) is the monopolist's optimal price and for any cost a monopolist facing price ceiling (floor) below (above) her unconstrained optimum will always choose to charge a price at that ceiling (floor). Conversely if D fails to satisfy weak MUC, even at single point, then there exists a cost $c \in \mathbb{R}$ such that there is a solution to (1) which is not the monopoly optimal price, given that cost.*

[5]Another way of looking at this restriction from a price theory perspective that was pointed out to me by Jeremy Bulow is that $MR'(Q) < P'(Q)$ or that the marginal revenue curve slopes down more steeply than inverse demand.

[6]Revenue as a function of quantity is $qD^{-1}(q)$ so marginal revenue is $p + \frac{D(p)}{D'(p)}$.

[7]This was pointed out to me by Jean-Charles Rochet.

Furthermore, so long as the monopolist has an optimal price given any cost, there exists some cost and some price ceiling or floor below or above respectively the monopolist's unconstrained optimal price given that cost such that the monopolist will choose a constrained price strictly below or above respectively that ceiling or floor. In other words, MUC is the weakest condition guaranteeing first-order solutions for all cost levels.

Proof. See Appendix A. □

C. Pass-through as elasticity and the quantity interpretation

The pass-through rate measures how sharply defined the monopolist's optimization problem is. If the optimal price is very sharply defined, then there is a rather rigid "price the market will bear" and increases in cost will not move the monopolist's optimal price much. If, on the other hand, the monopolist is close to indifferent between a range of prices, a small increase in cost can cause a dramatic shift in her optimal price. This can be seen formally by noting that (at the monopoly optimal price)

$$\rho = \frac{1}{-\frac{d^2\pi}{dm^2}\frac{m^2}{\pi}} \tag{3}$$

Thus pass-through is exactly the inverse of the second-order elasticity of profits[8] with respect to mark-up. This provides a simple way to interpret the theme, that pass-through is the analog of elasticity in monopoly problems, running through all that follows. In choosing their optimal price, monopolists take first-order effects (elasticities of demand) into account. Therefore the *level* of elasticity cannot drive the comparative statics of monopoly problems, as they do competitive problems. Instead, second-order effects[9], namely the pass-through

[8] I suspect that, given the role of second derivatives in statistical discrimination problems, that the pass-through is likely related to the optimal degree of price experimentation by a monopolist. An interesting topic for future research is to understand the relationship between pass-through and experimentation, as this may provide a way of generating testable implications of optimal experimentation, either with or without rational expectations.

[9] This leads to a natural question of what would govern a chain of such relationships. Imagine a tax authority trying to maximize revenue using a unit tax on a monopolist with a higher tax authority trying to raise a tax by a unit tax on the first authority and so forth. I would conjecture that the crucial parameter

rate, plays the role of the price elasticity of demand.

The fact that pass-through is an elasticity (a unit-less measure), rather than a derivative, means that it is relevant not only to the cost-price dimension of the monopolist problem. It is also the crucial parameter in the comparative statics of her optimal quantity decision. Imagine a (semi-)monopolist choosing an optimal quantity to produce, given that there exists some exogenous quantity $\tilde{Q}$ of the good already available. Let $Q^\star \equiv \tilde{Q} + Q_M$, the monopolist's optimal production, be the total industry production given that the monopolist optimizes. The natural quantity analog of the pass-through rate is the *quantity pass-through rate*

$$\rho_Q = \frac{\partial Q^\star}{\partial \tilde{Q}} \tag{4}$$

Proposition 2. *$\rho = \rho_Q$ at the monopoly optimal price.*

Proof. Profits are $Q_M m(Q_M + \tilde{Q}) \equiv Q_M \left[P(Q_M + \tilde{Q}) - c\right]$ where $P \equiv D^{-1}$ so by analogy to the standard monopoly pricing problem the first-order condition is

$$Q^\star - \tilde{Q} = \mu_Q(Q^\star) \equiv -\frac{m(Q^\star)}{m'(Q^\star)} \tag{5}$$

Again by analogy (dropping arguments)

$$\rho_Q = \frac{1}{1 - \mu_Q'} \tag{6}$$

So I just need to show that at the monopoly optimal prices

$$\mu_Q' = \mu'$$

To see this note

governing the optimization of the nth order authority is an nth order analog of pass-through. I have not explored such a model formally, as I am not sure whether it has any applicability.

$$\mu_Q' = \frac{m''m}{(m')^2} - 1 = -m(D')^2 \cdot \frac{D''}{(D')^3} - 1 = -\frac{mD''}{D'} - 1 = \frac{D''D}{(D')^2} - 1 = \mu'$$

where the first equality follows from differentiation, the second from the inverse function theorem (as $\mu' = P'$), the third from equation (1), the forth from the definition of market power and the final from differentiation again.

□

$\rho_Q - 1$ determines the strategic complementarity (if it is positive) or substitutability (if negative) in Cournot competition[10]. Therefore this result establishes a tight connection between the strategic dynamics of Cournot's two problems, quantity competition and double marginalization, given that, as I will show in the next section, pass-through determines the strategic dynamics of double marginalization. This provides intuition for the importance of pass-through in Cournot competition discussed in Subsubection IV.A.1 and reinforces the importance of pass-through as a parameter of the monopoly problem.

D. Pass-through and the division of surplus

The pass-through rate is also closely related to the division of surplus between consumers and producers under linear cost monopoly. I showed the most general form of this connection in joint work with Michal Fabinger . Consumer surplus when price p is charged is given by

$$V(p) \equiv \int_p^\infty D(q)dq$$

and producer surplus, the monopolist's profits, is $\mu(p)D(p)$ by her first-order conditions. Therefore the ratio of consumer to producer surplus is

$$r(p) \equiv \frac{\overline{V}(p)}{\mu(p)}$$

[10]The first work to (at a qualitative and implicit level) see the link between pass-through rates and the strategic interactions in Cournot competition was Seade (1986).

where the *average surplus* $\overline{V}(p) \equiv \frac{V(p)}{D(p)}$. In Fabinger and Weyl (2008) we show that, as long as $m' < 1$ and if demand is positive for all prices $\lim_{p\to\infty} m'(p) < 1$ then

$$r(p) = \frac{\int_p^\infty \nu(q)\rho(q)dq}{\int_p^\infty \nu(q)dq} \tag{7}$$

where $\nu(p) \equiv \frac{D(p)}{\rho(p)}$. Thus the ratio of consumer to producer surplus at monopoly optimal prices is just a weighted average of the pass-through rate at prices above the monopoly optimum. This somewhat mysterious result is driven by the fact that the log-curvature of a function is closely tied to the log-curvature of its integral. It has a number of useful, if trivial, corollaries. If pass-through is globally above (below) some threshold k, then so is the consumer-to-producer surplus ratio. For example, globally cost-absorbing demand always has greater producer than consumer surplus at monopoly optimal prices. If pass-through is globally increasing (decreasing) then the consumer-to-producer surplus ratio is always above (below) pass-through at monopoly optimal prices. This result is useful because the division of surplus plays a key role in situations where social planners or consumers must make choices about newly created monopolies, such as patent policy, auctioning natural monopoly rights or models of product choice by firms (Lancaster, 1975; Spence, 1976; Dixit and Stiglitz, 1977; Salop, 1979). It also plays an important role in two-sided markets as discussed below.

This, and other effects of pass-through below, make assumptions about pass-through being globally (or over some range of prices) above or below 1, or globally increasing or decreasing, extremely useful in identification.

Assumption 1. *When not otherwise stated, I assume that demand is either globally cost-absorbing, globally cost amplifying or globally constant mark-up. I also assume that demand has either globally increasing or globally decreasing pass-through as a function of costs.*

III. Double Marginalization

In this section I turn to my primary application here of pass-through, the classic double marginalization problem, which has two equivalent formulations.

A. Set-up

In the first (Cournot, 1838) two monopolists sell goods that are perfect complements in consumption. In the second (Spengler, 1950)[11] one firm sells an input to a second firm which sells to a consumer. The only difference[12] between these models is that in Cournot's the assembly is performed by the consumer and in Spengler's it is performed by the downstream firm. Each firm has linear cost and because the division of this between production stages is irrelevant (as I will show shortly) to the welfare of all participants, I will be agnostic as to the division and simply assume that the sum of these two costs is c_I. The natural benchmark against which to judge the monopolistic vertically separated organizations that Cournot, Spenler and I consider is that of a single vertically integrated monopolist Integrated who sets her markup $m_I^\star$ to solve equation (1), plugging in c_I for c.

The separated industrial organization envisioned by Cournot is shown in Figure 1. The two firms choose their prices simultaneously to a consumer for whom the goods are perfect complements in consumption. The first-order conditions for each firm i of the two is given by

$$m_i^\star = \mu(m_i^\star + m_j^\star + c_I) \tag{8}$$

Thus clearly at equilibrium both firms choose equal optimal markups $m^\star$ given by

$$m^\star = \mu(2m^\star + c_I) \tag{9}$$

[11]To be precise, Spengler considers the "triple", rather than double marginalization problem where there are three stages of production. It would be simple to extend the model here to that case, but for simplicity I stick to the case of two vertically related firms.

[12]As Spengler does not cite Cournot, it appears he was unaware of this connection.

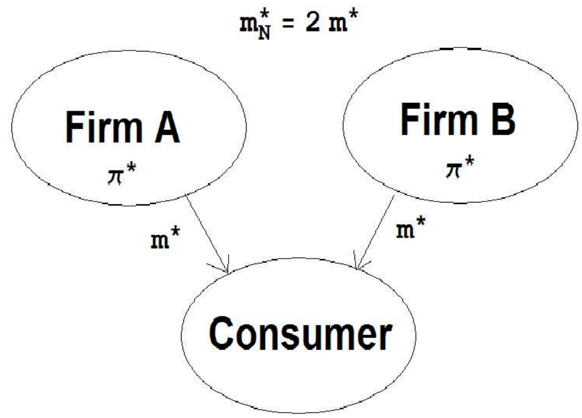

Figure 1: The Nash industrial organization

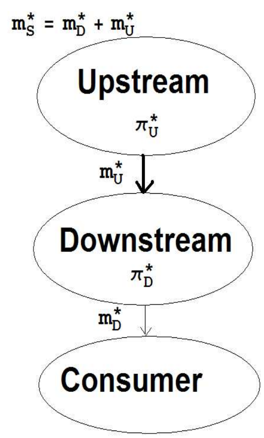

Figure 2: The Spengler-Stackelberg industrial organization

At equilibrium each firm earns profits $\pi^\star$ and the equilibrium total mark-up in the industry is $m_N^\star \equiv 2m^\star$.

Cournot's problem can also be formulated in Spengler's physical organization, shown in Figure 2, so long as the firms chose their mark-ups simultaneously. Thus it is the Nash timing that crucial to distinguishing the situation in Figure 1 (which I therefore call the Cournot-Nash organization) from that pictured in Figure 2 where, as Spengler originally assumed, Upstream commits to its price before Downstream chooses its price. Again it is the timing, rather than the physical organization, which is crucial here; Cournot's physical organization combined with price leadership in the spirit of von Stackelberg (1934) by one will yield the same outcomes as the organization in Figure 2. For any choice of mark-up m_U by Upstream, the optimal mark-up of Downstream is given, as Upstream's mark-up is just like an increased cost for Downstream, by

$$m_D^\star = \mu(m_U + m_D^\star + c_I) \tag{10}$$

Taking this into account, Upstream maximizes her profits $m_U D\big(m_U + m_D(m_U) + c_I\big)$ according to the first-order condition

$$m_U^\star = \frac{\mu\big(m_U^\star + m_D(m_U^\star) + c_I\big)}{\rho\big(m_U^\star + m_D(m_U^\star) + c_I\big)} \tag{11}$$

Equation (11) resembles Downstream's first-order condition, but takes into account the strategic effect of Upstream's choice on Downstream. Under cost absorbtion ($\mu' < 1$), mark-ups are strategic substitutes in the sense of Bulow et al. (1985): one firm raising its mark-up, which is equivalent to imposing a tax on the other firm, induces the other firm to absorb this increase and lower its mark-up. Conversely under cost amplification, mark-ups are strategic complements. Thus in the first (second) case Upstream has a strategic incentive to raise (lower) mark-ups. At equilibrium Upstream earns profits $\pi_U^\star$, the Downstream earns profits $\pi_D^\star$ and the total markup charged by the two firms is $m_S^\star \equiv m_U^\star + m_D^\star$.

B. Results and explanation

Table 2 summarizes my characterization of the relationship between these variables maintaining assumption 1[13]. The power of this assumption (along with the maintained assumptions

[13] It is reasonable to wonder whether cost-amplification is consistent with the stability of the equilibrium. In the Nash version of the game, the equivalent of MUC is a condition ensuring that any equilibrium is stable in myopic best responses. To see what this would be, imagine that one firm raises its mark-up by some small ϵ. The other firm will pass-through $\rho\epsilon$ of this; this corresponds to increasing its mark-up by $(\rho - 1)\epsilon$. This in turn will cause the original firm to increase its mark-up by $(\rho - 1)^2\epsilon$ and so on. Thus if $\rho - 1 \geq 1$ the equilibrium is not (strictly) stable. Therefore the appropriate condition is $\rho < 2$. The analog of this stability condition in the Stackelberg case is $(\rho_U - 1)(\rho - 1) < 1$, where the pass-through rate of Upstream $\rho_U \equiv \frac{1}{1-\mu'(1-\mu')+\mu'' m}$. Thus all types of demand considered are certainly possible even in a stable equilibrium.

	$\rho < 1$		$\rho > 1$	
	Cost absorption		Cost amplification	
	Decreasing pass-through		Decreasing pass-through	
	$m_U^\star$		$m^\star$	
ρ'	$\vee$	$\pi_U^\star$	$\vee$	$\pi_D^\star$
$\wedge$	$m_I^\star < m_N^\star < m_S^\star$	$\vee$	$m_D^\star$	$\vee$
0	$\vee$	$\pi^\star$	$\vee$	$\pi_U^\star$
	$m^\star$	$\vee$	$m_U^\star$	$\vee$
	$\vee$	$\pi_D^\star$	$\vee$	$\pi^\star$
	$m_D^\star$		$m_I^\star < m_S^\star < m_N^\star$	
	Cost absorption		Cost amplification	
	Increasing pass-through		Increasing pass-through	
	$m_I^\star < m_N^\star < m_S^\star$		$m^\star$	
ρ'	$\vee$	$\pi_U^\star$	$\vee$	$\pi_D^\star$
$\vee$	$m_U^\star$	$\vee$	$m_D^\star$	$\vee$
0	$\vee$	$\pi^\star$	$\vee$	$\pi_U^\star$
	$m^\star$	$\vee$	$m_I^\star < m_S^\star < m_N^\star$	$\vee$
	$\vee$	$\pi_D^\star$	$\vee$	$\pi^\star$
	$m_D^\star$		$m_U^\star$	

Table 1: A taxonomy of the Cournot-Spenler double marginalization problem

of known demand, model specification and equilibrium), though, is that it reduces $4! = 24$ possible rankings of firm mark-ups, $3! = 6$ possible rankings of firm profits, 2 possible rankings of Stackelberg versus Nash mark-ups and 4 ranges of values for pass-through rates and slope to only four possible cases. That is it predicts the empirical appearance of only 4 of 1152 possible clusters for results and thus has substantial identifying power, as discussed further below.

To understand the results, first note that $\pi_U^\star > \pi^\star$ as Upstream can always choose to imitate a Nash firm and obtain the same reaction from Downstream as a Nash firm would give. Most of the action in Table 1 happens across the vertical line (from the left hand side to the right hand). This is where demand moves from being cost-absorbing on the left to cost-amplifying on the right. This is crucial as it determines, as mentioned earlier, whether mark-ups are strategic substitutes (absorption) or complements (amplification). Knowledge of this furnishes all but one of the comparisons in Table 2.

1. Does a Nash firm or Integrated charge a higher mark-up? The only difference between

their incentives is that a Nash firm has a partner that has tacked on an additional mark-up. If it is optimal for the Nash firm to absorb this additional mark-up (which acts just like a cost) the Nash firm will choose a lower mark-up than Integrated; if it is optimal for the Nash firm to amplify this mark-up Nash will charge a higher mark-up than Integrated.

2. Does Upstream or Downstream charge a higher mark-up? Both face the same demand and therefore have the same market power. The only difference in their incentives is that Upstream's mark-up has a strategic effect on Downstream. Upstream has a strategic incentive to do whatever induces Downstream to reduce her mark-up. When mark-ups are strategic substitutes this involves Upstream increasing her mark-up and thus charging a higher mark-up than Downstream; when mark-ups are strategic complements this involves Upstream decreasing her mark-up and thus charging lower mark-up than Downstream. Because both face the same end-demand, this further determines the comparison of their profits[14].

3. Does a Nash firm or Upstream charge a higher mark-up? Starting from Nash prices, Upstream's incentives are driven entirely by the strategic effect; however, as Upstream moves her prices to induce Downstream to lower hers, she must react to these lower prices. However, these always reinforce her initial strategic incentive: in the strategic complements case when Upstream reduces her mark-up, the reduction in mark-up she induces to Downstream reinforces her incentive to reduce her mark-up. With strategic substitutes, Upstream's increase in her mark-up is reinforced by Downstream's decrease. Clearly this also means that Downstream will always choose a lower mark-up than a Nash firm, as Upstream always ends up acting so as to reduce her mark-up.

[14]This reasoning can also be seen as flowing in the opposite direction: because market power is the reaction function in this game, the equivalence in ranking between mark-ups and profits in this game means that these results can be seen as a consequences of Gal-Or (1985) and Dowrick (1986)'s classic observation that the slope of the reaction function determines whether the first mover is advantaged or disadvantaged in symmetric games. The application of Gal-Or's taxonomy to the vertical problem was first pursued by Lee and Staelin (1997); however as discussed above this does not imply, as the authors seem to infer, that a firm will only benefit from price leadership (relative to Nash timing) under cost-absorption.

4. Is the total mark-up higher under the Spengler-Stackelberg or Cournot-Nash organization? The real difference between these two is the additional strategic incentive to Upstream under Stackelberg. Thus with strategic complements the Stackelberg organization is preferable to (leads to a lower total mark-up than) the Nash organization and conversely under strategic substitutes. Because Upstream always earns higher profits than a Nash firm and total industry profits are always higher when mark-ups are lower as total mark-up is above the monopoly optimal level, this implies that when there are strategic substitutes the Nash firm must earn higher profits than Downstream.

These results are stated formally in the following proposition.

Proposition 3. *1. If demand is cost-absorbing then* $m_U^\star, m_I^\star > m^\star, m_D^\star$*,* $m_S^\star > m_N^\star > m_I^\star$ *and* $\pi_U^\star > \pi^\star > \pi_D^\star$.

2. If demand is cost-amplifying then $m^\star > m_D^\star > m_U^\star, m_I^\star$*,* $m_N^\star > m_S^\star > m_I^\star$ *and* $\pi_D^\star > \pi_U^\star > \pi^\star$.

3. If demand is constant mark-up, $m_U^\star = m_I^\star = m^\star = m_D^\star = \frac{m_N^\star}{2} = \frac{m_S^\star}{2}$ *and* $\pi^\star = \pi_U^\star = \pi_D^\star$.

Proof. See Appendix B. □

The one comparison that varies across the horizontal line (from top to bottom) in Table 1 is between $m_U^\star$ and $m_I^\star$. The reasoning behind this is a bit more subtle. Consider the case of cost absorption. There are two incentives facing the Upstream firm. On the one hand she would like to increase her mark-up, relative to what Integrated would charge, as she has a strategic incentive to induce Downstream to decrease her mark-up. On the other hand Upstream has an incentive to partially absorb Downstream's mark-up which Integrated does not confront; this leads Upstream to decrease her mark-up relative to what Integrated would charge. The first strategic incentive is marginal: its size is determined by how much, on the margin, a small increase in Upstream's price can induce Downstream to reduce her price. The second incentive, on the other hand, depends on the average rate at which Upstream should pass-through versus absorb Downstream's mark-up. The relative size of the average versus the marginal effect depends on whether pass-through is increasing or decreasing in cost. If it is decreasing the marginal incentive dominates the average, as on the margin Downstream will absorb more of the marginal Upstream increase in mark-up than it is optimal for Upstream to absorb of Downstream's mark-up. Conversely if pass-through is increasing, the opposite occurs. The analysis applies in the case of cost amplification for analogous reasons. The following result states this formally. Because the proof of this result is perhaps the most interesting of the lot I include it. It demonstrates the general strategy of ranking the relevant market power functions across firms and market structures that I use to prove all of the results.

Proposition 4. *1. If pass-through is decreasing* $m_U^\star > m_I^\star$.

2. If pass-through is increasing $m_I^\star > m_U^\star$.

3. If pass-through is constant $m_I^\star = m_U^\star$.

Proof. The strategy of the proof is to rank the effective market power functions faced by Upstream against that faced by Integrated. That is, I want to compare the value of the RHS of equation (11) and to the value of the RHS of equation (1) for a particular input value of mark-up for each of these expressions. The two expressions are respectively (evaluated at a common m):

$$\mu\big(m + m_D^\star(m) + c_I\big)\Big[1 - \mu'\big(m + m_D^\star(m) + c_I\big)\Big]$$

$$\mu(m + c_I)$$

If I can show that the first expression is greater for any m, then clearly $m_U^\star > m_I^\star$ as $\mu(m) - m$ and $\mu(m)\big(1 - \mu'[m]\big)$ are assumed decreasing by the second-order/stability conditions. If the second expression is greater for all m then the opposite result obtains; if they are equal, then Upstream and Integrated's optimal choice of mark-up are governed by the same equations. Because market power is always positive the relationship between these two expressions is the same as the relationship between:

$$1 - \mu'\big(m + m_D^\star(m) + c_I\big)$$

$$\frac{\mu(m + c_I)}{\mu\big(m + m_D^\star(m) + c_I\big)}$$

The second expression can be put in a more illuminating form by defining $p_S(m) \equiv m + m_D(m) + c_I$ and $p_I(m) \equiv m + c_I$:

$$\frac{\mu(p_I(m))}{\mu(p_S(m))} = \frac{\mu(p_S(m)) + \int_{p_S(m)}^{p_I(m)} \mu'(p)dp}{\mu(p_S(m))} = 1 - \frac{\int_{p_I(m)}^{p_S(m)} \mu'(p)dp}{\mu(p_S(m))}$$

Now note that for any $p_S(m) - p_I(m) = m_D(m)$ and that by equation (10) which defines $m_D(m)$ implicitly, $m_D(m) = \mu(p_S(m))$ so the above expression becomes:

$$1 - \frac{\int_{p_I(m)}^{p_S(m)} \mu'(p)dp}{p_S(m) - p_I(m)} = 1 - \overline{\mu'}|_{p_I(m)}^{p_S(m)}$$

where $\overline{\mu'}|_{p_I(m)}^{p_S(m)}$ is the average value of μ' over the interval $[\underline{p}(m), p_S(m)]$. Now note that if $\mu'' < (> / =) 0$ everywhere then clearly:

$$1 - \mu'\big(m + m_D^\star(m) + c_I\big) > (< / =) 1 - \overline{\mu'}|_{p_I(m)}^{\overline{p}(m)}$$

for any m. And thus if market power is concave (convex/linear) Upstream charges a higher (lower/same) margin than Integrated. And because $\rho = \frac{1}{1-\mu'}$, pass-through is decreasing (increasing/constant) exactly when market power is concave (convex/linear).

□

C. Applications and connections

As is usual in industrial organization, the table shows that to most questions one could ask the answer is "it depends". However, because reasonable empirical strategies exist, at least in some cases, for identifying what "it depends" on and because the double marginalization problem has a close relationship to several other theoretical problems, this taxonomy has a number of potential empirical and theoretical applications, which I briefly discuss here.

Clearly given enough information, the model here is significantly over-identified, allowing it to be tested. If pass-through can be identified[15] and it is known that one firm acts as the Stackelberg leader then the model can be tested by comparing mark-ups of the two firms. If the industrial organization changes (Mortimer, 2008) and mark-ups can be observed before and after, the model is further (essentially non-parametrically) over-identified.

The model can also be used to identify variables of interest. Identifying demand as cost-amplifying or absorbing allows the Stackelberg leader, if any, to be be identified from mark-up data. Conversely if one firm is known to be the Stackelberg leader and mark-up data is available the pass-through rate can be identified from these. Also pass-through rates can be used to identify whether the Stackelberg or Nash organization is preferable. Given that firms often establish price leadership through various forms of conduct overseen by antitrust authorities (such as aggressive advertising of prices or adjusting the durability of goods as discussed by Lee and Staelin (1997) and Lieberman and Montgomery (1988)), pass-through rates may be useful to inform antitrust policy towards vertical monopolies.

[15] Under Spengler-Stackelberg equilibrium the expression for pass-through rates is more complicated:

$$\frac{1}{1 + \mu\mu'' - \mu'(2 - \mu')}$$

But note that because $2 - \mu' > 0$, if one is willing assume demand is in or close to the constant pass-through class, then determining whether the equilibrium absorbs or amplifies cost pins down the sign of μ'. Furthermore this expression is decreasing (increasing) if $\mu''(2 - 3\mu') < \mu^{(3)}\mu$. If one is willing to assume that $m^{(3)}$ is small in magnitude, again the same test (that pass-through be decreasing in cost), will work. Thus while Table 2's pass-through assertions are most accurate in the case of an Integrated or Nash organization, they may be thought of as approximately right in the Stackelberg organization. Furthermore, it may be that pass-through rates can be estimated under monopoly or a Nash organization in one industry and then applied to an industry with plausibly similar demand that has a Spengler-Stackelberg organization.

One rich potential application is vertical relationships among tax authorities (Besley and Rosen, 1998b; Keen, 1998; Esteller-Moré and Solé-Ollé, 2001). Federal authorities clearly impose vertical externalities on local authorities when they levy commodity or value-added taxes. The wealth of data that such interactions afford would provide an interesting test of the theory and potentially a means of identifying properties of demand, such as the slope of pass-through. However, given the complexity of political motives, caution in viewing authorities as revenue maximizing is important. That being said work by Besley and Rosen (1998b) is consistent with the cost-amplifying demand.

The model additionally suggests some directions for understanding the endogenous determination of price leadership. Under cost-absorption, the industry as a whole is better off, both for consumers and the firms, under the Nash rather than the Stackelberg organization. However, Upstream earns greater profits than a Nash firm does and thus both firms may have an incentive to try to grab price leadership. Under cost-amplification, it is Pareto-superior that one firm acts as a price leader; however, Downstream earns greater profits than Upstream so each firm has an incentive to wait for the other become the price leader. The precise consequences of these observations would, of course, depend on the details of the model of this pre-play. Endogenous determination of first-mover positions has received some recent attention in the context of quantity competition (Hamilton and Slutsky, 1990; van Daame and Hurkens, 1999; Amir and Grilo, 1999) and if applied to this context would imply that price leadership emerges under cost amplification but not under cost absorption. However, because these games are largely geared toward the quantity competition context, theoretical work on endogenous timing in the double marginalization problem is still an open problem.

One problem with all such applications is that they rely on the inability of the firms to vertically integrate or find contractual (non-linear pricing) solutions to the double marginalization problem. To justify such a barrier, one would have to posit costs associated with vertical integration. I will not endeavor to speculate about the sources of such costs, but it is worth noting that if they do exist, the results here may have some implications for when solutions to the vertical problem arise or fail to arise endogenously. While the calculating the magnitude of the incentive to integrate would depend crucially on how the magnitude of costs vary with the size of the market, it seems likely that one driver of endogenous integration would be the size of the divergence between the $m_I^\star$ and, depending on the market structure under separation, $m_N^\star$ and $m_S^\star$. Precise calculation even of this would, again, require more specific assumptions about demand, but it seems clear from the above analysis that the primary driver of a large divergence between $m_I^\star$ and $m_N^\star$ is cost-amplification. On the other hand, divergence between $m_S^\star$ may also be driven by pass-through being strongly decreasing; cost-amplification may be more of a secondary factor in this case[16]. Because it provides rankings within the Stackelberg organization among firms' profits, the analysis above also provides bargaining points for the splitting of profits associated with integration and therefore potentially predictions about how such gains are split. Furthermore, because one source of costs associated with vertical integration may be intervention by antitrust authorities concerned that other products produced by the respective firms are substitutes, the use of pass-through to measure the magnitude of *social* benefits from vertical integration.

The results are also potentially applicable beyond the narrow context of the double marginalization problem. Whenever a firm sells a good that exhibits inter-temporal complementarity in consumption[17], questions like those explored here help determine the effect of an inability to commit to prices on their strategy. Because sequentiality is inherent in

[16]One promising route for exploring such issues would be to consider extending the constant pass-through class developed in Section VI to a linear pass-through class and calculating precise answers within that class. Within the constant pass-through class it is easy to see that the basic intuition that cost amplification drives divergence between $m_I^\star$ and $m_N^\star$, as well as (to a lesser extent) that between $m_I^\star$ and $m_S^\star$. Any demand in the constant pass-through class has market power that can be written as $a + \frac{\rho-1}{\rho}p$ where ρ is pass-through. $m_I^\star = a\rho + (\rho-1)c$, $m_N^\star = \frac{a\rho+(\rho-1)c}{2-\rho}$ and $m_S^\star = (1+\rho)\left[a\rho + (\rho-1)c\right]$. Assume that $\rho a + (\rho-1)c > 0$, which follows from demand being strictly positive and decreasing at $p = c$. Thus $\frac{m_N^\star}{m_I^\star} = \frac{1}{2-\rho}$ and $\frac{m_S^\star}{m_I^\star} = 1 + \rho$ both increase in ρ, but the first increases more quickly when demand is cost-amplifying: the derivative of the first is $\frac{1}{(2-\rho)^2} > 1$, the derivative of the second, when demand is cost-amplifying.

[17]This was first pointed out to me by Kevin Murphy. It presumably applies to goods that are inter-temporal substitutes (durable goods) as well, so long as the results are interpreted in the context of Cournot competition, as discussed below.

such problems, issues like those discussed here will help determine whether a monopolist selling an addictive good (Murphy and Becker, 1988) who cannot commit to future prices will charge a higher or lower first-period price relative to either a firm that can commit or has no intertemporal complementarity (like different versions of Integrated in my problem) or one that fails to take into account their effect on future pricing (like a Nash firm). This happens both through both forward (what is the effect of pricing today on pricing tomorrow?) and backward (what is the effect of the fact that future prices are higher on current prices?) effects, the first being determined by strategic effects like those above and the latter by the cost absorptions versus amplification type results. This seems particularly interesting because empirical work (Sumner, 1981) suggests that the cigarette industry exhibits, unusually, cost amplification. Similarly intra-firm effects on pricing without commitment in the market for goods with switching costs are similarly related to the double marginalization results here. While Klemperer (1987) assumes the effective analog of cost absorption and thus concludes that a firm will price higher initially to reduce its future incentive to mark-up, it seems likely that assuming the effective analog of cost amplification would yield the opposite result. The results here might help move away from the functional forms of demand assumed by him and others in the related literature to understand (and potentially identify empirically) the properties of demand associated with this effect going one way versus the other. The results might also be useful in the recent literature on tying products to goods which are complements to them in the future (Carlton et al., 2008), as comparisons like those between Upstream and Integrated help determine the relationship between current good component prices. Of course such extensions requires the extension of the results here to the case of imperfect complements that I am currently working out (Weyl, 2008a) and multiple firms in the Stackelberg case[18] (higher order derivatives of market power would likely be important), as well as careful identification of the analogs of log-concavity in each of these contexts.

[18] A extension of the Stackelberg model to the case of a long chain of strategic interactions has begun to be developed by Anderson and Engers (1992), but significantly more work would be necessary to fully extend the results here so as to permit the sort of analysis necessary for understanding multiperiod monopoly pricing with intertemporal complements and no commitment.

Another potential application[19] is to industries in which a taxing or subsidizing authority is subject to political pressure. In this case if demand is cost-absorbing, the firm will want to commit to pre-tax prices (gain Stackelberg leadership over the authority) so as to induce the authority to tax less (subsidize more). Thus the authority would do well to prevent it from making such a commitment, if it had the regulatory authority to do so. On the other hand if demand is cost-amplifying, the firm would like to commit to post-tax prices to lead the authority to play leader and the authority would do well to force it to lead by committing to pre-tax prices.

The results may also be useful in understanding common agency problem. Martimort and Stole (Forthcoming) show that the double marginalization problem carries over to the broader contract space of common agency problems with unknown reservation values: n principles simultaneously choosing contracts act like a single principle facing a demand (or in this case distribution of reservation values for contracting) with n times the market power. This suggests that my results may be useful in characterizing strategic interactions in stochastic common agency problems, especially given recent interest in the sequential common agency problem (Prat and Rustichini, 1998; Pavan and Calzolari, 2007; Calzolari and Pavan, Forthcoming). In fact, similar results to my comparison of the Stackelberg and Nash case in the cost absorption case for the common agency problem were obtained by Martimort (1996) and can be generalized by the techniques here to the case of cost amplification.

IV. Other Applications

The double marginalization is one among several problems where simple assumptions about pass-through provide powerful identification. Having developed this application in detail, I now discuss in a more cursory fashion a number of other application to demonstrate the breadth with which these techniques can be applied to commonly used economic models.

[19] This application was suggested to me by Bill Rogerson.

A. Some new results

I begin by discussing a few novel results.

1. Quantity competition

The other application most closely related to the double marginalization problem is that of quantity (Cournot) competition. As Sonnenschein (1968) pointed out, the quantity competition problem and the double marginalization problem are linked by duality: in the first case mark-up depends on the sum of outputs and in the second output depends on the sum of mark-ups. This means that quantity pass-through plays the same role[20], and lend the same identifying power, in the quantity competition problem that (cost) pass-through does in the double marginalization problem.

These results on their own are not very useful, as quantity pass-through is much harder to identify than is (cost) pass-through. Luckily, though, these two notions of pass-through are intimately linked to one another by Proposition 2. It is intuitive, though, not immediately obvious that, as the following proposition establishes formally, this connection extends to the equilibrium pass-through of a symmetric linear cost Cournot duopoly. Let $\rho_C \equiv \frac{dP_C^\star}{dc}$ be the equilibrium Cournot pass-through rate in a symmetric linear-cost model where $P_C^\star$ is the equilibrium market-clearing price.

Proposition 5. $\rho_Q > (< / =) 1 \iff \rho_C > (< / =) 2$ *and* $\frac{\partial \rho_Q}{\partial Q} > 0 \iff \frac{\partial \rho_C}{\partial c} < 0$

Proof. By duality with the double marginalization if we let $Q_C^\star$ be the equilibrium production by the Cournot duopoly then

$$Q_C^\star = -\frac{P(Q^\star) - c}{m'}$$

and therefore

[20] In some cases this argument is merely a simpler proof (in a simple case) of some classic results on quantity competition, such as those established by Dowrick (1986), Amir and Grilo (1999) and Amir and Lambson (2000). In others it provides novel results.

$$\frac{\partial Q_C^\star}{\partial c} = 2\frac{m' - \frac{\partial Q_C^\star}{\partial c}\left([m']^2 - m''m\right)}{(\mu')^2} = \frac{m'}{\frac{3}{2}(m)^2 - m''m}$$

where the second equality follows from simple solving out. Therefore as $P_C^\star = P(Q_C^\star)$ and because (again by duality with double marginalization) $\rho_Q = \frac{1}{2 - \frac{m''m}{(m')^2}}$, we have $\frac{m''m}{(m')^2} = \frac{2\rho_Q - 1}{\rho_Q}$ implying that

$$\rho_C = \frac{\partial P_C^\star}{\partial c} = \frac{(m')^2}{\frac{3}{2}(m')^2 - m''m} = \frac{2\rho_Q}{2 - \rho_Q}$$

This is an intuitive formula: the standard pass-through formula is, by the presence of Cournot duopoly is doubled by the presence of two firms and magnified or contracted by a stability-related term. The fact that this expression clearly increases in (and depends only on) ρ_Q yields the second result. Trivial inspection of the algebra yields the first. □

Thus because "everything of interest" (in the sense of discussed in the double marginalization problem) in the Cournot model depends on the quantity pass-through rate's comparison to unity and the sign of its slope, this proposition shows that the symmetric linear cost quantity competition model can be non-parametrically (over-)identified in precisely[21] the same manner as the corresponding double marginalizaiton model. This is particularly important in predicting the effects of mergers (Farrell and Shapiro, 1900) in homogeneous good Cournot markets[22]. As far as I know this is the first known feasible empirical test for the strategic dynamics of Cournot duopoly. Results of Seade (1986), discussed more extensively in Subsubsection C.1 below imply that these strategic dynamics can also be used to identify the effect of taxes on firm profits, as well as suggesting that these results, as well as through duality the double marginalization results, can be extended to the case of N firms.

[21]Of course, we again need to impose some boundedness conditions on curvature of, now, μ_Q'' in order to implement the empirical test starting at the Stackelberg organization.

[22]See Subsubsection C.2 below for the case of differentiated Cournot competition.

2. The effect of differentiated Bertrand competition on prices

Perloff and Salop (1985) considers a model of differentiated Bertrand competition where all consumers must purchase a good from one of n firms. Their utility of each good is drawn i.i.d. across consumer-good pairs from some distribution of valuations. Perloff and Salop then ask what the effect of an increase in "competition" (an increase in the number of independently owned brands) is on prices. This is not clear ex-ante because competition has two effects. On the one hand, it creates a greater number of choices for consumers, making it difficult for any given firm to hold onto a consumer and thereby decreasing their market power. However, it also creates a greater number of opportunities for a consumer to have one brand for which she has a very high valuation, encouraging the firms to raise prices to extract the surplus of this consumer. Which effect dominates depends on the distribution of the order statistics of the distribution of consumer utility. This makes the problem quite complex, and Perloff and Salop obtain results only for a very limited class of distributions.

Gabaix et al. (2005) overcome this difficulty by considering the behavior of prices as a function of competition when the market has a large number of participants using extreme value theory. They show that for large n, mark-ups are given by

$$\frac{1}{nf\left(F^{-1}\left[1-\frac{1}{n}\right]\right)}$$

where F is the cumulative distribution function of the consumer valuations and f is the density of these valuations. The authors explicitly solve for the value of this expression asymptotically for a variety of distributions. However a simpler approach is available based on pass-through (log-curvature) for distributions, like most typically considered, that are either globally log-concave or globally log-convex, as the following proposition shows.

Proposition 6. *Mark-ups in the Perloff and Salop (1985) model are decreasing (increasing) in the number of firms if $1-F$ is globally log-concave (convex). Equivalently they are decreasing (increasing) in the number of firms if the distribution of consumer preferences would* *give rise to a cost-absorbing (amplifying) demand function under monopoly with consumers having the option of not purchasing the good.*

Proof. Let's interpret things as demand functions. Let $D(p) \equiv F^{-1}(1 - F[p])$ and let $p(n) \equiv 1 - \frac{1}{n}$. Note that clearly $p' > 0$ so if I can show that

$$-\frac{D(p)}{D'(p)}$$

decreases (increases) in p this establishes that mark-ups decrease (increase) in competition. But this expression is exactly the market power, proving the result. □

Because nearly all distributions of consumer preferences typically considered are either globally log-concave or globally log-convex this result is analytically useful and further generalizes the Gabaix et al. (2005) results. However, it is does not provide a direct empirical test of that would allow us to use the Perloff and Salop (1985) model to predict the effects of competition on prices. The reason is that Perloff and Salop (1985) assumes that consumers do not have the option to consume no good and therefore market-power is cost independent, so that exogenous cost variations cannot be used to tie down the log-curvature of the distribution of consumer preferences. An interesting question for future research is whether, if this assumption is relaxed and consumers are allowed to choose to consume no good, the Gabaix et al. (2005) result, and therefore my Proposition 6, would continue to hold and whether equilibrium pass-through rates would identify log-curvature.

B. Results from my work on two-sided markets

A topic of recent interest in industrial organization has been the problem of so-called "two-sided markets". These are markets with network effects[23] that occur exclusively *between* rather than *within* two distinct groups of consumers. Typical examples are firms serving as

[23] Which resist direct Coasian bargaining (Rochet and Tirole, 2006).

a platform for transactions (payment cards), two-sided services (advertising, website access, video game playing) or matching (dating clubs or websites).

The classic approach of Rochet and Tirole (2006) posits that consumers in each group have an exogenous idiosyncratic utility function affine in the price they pay and the number of consumers participating on the other side of the market. This broad model is quite complicated to analyze. However if we specialize it by assuming that consumer utility and firm prices and costs on each side of the market are linear (no intercept) in the number of consumers on the other side of the market we arrive at the fairly tractable Rochet and Tirole (2003) (RT2003) model. If, on the other hand, we assume that all consumers on a particular side take the same value from participation of consumers on the other side of the market and the firms' costs on each side are proportional to the number of consumers participating on that side, we have the even more tractable Armstrong (2006) model. As I show in a series of papers, the comparative statics of both, particularly the first, depend crucially on the pass-through rate.

1. RT2003

As RT2003 formulate it, the basic problem of pricing in two-sided markets can be seen most clearly by considering a credit card company which charges a per-transaction price to card-carying consumers and card-accepting merchants. If for some exogenous reason the price they charge to merchants rises, this will give them a greater incentive to encourage consumers to use cards at stores by reducing the price to them. Conversely competition which drives down prices on one side of the market may have the perverse effect of raising prices to consumers on the other side of the market. In fact, the defining feature of the RT2003 model is that this is the *only* cross effect between pricing to the two sides of the market: each dollar earned on one side of the market (per-transaction) acts as a cross-subsidy of exactly one dollar to the other side of the market.

Two questions are crucial to the economics of a RT2003 two-sided market. First, does competition tend to reduce “overall prices” (the sum of prices on the two sides of the market)? Second, what are the normative effects on consumers on both sides of the market of prices rising to one group while falling to the other? As I show in Weyl (2008d) the pass-through rate (on both sides of the market) turns out to be crucial to answering both questions. On the positive side it determines how much of a cross-subsidy from one side of the market is passed through to the other. On the normative side, firms only internalize the benefits that marginal consumers gain from more partners joining on the other side of the market, as they cannot price discriminate. Average surplus measures the magnitude of this inefficiency and therefore the degree to which consumers on one side of the market might gain from being taxed to fund a reduction in prices to the other side of the market. Because pass-through determines the fraction of the two-sided benefit that is internalized, it is crucial to the normative economics of the RT2003 model. In fact all the basic economics of the model turn on whether demand on both sides of the market is cost absorbing or whether demand on one side is cost-amplifying.

Luckily, the distinction between the two cases is not only identified but actually over-identified, given exogenous cost variations. That is exogenous cost variations allow us not only to identify which case we are in, but also to test the model (Weyl, 2008c,d), given Assumption 1.

2. Double marginalization in RT2003

Two-sided markets are no more immune than any other markets to the problems of double marginalization arising from complementary products being produced by various firms. A prominent example is the separation[24] of the debit card industry into (for-profit) debit clearing networks such as Star and debit card-issuing banks. Other examples are a similar separation in the credit card industry, add-ons to video game consoles and relationships among internet service providers. This problem combines elements of both the results dis-

[24] Until recent partial integration undertaken by Visa through Interlink.

cussed in the previous subsubsection and those developed above on double marginalization.

As I show in Weyl (2008b), this combination of two problems which are over-identified by the same simple assumptions about pass-through leads to a model far more over-identified than either of the individual models. It furthermore allows a complete analysis of this problem, which shows that the benefits of vertical integration in two-sided markets are more robust than those of competition, because they largely avoid the harmful tendency of competition to reduce per-transation profits on one side of the market. I will not discuss this model in more detail here as those aspects of it not obvious from above are fairly messy. Nonetheless, it shows that, because the same properties of pass-through rates are crucial to identifying many simple models, more complicated ones need not be exponentially more complex (under-identified) given my identifying assumptions. In fact, it provides an example of a case where the opposite happens, and the same assumptions allow exponentially more *simplicity* (over-identification). This offers some hope that applying my approach, or ones like it, to more "realistic" models will not be as difficult as it might seem.

3. Armstrong two-sided markets

The most popular alternative model to the RT2003 approach is Armstrong's model. The analysis of this model (Weyl, 2008c) is quite a bit simpler than that of the RT2003 model as all consumers on a particular side of the market have the same benefit per consumer participating on the other side. This means that the platforms fully internalize consumers' two-sided benefits. This makes it much easier to sign all welfare-relevant comparative statics purely on the basis of theory. However, assumptions like 1 can still be used to test the model and make predictions about (welfare-irrelevant) observable prices.

C. From others' work

Several other economists have found other situations in which pass-through has significant identifying power.

1. Conjectural variations models

In a pair of papers that are perhaps those most closely related to mind, Seade (1980, 1986) explores the comparative statics of symmetric conjectural variations oligopolies. Clearly the Cournot case discussed above is a special case of these. Instead of pass-through, Seade uses as a primitive a number, the elasticity of the slope of inverse demand, that he calls $E = -\frac{QP''}{P'}$. This is related to pass-through in a very simple way by the inverse function theorem

$$E = \frac{DD''}{(D')^2} = \mu' + 1$$

So $E = \frac{2\rho - 1}{\rho}$. He shows that (translating things into my nomenclature) equilibrium cost-absorption/amplification are equivalent to quantity (primitive) pass-through being less (greater) than $\frac{N}{N+1}$ for a Cournot oligopoly with n firms. He further shows how E is related to whether taxes can benefit oligopolistic firms in equilibrium; in the duopoly case strategic complementarity is necessary and sufficient for this and therefore my test for strategic complementarity versus substitutability is also a test for this condition; a formula could likely be derived for the case of N firms. Though the mechanics he develops very much enable the sort of analysis developed in Subsubsection A.1 above, Seade does not emphasize the use of equilibrium pass-through rates to identify policy relevant variables, focusing more on the theoretical question of "reasonable" possibility of various effects. Furthermore, because there is no natural notion of sequentiality versus simultaneity in *general* conjectural variations models, some of the identifying power above is necessarily lost. However, Seade's results suggest that many derived here may be generalizable given careful analysis to the case of non-linear costs. This is a promising topic for future research.

2. Merger analysis

The two central elements of static merger analysis in differentiated product industries are the evaluation of anticompetitive effects and offsetting efficiencies (Willig, 1991). Both of these effects involve the shifting of costs faced by firms. Horizontal mergers tend to be anticompetitive as they increase the opportunity cost of sales faced by a firm, as after the merger it must take into account the lost sales "diverted" from the sale of a substitute product. Efficiencies, which may offset these anticompetitive effects, are reductions in firms' marginal costs as a result of productive synergies between the merging firms. This logic is developed by Froeb et al. (2005) and Farrell and Shapiro (2008) to argue that, under Nash-Bertrand competition, once the relative size of these two effects are determined, and therefore the sign of the price effect (Werden, 1996) of the merger tied down, the magnitude of the effects of a merger are determined by the pass-through rate.

This provides a non-parametric (local) foundation for merger analysis that avoids rampant sensitivity of merger analysis to the functional form (given the restrictions this places on pass-through, see below) used in the analysis, even given a collection of measured elasticities and cross-elasticities (Crooke et al., 1999). Nonetheless these analyses suffer from two major weaknesses. First, they ignore interaction between the anticompetitive effects on the two goods: as one good's price rises, so does the opportunity cost of a sale of the other. Second, they ignore the effects that changes in the price of one good have on those of others in a differentiated products industry with many firms. To overcome these problems while still avoiding the restriction of the pass-through rate, which is so crucial to the size of merger effects, I have formulated (to my knowledge) the first demand system allowing full variation in the matrix of gradients of market power functions with respect to industry prices.

This "constant pass-through demand system", which extends the Bulow and Pfleider (1983) single-firm demand function described below to differentiated product industries, has a number of other useful properties which I outline in a paper I am currently preparing (Weyl, 2008a). This system also can be used to investigate how my results generalize to this much broader complements/substitutes framework[25]. Crucially whether demand for a particular product is cost-absorbing or cost-amplifying determines whether the price of a good that complements or substitutes for it is a strategic complement or substitute for its price. Thus, at least within this demand system (which includes linear demand as a special case) whether the "conventional wisdom" (Tirole, 1988) that under competition in prices complementary goods are strategic substitutes and substitutable goods are strategic complements is driven by demand being cost-absorbing; under cost-amplification these are reversed. Because pass-through governs the quantity as well as the cost-price margin of monopoly problems, essentially the same approach can be applied to differentiated Cournot industries. This allows a unified approach to analyzing mergers in these two foundational models.

3. International macroeconomics

Exchange rate pass-through has been extensively explored in empirical international macroeconomics (Menon, 1995; Taylor, 2000; Campa and Goldberg, 2005; Gopinath and Rigobon, 2008). While this topic bears an obvious relationship to those discussed above, only recently have macroeconomists turned to industrial organization-based microeconomic modeling of exchange rate pass-through. In two recent papers, Gopinath et al. (2008) and Gopinath and Itskhoki (2008) show that pass-through rates are closely linked to two other phenomena of interest to macroeconomists: firms' choices of which currencies to price in and the frequency with which firms adjust their prices. Their results depart somewhat from the primary thrust here, as pass-through rates tell only part of the story they are interested in and therefore induce a correlation rather than full identification. Nonetheless they suggests a promising connection in modeling, rather than simply for identification of pass-through rates, between the sorts of results developed here and the understanding of exchange rate pass-through and

[25] Or more broadly if the ratio of the derivative of demand with respect to own price to its derivative with respect to another price is (nearly) constant. In this case, Assumption 1 can be invoked to identify strategic effects from pass-through rates.

other topics in the macroeconomics of international pricing.

V. Empirical Issues

Much of the motivation for the results above is the potential they hold for identifying policy relevant counter-factual from empirical data. In particular I argue that in many problems estimating pass-through is crucial to understanding the effects of various policy interventions. This can be accomplished in one of several ways, which I briefly discuss in the first subsection. It also implies that functional form assumptions which impose, rather than estimate, pass-through rates may be just as dangerous as those which impose rather than estimate elasticities. In the second subsection I show that many of the (single-product version of) common functional forms assumed for demand in applied analysis severely restrict pass-through and its slope. I briefly discuss problems this may cause in the interpretation of applied work.

A. Estimating pass-through

Pass-through can be estimated in several ways. The most natural and simplest is the regression of prices on variations in cost that are uncorrelated with shifts in demand shifts that change optimal prices. Note that this should not be mistaken for an assumption that the cost variations are uncorrelated with all shifts in demand: in a linear cost industry, multiplicative shifts in demand do not change optimal prices. These may be supplied directly by variations in exchange rates (Menon, 1995; Campa and Goldberg, 2005), technological shifts in an industry (Besank et al., 2005), input price shocks (Sijm et al., 2006) or variations in taxes across otherwise similar markets (Sidhu, 1971). A major problem with such analysis, however, is that the use of such “macro” cost shifters may undermine the exogeneity of the cost shifts, as they likely affect the costs of other firms selling related goods and therefore are likely to shift demand. This approach may therefore be more plausible (as discussed above), when a full set of industry gradients of market power are estimated simultaneously, as discussed in the previous section. An alternative is to recover pass-through by quantitative measurements of costs coupled with an instrumental variable, such as increases in the prices of other products of the same firm (Berry et al., 1995). As shown in Section III there are also cases in which pass-through rates can be recovered from properties of equilibrium along with data on, for example, mark-ups.

Pass-through may also be inferred from properties of demand through a structural estimation of demand (Kim and Cotterill, 2008). However this approach depends heavily on the functional form assumed, as it moves beyond the calibration of purely local properties of demand. In the absence of prior knowledge about functional form, a plausible structural estimation must allow for a wide variation in pass-through rates, if these rates are important to the sign and magnitude of the counterfactuals of interest. Further down this structural path is a total eschewing of the Marschakian approach, favoring instead a full demand estimation and computational simulation of counter-factuals in the estimated demand system. This approach has many advantages, allowing full quantitative (rather than sign) identification of arbitrary counter-factuals without much analytical work.

Of course in exchange for this, the computational structural approach also requires the adoption of stronger, and therefore less plausible, and often less easily interpretable assumptions about the shape of demand. In particular if this approach is to be plausible without ex-ante knowledge of the pass-through rate, it must be based on a functional form allowing a wide range of pass-through rates, independent of elasticity, at least when it is applied to problems where the comparative statics of monopoly pricing are crucial. In sequential problem, allowing a wide range of slopes for pass-through rates may also be important. Unfortunately, as the next subsection shows, most commonly used demand functions, at least in their single-product forms, severely restrict pass-through.

B. Functional forms and pass-through

Table 1 provides a taxonomy of the single product, homogeneous consumer version of some common demand classes. In the table I leave out constant pass-through (these include constant elasticity/Pareto demands) and constant mark-up (negative exponential) demands, as I discuss these extensively in the following section. I instead focus, for the most part, on common statistical distributions used to generate demand functions. The reader should understand by a probability distribution F a demand function $D(p) = A\big(1 - F[p]\big)$; because pass-through is scale-invariant, all categorization hold for arbitrary positive A. I also consider the common Almost Idea Demand System (AIDS) of Deaton and Muellbauer (1980) with constant expenditures. For a single product this can be written (over a particular range of prices as discussed below as

$$D(p) = \frac{a + b\log(p)}{p} \tag{12}$$

The range of prices over which this formula can be viewed as valid depends on whether b is positive or negative. With $b > 0$, demand behaves very strangely, sloping upwards for low enough prices. I therefore only consider the (more commonly used) case when $b \leq 0$. If $b = 0$ this is just constant elasticity demand with an elasticity of 1, which violates (strict) MUC as discussed below. With $b < 0$, formula (12) is valid only for $p \leq e^{-\frac{a}{b}}$; for prices above this, demand is 0. It is this demand function that is considered in the table below as AIDS.

Of course, there is no reason why a particular class should be uniformly, say, cost-absorbing and increasing pass-through: different parameter values and/or prices might well lead to different pass-through rates and slopes; Table 1 allows for these possibilities. However, strikingly many commonly used distributions *do* turn out to be simply classifiable according to this taxonomy. This perhaps provides a vague justification for the identifying assumptions I draw on. More persuasively they show how in problems where the level and slope of pass-

	$\rho < 1$	$\rho > 1$	Price- dependent	Parameter- dependent
ρ' $\wedge$ 0			AIDS with $b < 0$	
ρ' $\vee$ 0	Normal (Gaussian) Logistic Type I Extreme Value (Gumbel) Double Exponential Type III Extreme Value (Reverse Weibull) Weibull with shape $\alpha > 1$ Gamma with shape $\alpha > 1$		Type II Extreme Value (Fréchet) with shape $\alpha > 1$	
Price- dependent				
Parameter- dependent				
Does not globally satisfy MUC		Type II Extreme Value (Fréchet) with shape $\alpha < 1$ Weibull with shape $\alpha < 1$ Gamma with shape $\alpha < 1$		

Table 2: A taxonomy of some common demand functions

through are crucial to the economics, many commonly-used demand functions are overly restrictive, at least in the single-product case. If the distinction between cost-absorption and cost-amplification (or between increasing and decreasing pass-through) is crucial to the effects of policy in a particular problem, then assuming demand is of almost any of the common forms[26] is not an innocent simplifying assumption for computational purposes or even a questionable structural restriction. Instead, as it implies demand is cost-absorbing and has increasing pass-through or violates the monopolist's second-order conditions at some prices, it will drive the analysis of an "empirically estimated" model entirely independent of the data. Any single-product demand function with homogeneous price sensitivity estimated on the assumption that idiosyncratic valuations have one of these common distributions will lead to one of two possible situations:

1. In the case that the demand function does not globally satisfy the monopolist's second-order conditions it will lead to a prediction (entirely independent of the data) that there are some price caps (or floors) and cost levels such that the monopolist will charge a price below (above) a binding price cap (floor). In more complicated models, like two-

[26] With the exception of the Fréchet distribution with $\alpha > 1$ and AIDS . And even this is price dependent (neither is globally cost-amplifying).

sided markets or double marginalization where second-order conditions are generally more restrictive, such demand functions will lead to such strange effects more often and more severely.

2. In the case that the demand function is cost-absorbing and exhibits increasing pass-through, entirely independent of the data, the model will predict that reductions in cost will lead to a reduction in both prices in a two-sided market and that a Stackelberg leader in a quantity duopoly will produce more than a monopolist will produce, even though neither of these is not generally the case.

This has problematic implications for the standard approach to structural empirical work especially given empirical evidence (Sidhu, 1971; Sumner, 1981; Besley and Rosen, 1998a,b) that some real-world demand functions exhibit cost-amplification. Assumption that demand is Type I extreme value distributed implies that a Stackelberg leader in the double marginalization problem charges a lower mark-up than the integrated firm and that competition always decreases the price level in a two sided market, neither of which is generally true. These conclusions are entirely independent of empirical data used to estimate the demand system and thus in some cases calls into question the empirical nature of the structural exercise. However, it should be noted that my analysis here is restricted to the case of monopoly and the results are based on the assumption that the only source of consumer heterogeneity is idiosyncratic variations in valuations. If instead, as is commonly assumed in models based on the Berry et al. (1995) framework, consumers also vary in their price-sensitivity or if there are multiple goods, the effects of ancillary restrictions placed by the functional forms discussed here on pass-through rates are not known[27].

Developing demand systems with a wider range of pass-through rates can substantially alleviate this problem; I am currently writing up a first pass at this (Weyl, 2008a) for the case of differentiated Bertrand and Cournot markets and am working with Charles Fefferman on

[27] Though I am working currently on the former problem with Michal Fabinger and on the latter on my own.

developing a similar demand system that also obeys symmetry. In sequential problems, as naturally arise in applied dynamic analysis (Ericson and Pakes, 1995; Doraszelski and Pakes, 2007), allowing for a wide range of slopes of pass-through may also matter; I am currently working with Charles Fefferman to develop a single-variable demand function allowing for this.

My categorizations of demand functions into cost-absorbing and cost-amplifying come from the literature on log-concavity and log-convexity; see, for example, Bagnoli and Bergstrom (2005). The exception to this is AIDS; I am not aware of any previous classification of its log-curvature. The categorizations regarding violations of MUC and the slope of pass-through are, as far as I know, novel and are therefore stated and proved, along with the characterization of AIDS, in the following proposition.

Proposition 7. *For any shape parameter $\alpha < 1$ and for any non-degenerate value of other parameters, there exists a price in the range of the following probability distributions give rise (as described above) to demand functions such that mark-up contraction is violated at that price:*

1. *Type II Extreme Value (Fréchet) distribution with shape α*
2. *Weibull distribution with shape α*
3. *Gamma distribution with shape α*

The following distribution give rise to demand functions exhibiting increasing pass-through:

1. *Type I Extreme Value (Gumbel) distribution*
2. *Normal (Gaussian) distribution*
3. *Logisitic distribution*
4. *Double Expoential distribution*

5. *Type II Extreme Value (Fréchet) distribution with shape $\alpha > 1$*

6. *Type III Extreme Value (Reverse Weibull) distribution*

7. *Weibull distribution with shape $\alpha > 1$*

8. *Gamma distribution with shape $\alpha > 1$*

For any shape $\alpha > 1$, the Type II Extreme Value (Fréchet) distribution exhibits cost-absorbption at some prices and cost-amplifcation at others. For the single-product, constant-expenditure AIDS demand function of equation (12) above with $b < 0$, there are always some prices (yielding positive demand) at which the demand is cost-amplifying and others at which it is cost-absorbing. The demand function always exhibits decreasing pass-through.

Proof. See Appendix C. □

This taxonomy also helps reinforce the useful, if imprecise, rule of thumb[28] that (globally) cost-absorbing (log-concave) demand functions tend to have thin or no tails of consumers with high valuations, while (globally) cost-amplifying (log-convex) demand functions are characterized by thick tails. This provides some intuition for the link between pass-through and average surplus.

VI. Constant Pass-Through Demand

The identifying assumptions I employ above largely rely on the idea that pass-through does not move too dramatically over a particular range of prices and/or that it remains monotone over a range of prices. The logical extreme of the first assumption, which gives rise to a simple parametric class of demand functions, is that pass-through is constant. The logical extreme of the second assumption is that pass-through is linear (or exponential, or that market

[28]Global obedience of MUC seems to correspond roughly to uni-modality and some bound on the thickness of tails, though I do not establish or know of any formal result to this effect; investigation of this is an interesting topic for further research.

power is quadratic), which likely also gives rise to a (broader) parametric class of demand functions. I am working on developing these broader classes with Charles Fefferman. Here I focus on the narrower constant pass-through class, which allows, when pass-through rates are calibrated using exogenous cost variations, quantitatively precise estimates of counterfactuals that essentially spring from strengthening my identifying assumptions.

A. Form

This demand class was first considered by Bulow and Pfleider (1983)[29]. For cost-absorbing demand, it takes the form

$$D(p) = \begin{cases} \frac{(a-p)^{\frac{\rho}{1-\rho}}}{b} & p \leq a \\ 0 & p > a \end{cases} \tag{13}$$

with $b > 0$ and usually $a > 0$. They can be seen as roughly like linear demands, raised to an arbitrary positive exponent. Log-linear (constant mark-up) demand functions are given by

$$D(p) = be^{-ap} \tag{14}$$

for some $b > 0$ and $a > 0$. The set of cost-amplifying demand functions with constant pass-through and mark-up contration are

$$D(p) = \begin{cases} \frac{(a+p)^{-\frac{\rho}{\rho-1}}}{b} & p > -a \\ \infty & p \leq -a \end{cases} \tag{15}$$

for some $b > 0$ and (generally) $a \geq 0$[30]. Note that if $a = 0$ this is exactly the set of constant elasticity demand functions. Thus the cost-amplifying constant pass-through

[29]Though they did not consider cost-amplifying constant pass-though demand functions other than constant elasticity demand, did not discuss the fact that these demands have linear inverse hazard rates and linear monopoly pricing solutions and did not consider their average surplus properties.

[30]Otherwise demand can be infinite at strictly positive prices, which is a bit difficult to interpret, though perhaps not crazy in some applications where the cost is above this price.

demand functions are a generalization of the constant elasticity demand functions. This shows why linear demand and constant elasticity demand are both so easy to work with in monopolist pricing problems: both exhibit constant pass-through.

B. Useful properties

This class of demand functions has a number of useful properties. It is the set of demand functions which gives rise to linear monopoly pricing problems in cost and demand shifters. It therefore offers a useful set of examples to solve by hand, with one more degree of freedom than either linear demand or constant elasticity demand. Furthermore it allows the complete range of pass-through rates and elasticities independent of these pass-through rates, implying that if this class of demand functions is used to investigate an applied problem where the level of the pass-through rate and of elasticity are crucial, the assumption that demand lies in this class will not (immediately) bias the analysis. Finally, it also follows the logic of the relationship between average surplus and pass-through to its logical extreme.

Proposition 8. *If demand has constant pass-through then in the range of the demand function*

$$\overline{V}(p) = \rho\mu(p) = a + (\rho - 1)p$$

for some $a \in \mathbb{R}$.

Proof. If pass-through is constant, then by equation (7) $\overline{V}(p) = \rho\mu$. Calculating μ is trivial algebra given the functional forms above. □

One thing that should be noted about the constant pass-through demands is that one needs to treat them with a bit of care in order to interpret them as generated by probability distributions of idiosyncratic consumer valuations. The difficulty is that all of these demands are unbounded as price falls. In order to avoid this, one must assume that for sufficiently low prices demand stops rising as prices fall. Given that this can be done at arbitrary low prices (or in the case of cost-amplifying demands, arbitrarily close to the explosive price), probability distributions can approximate constant pass-through demands arbitrarily well.

Finally, it is worth noting that this class of demand functions has exactly the same number of degrees of freedom as typical statistical distributions used to generate demand functions, like the Logistic, Normal or Type I Extreme Value: it has a location parameter and, in place of a scale parameter, a pass-through parameter. Here the second parameter (rather than being used for the statistically, but not economically, intuitive notion of spread of consumer preferences) is instead used to allow freedom in pass-through rates.

VII. Conclusion

Understanding of pass-through, its applications and its identifying power is still at an early stage. This section is therefore devoted to discussing a few potential directions for future research. Because the empirical potential of the framework developed thus far was a dominant theme above, I here focus on potential theoretical extensions.

One promising direction is the further relaxation, formalization and clarification of the identifying assumptions discussed here. One natural direction would be to more thoroughly investigate the intuitive statistical foundations of these assumptions. For example, it may not be necessary to assume that pass-through is always on the same side of some barrier in order to achieve identification with high probability; it may be sufficient to assume that the observed pass-through rates in a regression of price on exogenous cost variation are randomly sampled from the set of pass-through rates over the relevant range. Then, in conjunction with an extension of the results here to a version with average pass-through rates, it might be possible to achieve identification with high probability based on weaker assumptions. Similarly, it would seem reasonable that our belief in the plausibility of identification of demand as cost-absorbing would grow the more cost-absorbing we observed demand to be. Therefore the interaction of the flavor of above results with explicit statistical analysis would be helpful.

Another extension along these lines would be the relaxation of the substantive economic assumptions that provide much of the identifying power of the models above. Because the approach here helps strip away many of the ancillary functional form assumptions typically used for identification, it helps reveal the identifying role played by basic economic assumptions in practical problems. This leads to natural questions: would the identification strategies proposed above still work if firms were uncertain about demand? Would they work if firms were not only uncertain, but had different priors? What if the firms did not know one another's costs? What if corporate finance imperfections exist within the firms? Is the exogenous cost variation technique of pass-through identification more or less robust than the direct use of micro data to relaxing these assumptions?

A perhaps more direct extension than any of these is simply the generalization of the results of Sections II, III and IV to cases of non-linear costs. My work here assumes constant marginal cost and focuses on the curvature of demand. Adding to this an analysis of identification with non-linear costs is crucial for realistic applied work.

Finally and further afield, I suspect that pass-through may have applications to a number of problems related to those discussed here. It is well known that auction theory is closely connected to monopoly pricing problems (Bulow and Roberts, 1989). I therefore wonder whether assumptions about pass-through rates might be useful to the empirical investigation of auctions. Furthermore, given that MUC plays a crucial role, as it is equivalent to the so-called "regular" case, in the theory of optimal auctions (Myerson, 1981), I wonder whether other properties of the pass-through rate might be useful in structuring preferences in auction theory. More broadly, unit-less measures of sharpness of definition of optimization problems are almost certainly useful parameters in studying the comparative statics of many optimization problems beyond the monopoly pricing problem, so long as shift parameters are "cost-like" in moving the effective level of the choice variable.

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A Proof of Proposition 1

The monopolist's first-order condition, again, is

$$\mu(p) - p + c = 0$$

If this is decreasing in p then the first-order condition is sufficient for optimization. Clearly MUC guarantees this. Furthermore if a floor or ceiling is imposed above (below) the unconstrained optimum, the monopolist will always charge this price as for every price below (above) $p^\star$, the unconstrained optimum, the derivative of profits with respect to price is positive (negative).

Conversely suppose that demand does not satisfy weak MUC. Note that for some c (possibly negative), any positive value of $p - c$ can be achieved for any chosen $p \in \mathbb{R}_+$. Thus for any point on $\mu(p)$ there exists some cost c such that $p - c$ intersects that point. Taking the second derivative of the monopolist's profit function with respect to price yields

$$2D'(p) + D''(p)(p - c) \tag{16}$$

Suppose one is at a solution to equation (1) where $\mu' > 1$, which from above can happen if there is even a single point with $\mu' > 1$. Then expression (16) becomes

$$2D'(p) + D''(p)\mu(p)$$

This expression is strictly positive if

$$\frac{D''(p)D(p)}{\left(D'(p)\right)^2} > 2$$

But

$$\mu' = \frac{D''(p)D(p)}{\left(D'(p)\right)^2} - 1 \tag{17}$$

And thus the second derivative of the monopolist's objective function is strictly positive if $\mu' > 1$ at this point, implying that this point is a local minimum and therefore clearly not an optimum. Let this point of intersection be $\tilde{p}$ and let the monopolist's unconstrained optimal price be $p^\star$ assuming this exists as the proof states. Either $\tilde{p} < p^\star$ or $\tilde{p} > p^\star$. Assume for the moment the first case. Then if a price ceiling is put on the monopolist at $\tilde{p}$ then the monopolist will always charge a price strictly below $\tilde{p}$ as she is prohibited from charging above it and $\tilde{p}$ is a local minimum of profits, so she can only do better by charging below it. The result in the case that $\tilde{p} > p^\star$ follows by the same logic.

B Proof of Proposition 3

First I analyze mark-ups, then profits. In all cases I use ranking of market power functions, as above, to rank mark-ups.

Consider the comparison of the Nash mark-ups $m^\star$ to the Integrated mark-up $m_I^\star$. Compare the RHS of equation (8) to that of (1). Under cost-absorption the second is greater for any input value of mark-up than the first as in this case m is decreasing and the only difference between them is that the first has added into its argument the strictly positive other-firm mark-up; the expressions are equal under constant mark-up demand; and the first is always greater under cost-amplification.

Now consider the comparison of $m_D^\star$ and $m_U^\star$. At equilibrium the arguments of the RHS of equations (10) and (11) must be identical. Therefore at equilibrium the second is higher if $\mu' < 0$ at this argument, equal if $\mu' = 0$ at this point and less if $\mu' > 0$ at this point, given that $m > 0$.

To compare $m_N^\star$ and $m_S^\star$ double equation (8) to obtain

$$m_N^\star = 2\mu(m_N^\star + c_I) \tag{18}$$

and sum together equations (10) and (11) to obtain

$$m_S^\star = \big(2 - \mu'[m_S^\star + c_I]\big)\mu(m_S^\star + c_I) \tag{19}$$

These are ranked (for any input m now rather than merely at equilibrium) in precisely the same way and for the same reason as the RHS of (10) and (11). This gives the desired comparison of $m_N^\star$ and $m_S^\star$. The comparison of these to $m_I^\star$ is already well-known, but can be seen here by the fact that $2 - \mu' > 1$ as $\mu' < 1$ by MUC.

Now compare $m^\star$ to $m_D^\star$ and $m_U^\star$. Consider the three cases:

1. Cost absorption: Now it is possible that $m^\star \geq m_U^\star > m_D^\star$, $m_U^\star > m^\star > m_D^\star$ or $m_U^\star > m_D^\star \geq m^\star$. I claim the middle case is true. To demonstrate this I prove by contradiction of the other two cases

   - Suppose $m^\star \geq m_U^\star > m_D^\star$. Then clearly $2m^\star > m_U^\star + m_D^\star$. Thus the RHS of equation (9) is great than the RHS of equation (10) as $\mu' < 0$; therefore $m^\star < m_D^\star$ which is a contradiction.
   - Suppose $m_U^\star > m_D^\star \geq m^\star$. Then $2m^\star < m_U^\star + m_D^\star$ and, by the same logic (reversed) as in the other case, $m^\star > m_D^\star$, which is a contradiction.

2. Constant mark-up: Now market power is constant so the RHS of (9) is always identical to that of (10) and (11). Thus $m_U^\star = m_D^\star = m^\star$.

3. Cost amplification: Now it is possible that $m^\star > m_D^\star > m_U^\star$, $m_D^\star \geq m^\star > m_U^\star$ or $m_D^\star > m_U^\star \geq m^\star$. The second two cases lead to contradictions:

   - Suppose $m_D^\star \geq m^\star > m_U^\star$. Then for any value of m put in the place of $m_D^\star$ on the RHS of equation (10) this must be less than the same value entered into equation

(8) if $m^j = m^\star$. Thus $m_D^\star < m^\star$, which is a contradiction.

- Suppose that $m_D^\star > m_U^\star \geq m^\star$. Then clearly $m_N^\star < m_S^\star$, contradicting my ranking of these in the cost amplification case.

In either the strictly cost-amplifying or strictly cost-absorbing case, $m^\star \neq m_U^\star$. Because Upstream could always choose to charge $m^\star$ and get a reaction of $m^\star$ from Downstream, his behavior must be preferable and therefore $\pi_U^\star > \pi^\star$ (unless there is constant mark-up, in which case $\pi_U^\star = \pi_D^\star = \pi^\star$ as all mark-ups are the same). Because they face the same demand but Downstream gets a higher mark-up in the cost amplifying case, here it must be that $\pi_D^\star > \pi_U^\star$. Finally because $m_S^\star > m_N^\star > m_I^\star$ and MUC ensures a first-order solution, it must be that total industry profits are greater under the Nash than the Stackelberg organization; thus the fact that $\pi_U^\star > \pi^\star \implies \pi^\star > \pi_D^\star$.

C Proof of Proposition 7

Before considering individual probability distributions, note that for any probability distribution of the form $\tilde{F}(x; m, \sigma) = F\left(\frac{x-m}{\sigma}\right)$, $\tilde{F}(\cdot; m, \sigma)$ will exhibit globally increasing (decreasing) pass-through for any $\sigma > 0$ and any real m if and only if F exhibits globally increasing (decreasing) pass-through. To see this note that the pass-through rate for a given price for $\tilde{F}$ is

$$\tilde{\rho}(x; m, \sigma) = \frac{1}{2 + \frac{\left(1-\tilde{F}[x;m,\sigma]\right)\tilde{f}'(x;m,\sigma)}{\left(f[x;m,\sigma]\right)^2}} = \frac{1}{2 + \frac{\left(1-F[z]\right)\frac{f'(z)}{\sigma^2}}{\frac{\left(f[z]\right)^2}{\sigma^2}}} = \frac{1}{2 + \frac{\left(1-F[z]\right)f'(z)}{\left(f[z]\right)^2}} = \rho(z)$$

where $\tilde{f}$ and f are the density functions of $\tilde{F}$ and F respectively, $z \equiv \frac{x-m}{\sigma}$ and ρ is the pass-through rate of F. Thus as z is clearly a positive monotone transformation of x it is order-preserving and $\tilde{\rho}$ is globally increasing (decreasing) for any $m \in \mathbb{R}$ and $\sigma > 0$ if and only if ρ is globally increasing. This is also obviously true of other properties of pass-through and of the slope of market power. This is useful as many of the probability distributions I consider below have scale and position parameters that this fact allows me to neglect.

I begin by considering the first part of the proof, that for any shape parameter $\alpha < 1$ the Fréchet, Weibull and Gamma distributions with shape α violate MUC at some price. Let me consider each in turn.

1. Type II Extreme Value (Fréchet) distribution: Up to scale and position (USP) this distribution is $F(x) = e^{-x^{-\alpha}}$. Simple algebra shows that

$$\mu'(x) = \frac{(e^{x^{-\alpha}} - 1)x^{\alpha}(1+\alpha) - e^{x^{-\alpha}}\alpha}{\alpha}$$

   As $x \to \infty$ and therefore $x^{-\alpha} \to 0$ (as shape is always positive), $e^{x^{-\alpha}}$ is well-approximated by its first-order approximation about 0, $1 + e^{x^{-\alpha}}$. Therefore the limit of the above expression is the same as that of

$$\frac{x^{-\alpha}x^{\alpha}(1+\alpha) - e^{x^{-\alpha}}\alpha}{\alpha} = \frac{1+\alpha - e^{x^{-\alpha}}\alpha}{\alpha} \to \frac{1}{\alpha}$$

   as $x \to \infty$. Clearly this is > 1 for $\alpha < 1$ so that for sufficiently large x MUC is violated.

2. Weibull distribution: USP this distribution is $F(x) = e^{-x^{\alpha}}$. Again simple algebra yields:

$$\mu'(x) = \frac{1-\alpha}{\alpha x^{\alpha}}$$

   Clearly for any $\alpha < 1$ as $x \to 0$ this expression goes to infinity, so that for sufficiently large x MUC is violated.

3. Gamma distribution: USP this distribution is $F(x) = \frac{\mu(\alpha,x)}{\Gamma(\alpha)}$ where $\mu(\cdot,\cdot)$ is the lower

incomplete Gamma function and $\Gamma(\cdot)$ is the complete Gamma function (note that here $\mu'(\cdot)$ is the derivative of market power and not a lower incomplete Gamma function):

$$\mu'(x) = \frac{e^x(1-\alpha+x)\Gamma(\alpha,x)}{x^\alpha} - 1$$

By definition, $\lim_{x\to 0}\Gamma(\alpha,x) = \Gamma(\alpha) > 0$ so

$$\lim_{x\to 0}\mu'(x) = +\infty$$

as $1-\alpha > 0$ for $\alpha < 1$. Thus clearly for small enough x, the Gamma distribution with shape $\alpha < 1$ violates MUC.

I now turn to the categorization of demand functions as increasing or decreasing pass-through. As price always increases in cost, this can be viewed as either pass-through as a function of price or pass-through as a function of cost.

1. Normal (Gaussian) distribution: USP this distribution is given by $F(x) = \Phi(x)$, where Φ is the cumulative normal distribution function. Simple algebraic computations show that for this distribution

$$\rho'(x) = \frac{\big(1-\Phi[x]\big)(1+x^2)\sqrt{2\pi}e^{\frac{x^2}{2}} - x}{\Big(e^{\frac{x^2}{2}}\sqrt{2\pi}x\big[1-\Phi(x)\big]-2\Big)^2} \tag{20}$$

This has the same sign as

$$\big(1-\Phi[x]\big)(1+x^2)\sqrt{2\pi}e^{\frac{x^2}{2}} - x \tag{21}$$

It is well known that

$$\frac{x}{\sqrt{2\pi}(1+x^2)e^{\frac{x^2}{2}}} < 1-\Phi(x)$$

so this expression (21) is greater 0.

2. Logistic distribution: USP this distribution is $F(x) = \frac{e^x}{1+e^x}$. Again simple algebra yields

$$\rho'(x) = \frac{e^x}{(1+e^x)^2} > 0$$

Thus the logistic distribution exhibits increasing pass-through.

3. Type I Extreme Value (Gumbel) distribution : USP this distribution has two forms. For the minimum version it is $F(x) = 1 - e^{-e^x}$. Simple algebra shows that for this distribution

$$\rho'(x) = \frac{e^x}{(1+e^x)^2}$$

Note that this is the same as for the logistic distribution; in fact the pass-through rates for the Gumbel minimum distribution are identical to the logistic distribution.

For the maximum version it is $F(x) = e^{-e^{-x}}$. Again algebra yields

$$\rho'(x) = \frac{e^{-x}\left(e^{e^{-x}}[1+e^x] + e^{2x}[e^{e^{-x}} - 1]\right)}{\left(1 + e^{e^{-x}} + e^x - e^{e^{-x}+x}\right)^2}$$

But clearly $e^{-x} > 0$ so $e^{e^{-x}} > 1$ an therefore the numerator and the entire expression is greater than 0 and the Gumbel distribution exhibits increasing pass-through.

4. Double exponential distribution: USP this distribution is

$$F(x) = \begin{cases} 1 - \frac{e^{-x}}{2} & x \geq 0 \\ \frac{e^x}{2} & x < 0 \end{cases}$$

For $x > 0$, $\rho = 1$ (so in this range pass-through is not strictly increasing). For $x < 0$

$$\rho'(x) = \frac{2e^x}{(2+e^x)^2} > 0$$

So the Double Exponential distribution exhibits globally weakly increasing pass-through, strictly increasing for prices below the mode. The pass-through rate for this distribution is $\frac{e^x}{2+e^x}$ as opposed to $\frac{e^x}{1+e^x}$ for Gumbel and Logistic...however these are very similar, again pointing out the similarities among pass-through functions assumed by common demand forms.

5. Type II Extreme Value (Fréchet) distribution with shape $\alpha > 1$: From the formula earlier it is easy to show that the derivative of the pass-through rate is

$$\rho'(x) = \frac{x^{-(1+\alpha)}\alpha^2\Big([1+\alpha]\big[x^{2\alpha}(e^{x^{-\alpha}}-1)+e^{x^{-\alpha}}x^{\alpha}\big]+\alpha e^{x^{-\alpha}}\Big)}{\big(\alpha[1+e^{x^{-\alpha}}]-[e^{x^{-\alpha}}]-1]x^{\alpha}(1+\alpha)\big)^2} > 0$$

as $x > 0$ in the range of this demand function and $e^x > 1$ for positive x. Thus this distribution, as well, exhibits increasing pass-through.

6. Type III Extreme Value (Reverse Weibull) distribution: UPS this distribution is $F(x) = e^{-(-x)^{\alpha}}$. Algebra shows

$$\rho'(x) = (-x)^{\alpha-1}\alpha^2 \frac{1-\alpha+e^{(-x)^{\alpha}}\Big([1-\alpha]\big[(-x)^{\alpha}-1\big]+[-x]^{2\alpha}\alpha\Big)}{\Big(\alpha-1+[-x]^{\alpha}\alpha+e^{[-x]^{\alpha}}\Big[1+\big([-x]^{\alpha}-1\big)\alpha\Big]\Big)^2}$$

which has the same sign as

$$1-\alpha+e^{(-x)^{\alpha}}\Big([1-\alpha]\big[(-x)^{\alpha}-1\big]+[-x]^{2\alpha}\alpha\Big) \tag{22}$$

Note that the limit of this expression as $x \to 0$ is

$$1 - \alpha - (1 - \alpha) = 0$$

and its derivative is

$$\frac{e^{(-x)^{\alpha}}(-x)^{2\alpha}\alpha\big(1 + \alpha + [-x]^{\alpha}\alpha\big)}{x}$$

which is clearly strictly negative for $x < 0$. Thus expression (22) is strictly decreasing and approaches 0 as x approaches 0. It is therefore positive for all negative x, showing that again in this case $\rho' > 0$.

7. Weibull distribution with shape $\alpha > 1$: As with the Fréchet distribution algebra from the earlier formula shows

$$\rho'(x) = \frac{x^{\alpha-1}(\alpha - 1)\alpha^2}{(\alpha - 1 + x^{\alpha}\alpha)^2}$$

which is clearly positive for $\alpha > 1$ as the range of this distribution is positive x. Thus the Weibull distribution with $\alpha > 1$ exhibits increasing pass-through.

8. Gamma distribution with shape $\alpha > 1$: Again using the formula calculated above for μ', a bit of algebra and a derivative yield:

$$\rho'(x) = \frac{\alpha - 1 - x + \frac{e^x}{x^{\alpha}}\big(x^2 - 2x[\alpha - 1] + [\alpha - 1]\alpha\big)\Gamma(\alpha, x)}{x\left(\frac{e^x}{x^{\alpha}}[1 + x - \alpha]\Gamma[\alpha, x] - 2\right)^2}$$

where $\Gamma(\cdot, \cdot)$ is the upper incomplete Gamma function. Because the Gamma distribution is only defined for positive x, this has the same sign as

$$\alpha - 1 - x + \frac{e^x}{x^{\alpha}}\big(x^2 + [\alpha - 2x][\alpha - 1]\big)\Gamma(\alpha, x) \tag{23}$$

A simple application of the quadratic formula, omitted here, shows that $x^2 + (\alpha - 2x)(\alpha - 1) > 0$ for $\alpha > 1$. Therefore so long as $x \leq \alpha - 1$ this is clearly positive. On the other hand when $x > \alpha - 1$ the proof depends on the following result of Natalini and Palumbo (2000):

Theorem (Natalini and Palumbo, 2000). *Let a be a positive parameter, and let q(x) be a function, differentiable on* $(0, \infty)$ *, such that* $lim_{x \to \infty} x^{\alpha} e^{-x} q(x, \alpha) = 0$. *Let*

$$T(x, \alpha) = 1 + (\alpha - x)q(x, \alpha) + x\frac{\partial q}{\partial x}(x, \alpha)$$

If $T(x, \alpha) > 0$ *for all* $x > 0$ *then* $\Gamma(\alpha, x) > x^{\alpha} e^{-x} q(x, \alpha)$.

Letting

$$q(x, \alpha) \equiv \frac{x - (\alpha - 1)}{x^2 + (\alpha - 2x)(\alpha - 1)}$$

$$T(x, \alpha) = \frac{2(\alpha - 1)x}{\left(\alpha^2 + x[2 + x] - \alpha[1 + 2x]\right)^2} > 0$$

for $\alpha > 1, x > 0$. So $\Gamma(\alpha, x) > x^{\alpha} e^{-x} q(x, \alpha)$. Thus expression (23) is strictly greater than

$$\alpha - 1 - x + x - (\alpha - 1) = 0$$

as, again, $x^2 + (\alpha - 2x)(\alpha - 1) > 0$. Thus again $\rho' > 0$.

This establishes the second part of the proposition. Turning to my final two claims, simple algebra shows that the pass-through rate for the Fréchet distribution is

$$\rho(x) = \frac{\alpha}{\alpha + e^{x^{-\alpha}}\big(\alpha - x^{\alpha}[1+\alpha]\big) + x^{\alpha}(1+\alpha)} = \frac{\alpha}{\alpha(1+e^{x^{-\alpha}}) - (e^{x^{-\alpha}} - 1)x^{\alpha}(1+\alpha)}$$

Note for any $\alpha > 1$ this is clearly continuous in $x > 0$. Now consider the first version of the expression. Clearly as $x \to 0$, $x^{\alpha} \to 0$ and $e^{x^{-\alpha}} \to \infty$ so the denominator goes to ∞ and the expression goes to 0. So for sufficiently small $x > 0$, $\rho(x) < 1$ and demand is cost-absorbing. On the other consider the second version of the expression. Its denominator is

$$\alpha(1+e^{x^{-\alpha}}) - (e^{x^{-\alpha}} - 1)x^{\alpha}(1+\alpha)$$

By the same argument as above with the Fréchet distribution the limit of the above expression as $x \to \infty$ is the same as that of

$$\alpha(1+e^{x^{-\alpha}}) - x^{-\alpha}x^{\alpha}(1+\alpha) = \alpha(1+e^{x^{-\alpha}}) - 1 - \alpha \to \alpha - 1$$

as $x \to \infty$. Thus

$$\lim_{x\to\infty} \rho(x) = \frac{\alpha}{\alpha - 1} > 1$$

and thus for sufficiently large x and any $\alpha > 1$, this distribution exhibits cost-amplifcation.

Finally, consider my claim about AIDS. First note that for this demand function

$$\mu'(p) = 1 + \frac{b\big(a - 2b + b\log[p]\big)}{\big(a - b + b\log[p]\big)^2} < 1$$

as $b < 0$ and $p \leq e^{-\frac{a}{b}} < e^{2-\frac{a}{b}}$.

$$\rho(p) = -\left(\frac{a}{b} + \log[p] + \frac{b}{a - 2b + b\log[p]}\right)$$

This is less than 1 iff

$$a^2 + 2ab\big(\log[p] - 2\big) + b^2\Big(1 + \log[p]\big[\log(p) - 2\big]\Big) < b^2\big(2 - \log[p]\big) - ab$$

or

$$\big(a + b\log[p]\big)^2 - b^2\big(\log[p] + 1\big) < 0$$

Clearly as $p \to 0$ the second term is positive; therefore there is always a price at which $\rho(p) > 1$. On the other hand as $p \to e^{-\frac{a}{b}}$ this expression goes to

$$0 - b^2\left(1 - \frac{a}{b}\right) = b(a - b) < 0$$

Thus there is always a price at which $\rho(p) < 1$.

$$\rho'(p) = \frac{b^2 - \big(a - 2b + b\log[p]\big)^2}{p\big(a - 2b + b\log[p]\big)^2}$$

which has the same sign as

$$b^2 - \big(a - 2b + b\log[p]\big)^2 < b^2 - (2b)^2 = -3b^2 < 0$$

Thus AIDS exhibits decreasing pass-through.