SIGNALING UNDER IMPERFECT COMPETITION: QUALITY ESCALATION AND VEBLEN EFFECTS

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Abstract. We analyze a duopoly model where firms sell conspicuous goods to horizontally- and vertically-differentiated consumers. These consumers care about both the intrinsic quality of the goods they purchase as well as the social status conveyed by these goods (namely, the social inference of their hidden wealth based on their purchase). Firms offer non-linear price and quality schedules that, in effect, screen consumers using a combination of two instruments: skewed markups (or "Veblen effects") and upward-distortions in quality. Our work differs from previous literature in that Veblen effects and quality distortions simultaneously arise, and it also provides a setting in which their interaction can be analyzed. The two screening instruments have very different welfare implications, with markups being welfare enhancing as they create an implicit market for status – mediated by the firms – that reduces the need for distortions in quality. We also show that once the forces of imperfect competition are considered, optimal corrective taxation differs significantly from standard proposals.

1. Introduction

It has long been recognized that the pursuit of social status in the form of prestige and peer recognition is a central determinant of behavior (e.g., Bentham, 1789, Veblen, 1899, Scitovsky, 1944, Duesenberry, 1949, Leibenstein, 1950, Marshall, 1962, Becker, 1974, Frank, 1985). Status is sought in a variety of ways, depending on individual skills, occupation, and surrounding social norms. Though notably absent in most academic circles, a manifestation of status-oriented behavior that stands out for its common occurrence and economic significance is the consumption of expensive goods for public display. As noted by Veblen, "In order to gain and to hold the esteem of men, wealth must be put in evidence ... [a frequent] effect of

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which is to hold the consumer up to a standard of expensiveness and wastefulness in his consumption of goods" (cited by Bagwell and Bernheim, 1996).

At first sight, the consumption of products with exaggerated characteristics and high prices may seem superfluous. However, once we recognize that conspicuous purchases typically stem from a deeply rooted need for social recognition, they become of general interest to the social sciences. Moreover, given the magnitude of these conspicuous expenditures, the associated waste of resources, and their determination in a market environment, they are also of specific interest to Economics.

In this paper, we revisit Hotelling's model of imperfect competition for the scenario in which consumers care about the goods they purchase for both their intrinsic properties as well as the social standing they convey. Our goal is to explore the impact of status-seeking behavior over the price profile and physical attributes of conspicuous goods supplied by spatially differentiated firms.

Our interest in an environment of imperfect competition stems from the fact that suppliers of conspicuous goods (such as motor vehicles, clothing, jewelry, sporting goods, and consumer electronics) commonly have some market power that arises from their brand name, but also face a number of close rivals. Thus, neither the case of perfect competition nor pure monopoly are entirely suitable. In fact, we show that under standard single-crossing preferences, only the case of imperfect competition can reproduce two phenomena commonly observed in the market for conspicuous goods: large markups and upward quality distortions.

In our model, two competing firms offer nonlinear menus of goods (i.e., product lines) to a continuous population of horizontally- *and* vertically-differentiated consumers. These menus implicitly allow consumers to purchase different levels of social status – with higher-priced goods leading to higher status. Importantly, any particular good can have a high price for two independent reasons: (1) because its intrinsic quality is high and therefore is expensive to produce, and (2) because the firm charges a high markup. As shown below, both these dimensions serve as strategic variables for the competing firms and they complement each other in nontrivial ways.

In equilibrium, high-end consumers who implicitly purchase high status pay supra-normal markups (i.e., higher than the standard Hotelling model), whereas the opposite occurs with low-end customers who end up with lower status. Overall profits, however, are not affected by the status motive. As a result, the skewed markups described above become a form of cross-subsidy among consumers, with high-end customers effectively subsidizing their low-end peers. In effect, these cross-subsidies constitute an implicit market for status in which the status race is partially settled through monetary transfers – mediated by the firms – among competing consumers. Importantly, this implicit market has a positive effect over welfare as it reduces the need to employ alternative means to settle the status competition.

Cross-subsidies, however, are not the only device employed by firms. The fact that markups are skewed means that a firm makes larger profits from its high-end consumers and therefore is eager to attract a larger fraction of these. Absent the status motive, the firm would do so by simply reducing the markup of its high-end products. This strategy, however, fails when the status motive is present: discounting a high-end product makes it accessible to lower-end consumers and therefore destroys it status appeal. Thus, the firm must recur to an alternative strategy: offering supra-normal quality levels at a price that only high types find attractive. In fact, as we show below, this argument extends to essentially all of the continuous vertical spectrum, with the implication that quality distortions are employed for all but a zero-measure subset of the population.

Two important benchmarks are useful for our analysis. The first is the case of perfect competition studied by Bagwell and Bernheim (1996) and Becker, Murphy, and Glaeser (2000). In these models, direct competition among firms drives all markups to zero (assuming single-crossing preferences). As a result, cross-subsidies across consumers are absent and status can only be obtained by high-end consumers through consumption of goods with excessive quality. This equilibrium is highly inefficient since all resources deployed in the status race are devoted to production costs. When a firm has market power, in contrast, it can use the additional instrument of cross-subsidies to induce high-end consumers to purchase their status, at least in part, through non-wasteful monetary transfers. Since this implicit market for status reduces the need for quality distortions, it expands the overall pool of surplus from which the firm draws its profits.

The second benchmark is the model of imperfect competition, also with horizontally- and vertically-differentiated consumers, studied by Rochet and Stole (2002). A special case of their model is a form of Hotelling competition similar to the one we study, but with no status motive. In fact, precisely because this status motive is absent, upward-quality distortions and cross-subsidies among consumers do not arise.

Our most subtle result concerns the firms' incentives to distort quality at different points of the vertical spectrum. Quality distortions, in effect, are used to differentially attract high-margin consumers at the high end of the spectrum *relative* to low-margin consumers at the low end. The resulting distortions have an inverted U-shape, with maximal distortions occurring at intermediate segments of the type space. Importantly, these quality distortions wane, and can disappear altogether, at the extremes of the vertical spectrum. In these extremes, markups are highly skewed and cross-subsidies become the primary form of status allocation.

Arguments for corrective taxation are commonplace in the literature on social status. Here we show that the optimal corrective tax schedule differs significantly from standard proposals that ignore forces of imperfect competition. In particular, since skewed markups – and the cross-subsidies behind them – substitute for quality distortions, they also serve as a substitute for corrective taxation. Accordingly, the need for taxation vanishes toward the extremes of the vertical spectrum where cross-subsidies become the firms' favored screening device.

In Sections 2-5 we present and analyze our model absent government intervention. In section 6 we study corrective taxation. Section 7 concludes.

2. **Model**

Consider a unit mass of consumers characterized by two types: a vertical type $\theta \in \Theta \subseteq \mathbb{R}_+$ that measures wealth, and a horizontal type $x \in [0,1]$ that measures spatial preference. We assume both types are private information and independently distributed. Let $F(\theta)$ and $G(x)$ denote the c.d.f. functions for each type, and let $f(\theta)$ and $g(x)$ denote their associated densities. Throughout, we assume $g(\cdot)$ is differentiable and symmetric around its midpoint $x = \frac{1}{2}$. We take Θ to be either a two-type set $\{\theta_L, \theta_H\}$ (section 3), or a continuous interval $[\theta_L, \theta_H]$ with a positive differentiable density $f(\cdot)$ (section 4).

Each consumer wishes to purchase a status good and can select between two supplying firms, $i \in \{A, B\}$, which are located at either extreme of the horizontal space $[0,1]$. A status good has three observable characteristics: a price $p \in \mathbb{R}$, intrinsic quality $q \in \mathbb{R}$, and brand name $i \in \{A, B\}$. In addition, the consumer of this good enjoys a social status level $s \in \Theta$, which equals the Bayesian point estimate of the consumer's vertical type θ based on the triple (p, q, i). The details of this Bayesian estimate are described in subsection 2.1 below.

The utility obtained by a consumer with types θ and x, who purchases a status good (p, q, i) and receives status s, is given by

$$\theta \cdot v(q, s) - p - T_i(x). \tag{1}$$

This expression is an extension of the quasi-linear utility functions used in the literature on non-linear pricing (e.g., Mussa and Rosen 1978, Maskin and Riley 1984, Rochet and Stole 2002). The first term $\theta \cdot v(q, s)$ represents gross utility derived from intrinsic quality q and social status s. We assume that the function v is smooth, increasing in both arguments, weakly concave in q, and has a non-negative cross partial v_{qs}. The second term means that utility is quasi-linear in money. Notice that both the marginal rate of substitution of money for quality $(\theta \cdot v_q)$, and money for status $(\theta \cdot v_s)$, are strictly increasing in the consumer's vertical type θ, which captures the notion that wealthier consumers are willing to pay more for both quality and status. The third term is a Hotelling transportation cost (Hotelling, 1929). The function $T_i(x)$ equals $t \cdot x$ if the consumer buys his good from firm $i = A$, and equals $t \cdot (1 - x)$ if he buys from firm B, where $t \in \mathbb{R}_+$ is an exogenous parameter that measures the degree of competition between the two firms.

Each firm i offers a menu of status goods (i.e., a product line) that discriminates consumers across the vertical spectrum. Expressed in direct-revelation form, a menu consists of a family of price-quality pairs $\langle p_i(\theta), q_i(\theta)\rangle_{\theta \in \Theta}$, where the pair $p_i(\theta), q_i(\theta)$ targets consumers with vertical type θ.[1] Also let $s_i(\theta)$ denote the status level associated with this pair. When designing their menus, firms must satisfy a standard truth-telling incentive constraint: for any pair of vertical types, θ, θ',

$$\theta \cdot v(q_i(\theta), s_i(\theta)) - p_i(\theta) \geq \theta \cdot v(q_i(\theta'), s_i(\theta')) - p_i(\theta'). \tag{IC}$$

This constraint says that once a consumer with vertical type θ has reached firm i, he must weakly prefer the pair $p_i(\theta), q_i(\theta)$ intended for him over any alternative pair $p_i(\theta'), q_i(\theta')$ from the firm's menu. Since the transportation cost $T_i(x)$ is additive, and must be paid regardless of the specific item chosen from the menu of firm i, it is absent from the constraint.

The marginal cost of producing each unit of a good with intrinsic quality q is given by $c(q)$, which is increasing, smooth, and convex in q. We also assume that

[1] In principle, firms could also condition their menus on the horizontal parameter x. However, given the additive nature of the transportation cost $T_i(x)$, there is no loss in restricting to menus that condition on the vertical type alone.

the function $c(\cdot)$ is the same for both firms. Following standard practice in the literature on non–linear pricing, we abstract away from fixed costs. Throughout, we assume that every consumer purchases a product from one of the two firms and that the appropriate participation constraints have slack. In other words, the relevant outside option for a consumer is to purchase from the competing firm. This assumption is without loss whenever the transportation cost parameter t is low relative to the gains from trade (i.e., the difference between values and costs).

Define $V_i(\theta) \equiv \theta \cdot v(q_i(\theta), s_i(\theta)) - p_i(\theta)$ (namely, the L.H.S. of (IC)). This *value function* $V_i(\theta)$ measures the optimized payoff (excluding transportation cost) that a consumer with vertical type θ obtains when consuming from firm i. From $V_A(\theta)$ and $V_B(\theta)$ we derive consumer demand. Assuming that any indifference is resolved in favor of firm A (a measure zero event), a consumer with types θ and x consumes from this firm if and only if

$$V_A(\theta) - T_A(x) \geq V_B(\theta) - T_B(x).$$

Since the L.H.S. of this inequality is strictly decreasing in x and the R.H.S. is strictly increasing, it follows that for each θ there exists a unique cutoff value $\widehat{x}(\theta) \in [0, 1]$ such that every consumer with $x \leq \widehat{x}(\theta)$ consumes from firm A. Accordingly, the fraction of consumers of type θ that buy from A and B are given, respectively, by the *demand functions*

$$D_A(\theta) \equiv G(\widehat{x}(\theta)) \text{ and } D_B(\theta) \equiv 1 - G(\widehat{x}(\theta)), \tag{2}$$

where $G(\cdot)$ is the c.d.f. for x. Notice that $D_A(\theta)$ and $D_B(\theta)$ depend implicitly on the value functions for both firms as well as the Hotelling parameter t.

Now define $\Omega_i(\theta) \equiv p_i(\theta) - c(q_i(\theta))$ (namely, the markup charged by firm i to consumers with vertical type θ). From $\Omega_i(\theta)$ and $D_i(\theta)$ we obtain the profits for firm i:

$$\int_{\theta \in \Theta} \{\Omega_i(\theta) \cdot D_i(\theta)\}\, dF(\theta). \tag{3}$$

The timing in the model is as follows. First, both firms simultaneously select their menus $\langle p_i(\theta), q_i(\theta) \rangle_{\theta \in \Theta}$. Next, after observing both menus, consumers simultaneously make their consumption decisions. Finally, social status is allocated and payoffs are realized.

2.1. **Social Status.**

The status level enjoyed by a consumer who selects the pair p, q from firm i is given by the posterior conditional expectation of θ based on the signal (p, q, i):

$$E[\theta \mid p, q, i] \equiv \frac{\int_{\theta\in\Theta} \theta \cdot D_i(\theta) dF(\theta)}{\int_{\theta\in\Theta} D_i(\theta) dF(\theta)}. \tag{4}$$

This expectation is based on the consumers' optimal behavior given the menus offered by the two firms, which is embedded in the demand functions $D_i(\theta)$.

We offer two interpretations as to why consumers value status. First, as modeled by Bagwell and Bernheim (1996), and Cole, Mailath, and Postlewaite (1995), consumers might have "social contacts," or peers, who take actions based on their inference of θ, with more favorable actions taken following higher inferences of θ. For instance, potential business partners or mates may be more inclined to form a match with a consumer who is believed to be wealthy. Second, as argued in the sociology literature (e.g., Bourdieu, 1984), consumers may value social admiration and deference for their own sake. Thus, in a culture or social group in which wealth is admired and the wealthy are deferred to, consumers will naturally seek to appear wealthy. Under both interpretations, status is instrumental (i.e., a means to eliciting a favorable response from peers) and therefore the assumption that it enters the utility function simply serves as a reduced-form representation for a more fundamental preference.[2]

2.2. **Full Separation and Monotonicity.**

We place two conditions on the class of menus that firms are allowed to offer.

Condition 1 (Full Separation). *For each firm i, the menu $\langle p_i(\theta), q_i(\theta)\rangle_{\theta\in\Theta}$ fully separates across vertical types, namely, for every pair of types θ, θ', either $p_i(\theta) \neq p_i(\theta')$ or $q_i(\theta) \neq q_i(\theta')$.*

Condition 1 implies that, for any θ, the status level associated with the pair $p_i(\theta)$, $q_i(\theta)$ (which, in equilibrium, is only chosen by consumers with type θ) is exactly

[2] The reader may wonder why an average consumer who wishes to signal his wealth has to bother with purchasing conspicuous goods rather than simply showing the balance of his bank account or tax return. One possibility is that the consumer seeks to signal his future expected wealth as opposed to its current level. Another possibility is that conspicuous goods are more visible and, depending on their features, less prone to counterfeiting.

equal to θ. In other words, $s_i(\theta) = \theta$ for all types. As a result, the truth telling constraint (IC) simplifies to

$$\theta \cdot v(q_i(\theta), \theta) - p_i(\theta) \geq \theta \cdot v(q_i(\theta'), \theta') - p_i(\theta') \text{ for all } \theta, \theta'. \qquad (IC\text{-}FS)$$

Conversely, this new constraint (IC-FS) automatically implies that condition 1 is satisfied.

Condition 2 (Monotonicity)**.** *For each firm i, the schedule $q_i(\theta)$ is nondecreasing in θ.*

Notice that the (IC-FS) constraint, in and of itself, implies that the composite function $v(q_i(\theta), \theta)$ is non-decreasing in θ, which is a weaker restriction than condition 2. We refer to a menu $\langle p_i(\theta), q_i(\theta)\rangle_{\theta\in\Theta}$ satisfying condition 2 as *monotonic*.

We provide the following motivation for conditions 1 and 2. Suppose that, in addition to purchasing a status good form one of the two firms, consumers have the option of burning any amount of money they desire in public. This money burning need not be literal. It could represent, for example, the purchase of additional expensive goods, visible to society, that deliver no intrinsic value to the consumer.

The possibility of burning money in public, combined with a suitable refinement concerning off-equilibrium beliefs, expands the ability of consumers with high types to separate from the rest. It turns out that once firms anticipate this separation ability, they will optimally decide to offer menus satisfying Conditions 1 and 2. We derive this result formally in Appendix A. Here we simply provide intuition for the case of two vertical types θ_L and θ_H (as shown in the appendix, a similar argument applies when types are continuous):

Consider first condition 1. Suppose a firm attempts to pool consumers with types θ_L and θ_H by offering them the same price-quality pair. If the consumers in this pool did not engage in any money burning, their status level would equal the average type in the pool. Alternatively, consumers with type θ_H could burn just enough money so that society infers that they have type θ_H rather than the pool average. Under single-crossing preferences, standard equilibrium refinements imply that the latter option is more attractive, and therefore these high-type consumers would indeed burn some money. However, the firm can preempt this response by adding whatever amount of money high-type consumers would have chosen to burn to the price of their item in the menu. In this way, all signaling occurs through the firm and profits are increased.[3]

A related argument applies for condition 2. Suppose firm i offers quality levels $q_i(\theta_L)$ and $q_i(\theta_H)$ such that $q_i(\theta_L) > q_i(\theta_H)$. Consumers with type θ_H could either select the option intended for them or, alternatively, they could consume the higher quality level $q_i(\theta_L)$ while supplementing this purchase with a sufficient amount of money burning so that they are not mistaken with types θ_L. Since preferences are single-crossing in quality, the latter option constitutes a more effective way to separate from the low types (i.e., it is less attractive for low types to imitate high types when the latter consume a higher quality level). As a result, the high-type consumers would simply not accept the lower quality $q_i(\theta_H)$ that the firm had originally intended for them.

2.3. **Preliminaries.**

An *equilibrium* is a pair of monotonic menus $\langle p_i(\theta), q_i(\theta)\rangle_{\theta\in\Theta}$ satisfying $(IC\text{-}FS)$ such that, given optimal consumer behavior, no firm can increase its profits by unilaterally deviating to an alternative monotonic menu satisfying $(IC\text{-}FS)$.

In what follows, we focus on symmetric equilibria in which both firms offer the same menus and cover half of the consumers with any given vertical type. Given the assumption that both firms are symmetric in terms of their production costs, location, and consumer distributions, restricting to symmetric equilibria is without loss. We denote a symmetric-equilibrium menu by $\langle p^*(\theta), q^*(\theta)\rangle_{\theta\in\Theta}$.

For future reference, define

$$S(q,\theta) \equiv \theta \cdot v(q,\theta) - c(q).$$

This function measures the social surplus, excluding transportation cost, that is generated when a consumer with vertical type θ consumes quality q while enjoying status level θ. Accordingly, define the first-best quality for type θ, denoted by $q^{FB}(\theta)$, as the value of q that maximizes $S(q,\theta)$. We assume that $q^{FB}(\theta)$ is interior for all types. It follows that $S_q(q^{FB}(\theta),\theta) = 0$ for all types, and $q^{FB}(\theta)$ is continuous and strictly increasing.

Provided transportation costs are small, the unique socially-efficient allocation (up to a zero measure subset) prescribes $q^{FB}(\theta)$ for every consumer of type θ, with

[3] Bagwell and Bernheim (1996) present a related argument, the difference is that competitors offer the separating signal as apposed to this signal being money burning.

all consumers with horizontal types $x \leq \frac{1}{2}$ purchasing from firm A, and the rest purchasing from firm B.

Before we proceed, following standard practice, it useful to restate the firms' problem directly in terms of the value $V_i(\theta)$ offered to each type of consumer. From the definition $V_i(\theta) \equiv \theta \cdot v(q_i(\theta), s_i(\theta)) - p_i(\theta)$ it follows that, conditional on $q_i(\theta)$ there is a one-to-one mapping between $p_i(\theta)$ and $V_i(\theta)$. As a result, firms can simply choose menus of the form $\langle V_i(\theta), q_i(\theta)\rangle_{\theta\in\Theta}$ while sending prices to the background.

Stated in terms of $V_i(\theta)$, the incentive constraint (IC-FS) takes a simple form. For the case of two types, it becomes

$$v(q_i(\theta_H), \theta_H) \geq \frac{V_i(\theta_H) - V_i(\theta_L)}{\theta_H - \theta_L} \geq v(q_i(\theta_L), \theta_L), \tag{5}$$

where the first and second inequalities correspond, respectively, to the upward and downward truth-telling constraints.

On the other hand, when types are continuous, the envelope theorem (e.g., Milgrom and Segal, 2002) implies that, for any monotonic schedule $q_i(\cdot)$, constraint (IC-FS) is equivalent to the condition that

$$V_i'(\theta) = v(q_i(\theta), \theta) \text{ for all } \theta, \tag{6}$$

where the R.H.S. of this equality equals the direct derivative of $V_i(\theta)$ with respect to the consumers true type alone.[4]

Constraints (5) and (6) capture a simple but important fact: the underlying consumer valuations $v(q_i(\theta), \theta)$ place an upper and lower bound on the rate at which $V_i(\theta)$ can change across types. All our results are driven by the interaction between these bounds and the horizontal competition across firms.

We can now state firm i's problem more compactly:

$$\max_{V_i(\cdot),\, q_i(\cdot)} \int_{\theta\in\Theta} \underbrace{[S(q_i(\theta), \theta) - V_i(\theta)]}_{\Omega_i(\theta)} \cdot D_i(\theta) dF(\theta) \tag{$P1$}$$

$$s.t.\ q_i(\cdot) \text{ is nondecreasing, and}$$

$$(IC\text{-}FS),$$

[4]Strictly speaking, (IC-FS) only requires that $V_i(\cdot)$ is absolutely continuous and $V_i'(\theta) = v(q_i(\theta), \theta)$ over a full-measure subset of types, which is a weaker requirement than condition (6). However, imposing (6) is without loss because altering the derivative $V_i'(\cdot)$ over a zero-measure subset of types has no impact over $V_i(\cdot)$, and therefore has no impact over the firm's objective.

where the constraint (IC-FS) corresponds to either (5) or (6) depending on the number of vertical types, and the markup $\Omega_i(\theta)$ has been directly expressed as a function of the firm's decision variables $V_i(\theta)$ and $q_i(\theta)$ using the accounting identity

$$S(q_i(\theta), \theta) \equiv V_i(\theta) + \Omega_i(\theta). \tag{7}$$

3. **Two Vertical Types**

We begin with the simplest case of two types, $\theta \in \{\theta_L, \theta_H\}$. This is a useful starting point because it allows us to introduce the basic driving forces of the model and present some initial results. In section 4 below, we study the richer case of the continuum.

In the two-type case, the firms' problem specializes to

$$\max_{V_i(\cdot),\, q_i(\cdot)} \sum_{\theta \in \{\theta_L, \theta_H\}} \underbrace{[S(q_i(\theta), \theta) - V_i(\theta)]}_{\Omega_i(\theta)} \cdot D_i(\theta) f(\theta) \tag{$P2$}$$

$$s.t.$$

$$v(q_i(\theta_H), \theta_H) \geq \frac{V_i(\theta_H) - V_i(\theta_L)}{\theta_H - \theta_L} \geq v(q_i(\theta_L), \theta_L), \tag{IC-FS}$$

where $f(\theta) > 0$ represents the proportion of consumer with type θ, and constraint (IC-FS) has been specialized to the case of two types using (5). Notice that the monotonicity constraint (i.e., condition 2) has been omitted. The reason is that, when there are only two vertical types, as will become clear below, this monotonicity constraint does not bind.

As a benchmark, consider the hypothetical case in which the incentive constraint is ignored. This case represents an environment in which the firms directly observe θ and discriminates accordingly. Remark 1 characterizes the equilibrium.

Remark 1. *Consider the relaxed environment in which (IC-FS) is ignored. In equilibrium, firms offer the fist-best quality $q^{FB}(\theta)$ for each type θ. In addition, they charge a constant markup across types equal to the inverse semi-elasticity of demand:*

$$\Omega_i(\theta) = \frac{t}{g'(\frac{1}{2})} \equiv \tau.$$

Proof. See appendix B. □

This result replicates the standard Hotelling equilibrium for each vertical type. Once (IC-FS) is ignored, firms optimally offer the first best-quality levels $q^{FB}(\theta)$ for each θ because this quality level uniquely maximizes the surplus function $S(q_i(\theta), \theta)$

in their objective. On the other hand, when selecting $V_i(\theta)$, firm i faces a trade-off between the fraction of type-θ consumers it attracts (which increases with $V_i(\theta)$) and the markup it extracts from each of these consumers (which decreases with $V_i(\theta)$). This trade-off is resolved by setting $V_i(\theta)$ such that the resulting markup is equal to the inverse semi-elasticity of demand – denoted here by the exogenous parameter τ. We refer to τ as the *Hotelling markup*.

Definition 1. *From this benchmark, we define the Veblen effect imposed by firm i over type θ as the supra-normal markup $\Omega_i(\theta) - \tau$.*

As show below, when consumers place high value on status, Veblen effects become an important component of the firms' strategy.

We now proceed with some preliminary results that simplify the original constrained problem $(P2)$. Lemma 1 shows that firms can safely ignore the downward incentive constraint (namely, the second inequality in $(IC\text{-}FS)$).

Lemma 1. *In equilibrium, the downward incentive constraint in $(IC\text{-}FS)$ does not bind.*

Proof. See Appendix B. □

Remark 1 provides intuition for why this is the case. Starting from the relaxed allocation, high types have two reasons not to imitate low types. First, low types are offered a quality strictly lower than the high-types' ideal level with no corresponding reduction in markup. Second, a deviating high type would lose social status.

Once the downward constraint is ignored, the low-type quality only enters problem $(P2)$ through the surplus function $S(q_i(\theta_L), \theta_L)$ in the objective. As a result, low types are always offered first-best quality $q^{FB}(\theta_L)$.

Remark 2. *In equilibrium, low types θ_L receive quality $q^{FB}(\theta_L)$.*

The upward incentive constraint, in contrast, potentially binds. The reason is a low-type consumer who pretends to be a high type enjoys higher status, and this benefit may offset the loss he experiences from consuming the higher price-quality level intended for high types.

Lemma 2. *In equilibrium, the upward incentive constraint in $(IC\text{-}FS)$ binds if and only if*

$$\underbrace{\theta_L v(q^{FB}(\theta_H), \theta_H) - \theta_L v(q^{FB}(\theta_H), \theta_L)}_{A} \;>\; \underbrace{S(q^{FB}(\theta_L), \theta_L) - S(q^{FB}(\theta_H), \theta_L)}_{B}. \quad (8)$$

Proof. See Appendix B. □

Starting from the relaxed allocation, term A measures the benefit that a deviating low-type consumer derives from status θ_H rather than θ_L. Term B represents the degree to which the high-type quality $q^{FB}(\theta_H)$ differs from the low-type ideal $q^{FB}(\theta_L)$ as measured by the surplus function $S(q, \theta_L)$.

Term A exceeds B when the marginal utility of status v_s is large. Term A also exceeds B when the two types θ_L and θ_H are sufficiently close to each other. In particular, as the distance $\theta_H - \theta_L$ approaches zero, the deviating low-type consumer experiences a first-order benefit from higher status, but only a second-order loss from consuming the higher quality $q^{FB}(\theta_H)$ (since this quality is now only marginally higher than the low-type ideal).[5]

In contrast, when the marginal value of status v_s is small relative to the distance between types $\theta_H - \theta_L$, the upward incentive constraint can be safely ignored by the firms. In this case, since the downward constraint also has slack, the relaxed allocation in Remark 1 becomes the equilibrium. This result, which holds for any number of vertical types, has been previously shown by Rochet and Stole (2002) for the limiting case in which status has no value ($v_s = 0$).

3.1. **Quality Distortions and Veblen Effects.**

Propositions 1 and 2 show how the signaling motive affects the equilibrium.

Proposition 1. *Suppose there are two vertical types. If the upward incentive constraint binds (i.e., inequality* (8) *holds), in equilibrium both firms offer a quality schedule* $q^*(\cdot)$ *such that*

$$q^*(\theta_H) > q^{FB}(\theta_H) \text{ and } q^*(\theta_L) = q^{FB}(\theta_L),$$

and they offer a markup schedule $\Omega^*(\cdot)$ *such that*

$$\Omega^*(\theta_H) > \tau > \Omega^*(\theta_L).$$

In other words, high-type consumers receive excessive quality and experience a positive Veblen effect, while low-type consumers receive first-best quality and experience a negative Veblen effect.

[5]Formally, for small $\theta_H - \theta_L$, the left hand side of (8) is approximately $\theta_L v_s(q^{FB}(\theta_H), \theta_L) \cdot (\theta_H - \theta_L)$ (which is strictly positive) whereas the right hand side is approximated by $S_q(q^{FB}(\theta_L), \theta_L) \cdot (q^{FB}(\theta_L) - q^{FB}(\theta_H))$ (which, from the definition of $q^{FB}(\theta_L)$, is zero).

On the other hand, if the upward incentive constraint has slack, in equilibrium both firms offer the first-best quality to each consumer and all Veblen effects are zero.

Proof. See Appendix B. □

Proposition 2. *Suppose there are two vertical types. In equilibrium, firms impose a markup schedule $\Omega^*(\cdot)$ such that the average Veblen effect across consumers is zero:*

$$\sum_{\theta \in \{\theta_L, \theta_H\}} \left[\Omega^*(\theta) - \tau\right] f(\theta) = 0.$$

As a result, firms earn an average profit per consumer equal to the Hotelling benchmark τ.

Proof. See Appendix B. □

Proposition 1 tells us that, when the upward incentive constraint binds, firms use a combination of two instruments to separate high-type consumers from their low-type peers. First, akin to the competitive firms in Bagwell and Bernheim (1996) and Becker et al. (2000), they offer excessive quality to high types, making it relatively less attractive for a low type consumer to purchase such quality. Second, they increase the markup for the high-type product above τ while simultaneously reducing the markup for the low type product below τ, which further deters low types from deviating to the high-type allocation. Moreover, proposition 2 tells us that the Veblen effect imposed over high types is not translated into additional profits for the firms. Instead, this Veblen effect is used in its entirety to cross-subsidize low-type consumers through a reduced markup. We provide intuition for these results below.

The two instruments used by the firms differ in their efficiency implications. While quality distortions lead to reduced social surplus, Veblen effects merely represent implicit transfers among consumers. In fact, these cross-subsidies can be interpreted as a market for social status, mediated by the firms, in which high-type consumers effectively purchase part of their high status from low-type consumers through a monetary transfer. Since this implicit market reduces the need for quality distortions, it enhances social surplus.

3.2. **Intuition for Propositions 1 and 2.**

We provide intuition for proposition 1 using two hypothetical benchmarks. First, the firms could in principle resolve the screening problem by offering the first-best quality to both types of consumers, while simultaneously imposing a sufficiently large cross-subsidy across these consumers via a large positive Veblen effect for high types, and a large negative Veblen effect for low types. This allocation would maximize social surplus. However, under this allocation, high-type consumers would become highly profitable for the firms, and the opposite would be true for low types. As a result, following standard Hotelling logic, each firm would attempt to attract a larger fraction of high types away from its competitor without simultaneously attracting more low types. Under single-crossing preferences, this goal is precisely achieved by offering excessive quality to high types at a price that they consider a bargain but that low types consider unattractive.[6]

Second, the firms could resolve the screening problem by imposing equal markups across consumers (thus eliminating all cross-subsidies), while simultaneously imposing a sufficiently large distortion in the high-type quality in order to allow high types to separate based on this distortion alone. This hypothetical allocation is analogous to the competitive equilibrium in Bagwell and Bernheim (1996) and Becker et al. (2000), where all markups are zero and therefore all screening occurs through an escalation of quality. However, once firms have market power, they can capture a fraction of the surplus they create. Accordingly, if a firm unilaterally replaces part of the quality distortion with a cross-subsidy among consumers via the creation of Veblen effects, it expands the overall pool of surplus from which its profits are drawn.

The result in proposition 2 follows from a particular strategy available to the firms. Each firm can change the entire value schedule $V_i(\theta)$ by a constant amount ε while keeping the quality schedule fixed (which means that all prices are simultaneously increased or decreased by ε). Since this change is constant across θ, the new schedule remains incentive compatible. Moreover, this change leads to the same fundamental trade-off present in Hotelling's model: a higher average markup per consumer is traded-off against a lower demand. From this trade-off it follows that the average

[6]Formally, in order to attract more high types relative to low types, the firm must increase the difference in values $V_i(\theta_H) - V_i(\theta_L)$. Under a binding upward incentive constraint, this can only be achieved by increasing $v(q_i(\theta_H), \theta_H)$ (i.e., the upper-bound on the variability of $V_i(\cdot)$) through an increase in $q_i(\theta_H)$. Moreover, that it is desirable to expand $q_i(\theta_H)$ beyond $q^{FB}(\theta_H)$ follows from the fact that a marginal increase in $q_i(\theta_H)$ above $q^{FB}(\theta_H)$ has a first-order impact over the incentive constraint while creating only a second-order loss over the surplus function in the objective.

markup per consumer, $\sum_{\theta\in\{\theta_L,\theta_H\}}\Omega^*(\theta)f(\theta)$, is optimally set equal to the Hotelling benchmark τ.

4. A Continuum of Vertical Types

We now turn to our main case of interest. We assume θ is distributed according a positive differentiable density $f(\cdot)$ over the interval $[\theta_L,\theta_H]$. The firms' problem now becomes

$$\max_{V_i(\cdot),\, q_i(\cdot)} \int_{\theta_L}^{\theta_H} \underbrace{[S(q_i(\theta),\theta)-V_i(\theta)]}_{\Omega_i(\theta)} \cdot D_i(\theta)f(\theta)d\theta \qquad (P3)$$

$$s.t.\ q_i(\cdot) \text{ is nondecreasing, and}$$

$$V_i'(\theta) = v(q_i(\theta),\theta) \text{ for all } \theta, \qquad (IC\text{-}FS)$$

where the $(IC\text{-}FS)$ constraint has been expressed using (6).

Definition 2. *As before, we define the Veblen effect for type θ as the supra-normal markup $\Omega_i(\theta)-\tau$.*

Definition 3. *In addition, we define the marginal Veblen effect for type θ as the local rate of change in the markup $\Omega_i'(\theta)$.*

Since the incentive constraints for the continuum are local, these marginal Veblen effects will play a central role in the model.

We begin be extending proposition 2 to the case of continuos types.

Proposition 3. *Suppose there is a continuum of vertical types. In equilibrium, firms impose a markup schedule $\Omega^*(\cdot)$ such that the average Veblen effect across consumers is zero:*

$$\int_{\theta_L}^{\theta_H} [\Omega^*(\theta)-\tau]\, f(\theta)d\theta = 0.$$

Proof. See appendix C. □

The logic behind this result is identical to the two-type case: the fact that firms can shift the entire value schedule $V_i()$ by a constant amount without affecting the incentive constraint leads, in equilibrium, to an average markup across consumers equal to the Hotelling benchmark τ.

We now turn to the incentive constraint. Ideally, firms would like offer the first-best quality for all types and set all Veblen effects to zero, as occurred in the two-type case. However, as long as status has positive marginal value ($v_s > 0$), this ideal allocation always fails to be incentive compatible when types are continuous. To see this, it is useful to express (IC-FS) in a more intuitive way:

Remark 3. *At any point of differentiability of $q_i(\cdot)$, the equality $V_i'(\theta) = v(q_i(\theta), \theta)$ is equivalent to*

$$\underbrace{\theta \cdot v_s(q_i(\theta), \theta)}_{I} \;=\; \underbrace{-S_q(q_i(\theta), \theta) \cdot q_i'(\theta)}_{II} \;+\; \underbrace{\Omega_i'(\theta)}_{III}. \tag{9}$$

Proof. The desired equality follows from totally differentiating the accounting identity (7) w.r.t. θ, setting $V_i'(\theta) = v(q_i(\theta), \theta)$, and rearranging terms. □

Equation (9) tells us that, in order for an allocation to be incentive compatible, the marginal willingness to pay for additional status must equal the marginal cost of acquiring it. Term I measures type θ's marginal utility of status, and therefore his marginal willingness to pay. Terms II and III, on the other hand, measure the marginal cost of obtaining higher status. In particular, if type θ wishes to acquire higher status, he must potentially consume a higher quality level (as indicated by the derivative $q_i'(\theta)$) and must also potentially pay a higher markup (as indicated by the marginal Veblen effect $\Omega_i'(\theta)$). Consuming higher quality is costly to the extent that the prescribed quality $q_i(\theta)$ is distorted beyond first best to begin with (as indicated by the extent to which $S_q(q_i(\theta), \theta)$ is negative), therefore giving rise to term II, and paying a higher markup is directly translated into lower utility, therefore giving rise to term III.

Notice that when quality is first-best and markups are constant across types, terms II and III are zero (i.e., $S_q(q^{FB}(\theta), \theta) = 0$ and $\Omega_i'(\theta) = 0$), which means that consumers would be able to acquire additional status for free by simply deviating to the allocation of a marginally higher type. As a result, the relaxed allocation necessarily fails (IC-FS).

4.1. **Feasible Menus.**

From remark 3 we learn that firms have a variety of ways to meet the incentive constraint. For any given θ, they can impose either a sufficiently high quality distortion, or a sufficiently large marginal Veblen effect, or any combination of the two. We illustrate this flexibility using the two simplest options available.

In one extreme, firms could set the entire quality schedule equal to the first-best $q^{FB}(\cdot)$ (so that term II in eq. (9) is zero), while setting $\Omega_i'(\theta)$ equal to the marginal utility of status $\theta \cdot v_s(q^{FB}(\theta), \theta)$ at every point of the vertical spectrum.

This is the most efficient allocation possible since all status is purchased through money transfers. In fact, when the average Veblen effect across consumers is zero (Proposition 3), these transfers take the form of pure cross-subsidies from high-end to low-end consumers, with the property that the magnitude of the net subsidy for type θ is continuously increasing in θ at a rate equal to the local marginal willingness to pay for status. Since the corresponding marginal Veblen effects capture the full marginal value of status, this implicit market mechanism eliminates all need for quality distortions. This allocation is illustrated in Figure 2.

In the opposite extreme, firms could impose a constant markup τ across consumers. In this case, since all marginal Veblen effects are zero, screening must occur exclusively through quality distortions. This resulting quality schedule, denoted $q^C(\cdot)$, must satisfy the differential equation

$$\theta \cdot v_s(q^C(\theta), \theta) = -S_q(q^C(\theta), \theta) \cdot \frac{d}{d\theta} q^C(\theta) \text{ for all } \theta,$$

which simply states that the full marginal utility of status must be translated into quality distortions. The schedule $q^C(\cdot)$ is depicted in Figure 2 under the initial condition $q^C(\theta_L) = q^{FB}(\theta_L)$, so that quality is distorted upward for all but the lowest type (this schedule is denoted $q_{BB}(\cdot)$ in the figure). This specific schedule is of interest because it corresponds to the equilibrium of a perfectly competitive environment in which all markups, and therefore all marginal Veblen effects, are zero. In particular, it represents the continuous analogue of the equilibrium allocation in Bagwell and Bernheim (1996). This allocation is highly inefficient because pure transfers are in no way used as a signaling device.

In addition to the two extreme options described above, firms can use a combination of quality distortions and marginal Veblen effects to screen their consumers, and the weight placed on each of these instruments can vary along the vertical spectrum.

5. **Analysis**

We derive the equilibrium using optimal control methods. For expositional purposes, it is useful to separate the analysis into two cases, according to whether or not the monotonicity constraint in the firms' problem $(P3)$ binds. As we will see, the role of this constraint is intimately related to the degree of competition between the two firms. Moreover, a binding constraint will have a non-trivial impact over the allocation. We begin with the case in which the monotonicity constraint is slacked.

5.1. **Non-Binding Monotonicity Constraint.**

For this case, we treat quality $q_i(\theta)$ as a control variable for firm i and consumer value $V_i(\theta)$ as the corresponding state. From the incentive constraint, this state evolves according to the law of motion $V_i'(\theta) = v(q_i(\theta), \theta)$.

Dropping the i subindex for notational simplicity, the corresponding Hamiltonian is given by

$$H(q, V, \lambda, \theta) \equiv \underbrace{[S(q(\theta), \theta) - V(\theta)]}_{\Omega(\theta)} \cdot D(\theta) f(\theta) + \lambda(\theta) \cdot v(q(\theta), \theta), \qquad (H1)$$

where the first term is the integrand in the objective of problem $(P3)$ and $\lambda(\theta)$ represents the co-state variable for $V(\theta)$. Intuitively, for any given type $\widehat{\theta}$, $\lambda(\widehat{\theta})$ measures the gain experienced by the firm when *increasing* consumer value $V(\theta)$ by a small amount ε for all types θ higher than $\widehat{\theta}$ without changing the value for lower types. Equivalently, $\lambda(\widehat{\theta})$ measures the marginal gain experienced by the firm when *reducing* consumer value $V(\theta)$ by a small amount ε for all types θ lower than $\widehat{\theta}$ without changing the value for higher types.[7]

The control $q(\theta)$ only enters this Hamiltonian through the surplus function $S(q(\theta), \theta)$ and through the function $v(q(\theta), \theta)$ that determines the rate of change of the state. Since $v_q > 0$, the firm will optimally set $q(\widehat{\theta})$ higher (resp. lower) than $q^{FB}(\widehat{\theta})$ whenever $\lambda(\widehat{\theta})$ is positive (resp. negative), and $q(\widehat{\theta}) = q^{FB}(\widehat{\theta})$ whenever $\lambda(\widehat{\theta}) = 0$.

Theorem 1 characterizes the equilibrium.

Theorem 1. *Suppose there is a continuum of vertical types. If the monotonicity constraint does not bind, in equilibrium both firms offer a quality schedule $q^*(\cdot)$ such that*

$$q^*(\theta) > q^{FB}(\theta) \text{ for all interior types } \theta \in (\theta_L, \theta_H), \text{ and}$$
$$q^*(\theta) = q^{FB}(\theta) \text{ for both extremes types } \theta_L \text{ and } \theta_H.$$

In addition, firms offer a markup schedule $\Omega^*(\cdot)$ such that, for any given interior type $\widehat{\theta} \in (\theta_L, \theta_H)$,

$$E\left[\Omega^*(\theta) \mid \theta \geq \widehat{\theta}\right] > \tau > E\left[\Omega^*(\theta) \mid \theta \leq \widehat{\theta}\right].$$

[7] The reason for this equivalence is that, in equilibrium, the marginal gain from increasing or decreasing the entire schedule $V_i(\cdot)$ by ε must be zero (recall that such a change can always be performed without affecting the incentive constraint), and therefore the marginal gain from increasing $V_i(\theta)$ by ε for all $\theta \geq \widehat{\theta}$ equals the marginal loss from increasing (or the marginal gain from reducing) $V_i(\theta)$ by ε for all $\theta < \widehat{\theta}$.

As a result, on average, high types experience positive Veblen effects and low types experience negative Veblen effects.

Proof. See Appendix C. □

Figure 2 presents an example of this equilibrium. The quality and markup schedules are plotted in panels **a** and **b**, and the associated consumer values, denoted $V^*(\theta)$, are presented in panel **c**.

As in the two-type case, firms use a mixture of quality distortions and cross subsidies to screen their consumers. Accordingly, the price of obtaining higher status comes partially in the form of wasteful signaling and partially in the form of efficient money transfers, within an implicit market for status, to low-type consumers.

However, unlike the two-type case, quality distortions vanish toward the high end of the spectrum. Indeed, the highest-type consumer purchases his last unit of status in the most efficient way possible via a marginal Veblen effect that equals his full marginal value of status. This efficiency at the high end represents a qualitative departure from the competitive case, where status is always allocated inefficiently. We return to this point in below, once we discuss the potential impact of a binding monotonicity constraint.

A central feature of the equilibrium is that the consumer value schedule $V^*(\cdot)$ is rotated counter-clockwise with respect to the first-best schedule $V^{FB}(\cdot)$.As explained in detail below, this rotation of $V^*(\cdot)$ results from an attempt by each firm to attract a larger mass of high-type, high-margin, consumers away from its rival while simultaneously attracting a smaller mass of low-end, low-margin, consumers. In fact, all quality distortions – the means by which the slope of $V^*(\cdot)$ is increased – originate from this attempt.

5.2. **Intuition for Theorem 1.**

As a preliminary step, it is useful to describe the relation between the co-state variable λ and the firm's markup schedule Ω. For any given type $\widehat{\theta}$, the co-state $\lambda(\widehat{\theta})$ can be expressed as[8]

$$\lambda(\widehat{\theta}) = \frac{1 - F(\widehat{\theta})}{2\tau} \left\{ E\left[\Omega(\theta) \mid \theta \geq \widehat{\theta} \right] - \tau \right\}. \tag{10a}$$

Consider an interior type $\widehat{\theta}$. The term $\frac{1-F(\widehat{\theta})}{2\tau}$, which is proportional to the mass of consumes above $\widehat{\theta}$, is strictly positive. The term in braces, on the other hand, is

[8]See Appendix C, eq. C4.

positive whenever the average markup for types higher than $\widehat{\theta}$ exceeds the Hotelling ideal τ. In this case, $\lambda(\widehat{\theta})$ is also positive, which indicates that the firm would benefit from increasing $V(\theta)$ for all types higher than $\widehat{\theta}$. The reason is precisely that, in doing so, the firm would attract more of these high-margin consumers. (Of course, as indicated by the Hamiltonian $(H1)$, this benefit must be traded-off against the loss caused by the quality distortions needed to alter the shape of $V(\theta)$.)

Moreover, the term $\frac{1-F(\widehat{\theta})}{2\tau}$ tells us that the above benefit is directly proportional to the mass $1-F(\widehat{\theta})$ of the affected high-margin consumers, and inversely proportional to the elasticity parameter τ because a lower value for this parameter means that demand is more elastic and therefore consumers react more aggressively to changes in $V(\theta)$.

A symmetric reasoning follows for consumers on the low end of the spectrum. Since the unconditional average markup $E[\Omega(\theta)]$ always equals τ, equation $(10a)$ can be equivalently expressed as

$$\lambda(\widehat{\theta}) = \frac{F(\widehat{\theta})}{2\tau}\left\{\tau - E\left[\Omega(\theta) \mid \theta \leq \widehat{\theta}\right]\right\}. \tag{10b}$$

Consider again an interior type $\widehat{\theta}$. The term in braces is now positive, leading to a positive $\lambda(\widehat{\theta})$, whenever the average markup for types lower than $\widehat{\theta}$ falls *below* the Hotelling ideal τ. In this case, the firm would benefit from *decreasing* $V_i(\theta)$ for all types lower than $\widehat{\theta}$ because, by doing so, it would attract fewer of these low-margin consumers. As before, the term $\frac{F(\widehat{\theta})}{2\tau}$ indicates that this marginal gain is directly proportional to the abundance of the affected consumers, and is inversely proportional to τ.

We are now ready to provide intuition for the theorem. In order to see why firms distort quality (the first part of the theorem), consider the benchmark case in which firms offer quality $q^{FB}(\theta)$ for all types. Recall that, in this case, the marginal Veblen effect would be strictly positive for all types and therefore the associated markup schedule, denoted $\Omega^{FB}(\theta)$, would be strictly increasing. As a result, for any interior type $\widehat{\theta}$, we obtain

$$E\left[\Omega^{FB}(\theta) \mid \theta \geq \widehat{\theta}\right] > \tau > E\left[\Omega^{FB}(\theta) \mid \theta \leq \widehat{\theta}\right].$$

These inequalities imply that, under first best-quality, the terms in braces in equations $(10a)$ and $(10b)$ would be strictly positive. But this fact tells us that each firm would experience a first-order gain from rotating the value schedule $V(\cdot)$ counterclockwise around type $\widehat{\theta}$ (by increasing the derivative $V'(\widehat{\theta})$) in order to attract more high-margin consumers while simultaneously attracting fewer low-margin ones. From the law of motion $V'(\widehat{\theta}) = v(q(\widehat{\theta}), \widehat{\theta})$, this rotation is achieved by increasing the quality of type $\widehat{\theta}$ beyond $q^{FB}(\widehat{\theta})$, which leads only to a second-order reduction of surplus and therefore is always beneficial to the firm.

Notably, the gain from distorting quality fades as we approach either extreme of the vertical spectrum, and disappears altogether for the extreme types θ_L and θ_H. Consider, for instance, a type $\widetilde{\theta}$ close to θ_H. For this type, increasing the derivative $V'(\widetilde{\theta})$ through a distorted quality can be used to either selectively attract more high types $\theta > \widetilde{\theta}$, or selectively attract fewer low types $\theta < \widetilde{\theta}$, or any combination of the two. But, from equation $(10a)$ we learn that the former is of limited value because there is only a small remaining mass of high types ($1 - F(\widetilde{\theta})$ is close to zero). And from equation $(10b)$ we lean that the latter is also of limited value because the average markup for types lower than $\widetilde{\theta}$ is close to the unconditional expectation $E[\Omega^{FB}(\theta)]$, and therefore close the Hotelling ideal τ. In fact, when $\widetilde{\theta} = \theta_H$ there is zero mass of higher types to attract and the average markup among types lower than this type is precisely the Hotelling ideal. As a result, distorting quality for θ_H no longer has any value. A symmetric reasoning applies for types at the low end of the spectrum.

In order to see why firms employ cross-subsidies as opposed to relying exclusively on quality distortions (the second part of the theorem), consider the benchmark case in which all markups are constant and equal to τ. In this case, the quality schedule corresponds to the distorted competitive schedule $q^C(\cdot)$. Starting form this benchmark, suppose a firm unilaterally reduces the quality distortion for an arbitrary interior type $\widehat{\theta}$. This reduced distortion has two effects over profits: (1) a direct beneficial effect via an expansion in the surplus level $S(q(\widehat{\theta}), \widehat{\theta})$, and (2) an indirect effect via a reduction in the slope $V'(\widehat{\theta})$, which means that the firm would attract more low-type consumers relative to high types. While the first effect has a first-order magnitude, the second effect is only second-order. The reason for the latter is that, starting from a situation in which all consumers pay the same markup τ, making every type of consumer equally profitable, the firm experiences no loss, in the margin, when replacing high-type with low-type consumers. As a result, each firm finds it beneficial to reduce quality, at least marginally, below the competitive level.

5.3. **Binding Monotonicity Constraint.**

Theorem 2 considers equilibria in which the monotonicity constraint (*ii*) binds for some subset of types:[9]

Theorem 2. *Consider a symmetric separating equilibrium with full market coverage. If the monotonicity constraint does not bind for the highest type θ_H, this equilibrium must satisfy all properties described in theorem 1.*

On the other hand, if the monotonicity constraint does bind for the highest type, this equilibrium must satisfy the properties described in theorem with the following exceptions:

(1) *Quality is distorted upward for the highest type.*
(2) *All types in a neighborhood of θ_H experience marginal Veblen effects equal to the marginal utility of status $\theta \cdot v_s(q^*(\theta), \theta)$.*

6. **Corrective Taxation**

The inefficiencies arising from status seeking suggest a role for government intervention. Here we consider the use of corrective taxation. We discuss two cases. First, under the hypothetical assumption that production costs $c(q)$ are observable, we consider taxes that are directly imposed on these production costs. Since this instrument attacks the direct source of the inefficiency (i.e., over–investment in quality), the first best can indeed be achieved. Nevertheless, we show that given the non–monotonic nature of the quality distortions, the optimal tax-schedule will not have a conventional shape.

Second, we consider the more realistic case in which taxes are imposed over prices instead of over costs. Since such a policy instrument does not only affect the quality distortions (our target), but they also alter the efficient Veblen effects, achieving first-best with this instrument may not be feasible. This conclusion casts doubt over simplistic proposals that luxury goods should be heavily taxed.

[9] The monotonicity constraint is guaranteed to bind at the high end of the spectrum whenever τ is close to zero and, therefore, the elasticity of demand is close to infinity. In this case, firms have a strong incentive to distort quality in order to increase their share of high-margin consumers, which means that quality for intermediate types eventually exceeds the first-best quality for the highest type.

6.1. **Taxes on Production Costs.**

Suppose that whenever a firm produces a good with quality q, in addition to incurring the cost $c(q)$, it is required to make a tax payment equal to $\alpha(q)$. In this case, the firm's problem is identical to the original problem except for the fact that it now faces a higher effective cost function given by:

$$\widetilde{c}(q) \equiv c(q) + \alpha(q).$$

The goal is to find a function $\alpha(q)$ such that the equilibrium quality that arises under the new cost $\widetilde{c}(q)$ corresponds to the first best.

Let $\widetilde{\Omega}(\theta) \equiv p(\theta) - \widetilde{c}(q)$ denote the firm's after–tax markup, which equals the gross markup minus the tax: $\Omega(\theta) - \alpha(q)$. We refer to $\widetilde{\Omega}(\theta)$ as the firm's (after–tax) Veblen effect and to $\Omega(\theta)$ as the gross Veblen effect experienced by consumers. From the envelope condition $\Omega'(\theta) = \theta \cdot v_s(q(\theta), \theta) + q'(\theta) \cdot S_q(q(\theta), \theta)$, in order for the tax schedule to implement the first-best ($S_q(q_i(\theta), \theta) = 0$) we require that:

$$\Omega_i'(\theta) = \widetilde{\Omega}'(\theta) + \alpha'(q(\theta))q'(\theta) = \theta \cdot v_s(q_i(\theta), \theta), \tag{11}$$

where $\alpha'(q)$ denotes the marginal tax on quality.

The relationship in (11) implies that the marginal utility for status must be fully translated into monetary transfers (as opposed to quality distortions), and these transfers must either go to the firm (through a positive marginal Veblen effect $\widetilde{\Omega}'(\theta)$) or to the government (through the marginal tax rate $\alpha'(q)$). In other words, taxes are necessary only insofar as the firms do not impose sufficiently high marginal Veblen effects $\widetilde{\Omega}'(\theta)$ to begin with, and the optimal marginal tax $\alpha'(q)$ precisely supplements the firm's marginal Veblen effects in such a way that the gross Veblen effects experienced by the consumers equal their full marginal utility for status.

The following corollary of theorem 1 describes the optimal tax schedule.

Corollary 1. *Suppose the marginal tax schedule $\alpha(q)$ implements the first-best quality schedule under an equilibrium with full market coverage. Then, for all θ, the marginal tax $\alpha'(q)$ is such that:*

- **a.** *For both extreme types, $\alpha'(q^{FB}(\theta)) = 0$.*
- **b.** *For all interior types, $\alpha'(q^{FB}(\theta)) = \frac{1}{f(\theta)} v_q(q^{FB}(\theta), \theta) \int_0^\theta \left[1 - \frac{\widetilde{\Omega}(z)}{\tau}\right] f(z)dz$, which is positive.*

Proof. See Appendix C. □

This result tells us that only the quality sold to the interior types must be taxed in the margin. The reason is that, from theorem 1, in any equilibrium with a monotonic quality schedule, the firms are tempted to impose quality distortions (and low marginal Veblen effects) only for these interior types (and the first-best quality schedule is, by assumption, monotonic). Panel (b) of Figure 3 depicts the optimal tax schedule for the special case in which $f(\theta)$ is uniformly distributed on $[1,2]$, $v(q,s) = q + s$, and $\widetilde{c}(q) = \frac{1}{2}q^2$.

6.2. **Taxes on Prices .** Now suppose that whenever a firm sells a good of price p, it is required to make a tax payment equal to $\beta(p)$. As before, the goal is to find a function $\beta(p)$ that induces the first best.

Let $\widetilde{\Omega}(\theta) \equiv p(\theta) - \beta(p(\theta)) - c(q)$ denote the firm's after–tax markup, which again equals the gross markup minus the tax: $\Omega(\theta) - \beta(p(\theta))$. The following theorem characterizes the optimal tax.

Theorem 3. *Suppose $\beta(p)$ implements $q^{FB}(\theta)$ in an equilibrium with full market coverage. Then, the marginal tax $\beta'(p)$ is such that:*

- **a.** $\beta'(p^*(\theta_L)) = \beta'(p^*(\theta_H)) = 0$.
- **b.** *For all interior types $\widehat{\theta}$,*

$$\beta'(p^*(\widehat{\theta})) = \int_0^{\widehat{\theta}} \left[\tau - \widetilde{\Omega}(\theta)\right] f(\theta) d\theta \left(\frac{1}{\tau\widehat{\theta} f(\widehat{\theta})}\right) - \int_0^{\widehat{\theta}} \beta'(p^*(\theta)) f(\theta) d\theta \left(\frac{1}{\widehat{\theta} f(\widehat{\theta})}\right) \tag{12}$$

Proof. See Appendix C. □

As before, if the tax is to implement the first–best, only interior types must be taxed in the margin. However, unlike the case in which quality was directly taxed, a tax on prices plays a dual role in our environment. On the one hand, since firms have an incentive to distort quality upward – and charge their consumers higher prices for this additional quality – high prices should be taxed. On the other hand, by creating Veblen effects, high prices serve as a substitute screening device for quality distortions, and therefore enhance efficiency. This second role suggests that higher prices should be subsidized. Because of these two opposing goals, the optimal tax schedule has a more elaborate structure than before, as reflected by the second term in equation (12).

7. Conclusion

We have studied the emergence of two frequently observed phenomena in markets for conspicuous goods: upward quality distortions and Veblen effects. In our model, two firms offer conspicuous goods to a heterogeneous collection of consumers with standard single-crossing preferences. This model combines elements of both screening and signaling. Namely, firms offer individually–targeted products using non-linear pricing schemes, and when purchasing these products, consumers' signal their hidden characteristics.

The firms' strategies are driven by two competing goals: (1) satisfying incentive-compatibility constraints in order to screen among different types of consumers, and (2) seeking an appropriate balance between market share and price markups. As a result, they adopt an mix of quality distortions (which attract more consumers while satisfying their incentive constraints) and cross–subsidies among consumers (which deliver an optimal balance between market share and profits per customer).

The use of cross–subsidies creates an implicit market for status, mediated the firms, in which high-ranking consumers effectively purchase status from their low-ranking peers. Unlike quality distortions, this market mechanism is an efficient way of allocating status. However, since firms are eager to gain a larger market share for high-margin consumers, quality distortions are also employed.

The novelty of our model resides in providing a rationale for the simultaneous presence of the two above phenomena under single-crossing preferences, as well as a framework for analyzing their interaction. In addition, the model uncovers clues for optimal corrective taxation. Contrary to informal prescriptions, high-end products with high markups do not require large taxes. In fact, it is precisely because of these high prices that the status competition is resolved efficiently (through cross-subsidies across consumers) as opposed to being resolved through a wasteful over-provision of quality.

8. **Appendix A: Full Separation and Monotonicity**

Proofs available from the authors.

9. **Appendix B: Two Types**

Proof of Remark 1

Consider first the choice of quality. Notice that, for each vertical type and each firm, the quality level $q_i(\theta)$ only enters the objective in $(P2)$ through the function $S(q_i(\theta), \theta)$. But, by assumption, this function is uniquely maximized at $q^{FB}(\theta)$.

Consider now the choice of consumer value. Since $V_i(\theta)$ only enters the firm's objective through the integrand $[S(q_i(\theta), \theta) - V_i(\theta)] \cdot D_i(\theta)$, we obtain the following first-order condition for this value:

$$-D_i(\theta) + \Omega_i(\theta) \cdot \frac{\partial}{\partial V_i(\theta)} D_i(\theta) = 0. \qquad (B1)$$

In addition, from the demand functions in (2) we obtain $\frac{\partial}{\partial V_i(\theta)} D_i(\theta) = g'(\widehat{x}(\theta)) \cdot \frac{1}{2t}$, and from the symmetry between firms we obtain $\widehat{x}(\theta) = D_i(\theta) = \frac{1}{2}$. The desired result follows from combining these equalities with $(B1)$ and rearranging terms.

Proof of Lemma 1

We show that each firm can ignore the downward incentive constraint without loss. Suppose firm i ignores this constraint. There are two cases to consider, depending on whether or not the upward incentive constraint binds for this firm.

If the upward constraint does bind, it follows from standard optimization rules that the first inequality in $(IC\text{-}FS)$ must hold with equality. But since the monotonicity constraint $q_i(\theta_H) \geq v(q_i(\theta_H)$ implies that $v(q_i(\theta_H), \theta_H) > v(q_i(\theta_L), \theta_L)$, the downward constraint is automatically met.

If the upward constraint does not bind, the equilibrium, by hypothesis, is described by remark 1. From this remark and the accounting identity (7), it follows that

$$V_i(\theta_H) - V_i(\theta_L) = S(q^{FB}(\theta_H), \theta_H) - S(q^{FB}(\theta_L), \theta_L) >$$
$$S(q^{FB}(\theta_L), \theta_H) - S(q^{FB}(\theta_L), \theta_L) > (\theta_H - \theta_L) \cdot v(q^{FB}(\theta_L), \theta_L).$$

Combining the first and last expression, we again see that the downward constraint is automatically met.

Proof of Lemma 2

Since the downward incentive constraint has slack (Lemma 1), it follows from remark 1 that the upward incentive constraint binds in equilibrium if and only if the relaxed allocation (with $q_i(\theta) = q^{FB}(\theta)$ and $\Omega_i(\theta) = \tau$ for all i and all θ) fails to satisfy the constraint

$$v(q_i(\theta_H), \theta_H) \geq \frac{V_i(\theta_H) - V_i(\theta_L)}{\theta_H - \theta_L}.$$

This failure occurs if and only if

$$v(q^{FB}(\theta_H), \theta_H) < \frac{S(q^{FB}(\theta_H), \theta_H) - S(q^{FB}(\theta_L), \theta_L)}{\theta_H - \theta_L},$$

where, for each type, $q_i(\theta)$ has been set equal to $q^{FB}(\theta)$, and, using the accounting identity (7), $V_i(\theta)$ has been set equal to $S(q^{FB}(\theta), \theta) - \tau$. After rearranging terms, the above equality is equivalent to (8).

Proof of Proposition 1

Consider problem $(P2)$. Since the downward incentive constraint has slack (Lemma 1), any solution to this problem must satisfy the following first-order conditions:

$$S_q(q^*(\theta_L), \theta_L) \cdot \frac{1}{2} f(\theta_L) = 0, \tag{A1}$$

$$S_q(q^*(\theta_H), \theta_H) \cdot \frac{1}{2} f(\theta_H) = -\lambda \cdot v_q(q^*(\theta_H), \theta_H), \tag{A2}$$

$$-\frac{1}{2} f(\theta_L) + \Omega^*(\theta_L) \cdot g'\left(\frac{1}{2}\right) \frac{1}{2t} f(\theta_L) = -\frac{\lambda}{\theta_H - \theta_L}, \tag{A3}$$

$$-\frac{1}{2} f(\theta_H) + \Omega^*(\theta_H) \cdot g'\left(\frac{1}{2}\right) \frac{1}{2t} f(\theta_H) = \frac{\lambda}{\theta_H - \theta_L}. \tag{A4}$$

where λ denotes the Lagrange multiplier for the upward incentive constraint, the first two equations are the first-order conditions, respectively, for $q_i(\theta_L)$ and $q_i(\theta_H)$, and the last two equations are the first-order conditions, respectively, for $V_i(\theta_L)$ and $V_i(\theta_H)$. When deriving these first-order conditions, we have set $\frac{\partial}{\partial V_i(\theta)} D_i(\theta) = g'(\widehat{x}(\theta)) \cdot \frac{1}{2t}$ (from the demand functions in (2)), as well as $\widehat{x}(\theta) = \frac{1}{2}$ and $D_i(\theta) = \frac{1}{2}$ (from the symmetry between firms).

Suppose the upward incentive constraint binds (i.e., $\lambda > 0$). Equation $(A1)$ implies that $S_q(q^*(\theta_L), \theta_L) = 0$ and therefore $q^*(\theta_L) = q^{FB}(\theta_L)$. Equation $(A2)$, combined with the fact that $\lambda > 0$ and $v_q > 0$, implies that $S_q(q^*(\theta_H), \theta_H) < 0$ and therefore $q^*(\theta_H) > q^{FB}(\theta_H)$.

On the other hand, $\lambda > 0$ implies that the L.H.S. of equation $(A3)$ is negative and the L.H.S. of $(A4)$ is positive. As a result, using the definition $\tau \equiv g'\left(\frac{1}{2}\right)\frac{1}{2t}$, we obtain

$$\left[-1+\frac{\Omega^*(\theta_L)}{\tau}\right]\cdot f(\theta_L) < 0 \text{ and } \left[-1+\frac{\Omega^*(\theta_H)}{\tau}\right]\cdot f(\theta_H) > 0.$$

From these inequalities it follows that $\Omega^*(\theta_L) < \tau$ and $\Omega^*(\theta_H) > \tau$, as desired.

Now suppose the upward incentive constraint does not bind (i.e., $\lambda = 0$). In this case, $(A1)$ and $(A2)$ imply that both quality levels are first best, while $(A3)$ and $(A4)$ imply that both markups equal τ.

Proof of Proposition 2

This result follows from adding equations $(A3)$ and $(A4)$ (contained in the proof of Proposition 1), and rearranging terms.

10. **Appendix C: The Continuum**

Proof of Proposition 3

Consider a symmetric equilibrium in which both firms offer the same menu $\langle V^*(\theta), q^*(\theta)\rangle_{\theta\in[\theta_L,\theta_H]}$, and therefore $\widehat{x}(\theta) = \frac{1}{2}$ for all θ. Accordingly, from (2), firm L's equilibrium payoff is given by

$$\begin{aligned}\Pi_L &\equiv \int_{\theta_L}^{\theta_H} \Omega_L(\theta)D_L(\theta)dF(\theta)\\ &= \int_{\theta_L}^{\theta_H} \left[S(q^*(\theta),\theta) - V^*(\theta)\right]G\left(\frac{1}{2}\right)dF(\theta),\end{aligned}$$

where the markup $\Omega_L(\theta)$ has been expressed as $S(q^*(\theta),\theta) - V^*(\theta)$. Now consider an alternative menu for firm L given by $\langle V^*(\theta)+\varepsilon, q^*(\theta)\rangle_{\theta\in[\theta_L,\theta_H]}$ for some small ε (perhaps negative), which is is identical to the original menu except for the fact that all consumers are offered a payoff that is higher or lower by a constant amount ε. Since this change is constant across θ, and quality is unaffected, the new menu remains incentive compatible. The payoff obtained by firm L under this new menu becomes

$$\int_{\theta_L}^{\theta_H} \left[S(q^*(\theta),\theta) - V^*(\theta) - \varepsilon\right]G\left(\frac{1}{2}+\frac{\varepsilon}{2t}\right)dF(\theta), \qquad (C1)$$

where, from equation $(A1)$, the horizontal cutoff $\widehat{x}(\theta)$ has now increased to $\frac{1}{2}+\frac{\varepsilon}{2t}$. Notice that the derivative of $(C1)$ with respect to ε evaluated at $\varepsilon = 0$ is given by

$$\int_{\theta_L}^{\theta_H} \left\{-G\left(\frac{1}{2}\right) + \left[S(q^*(\theta),\theta) - V^*(\theta)\right]g\left(\frac{1}{2}\right)\frac{1}{2t}\right\}dF(\theta). \qquad (C2)$$

Since the original schedule constitutes an equilibrium, it must be the case that the new payoff $(C1)$ is maximized, with respect to ε, when $\varepsilon = 0$. But this in turn implies that $(C2)$ must be equal to zero, which is equivalent to the desired equality $\int_{\theta_L}^{\theta_H} \Omega_L(\theta)dF(\theta) = \tau$. The analysis is symmetric for firm R.

Proof of Theorem 1

When the monotonicity constraint does not bind, the Hamiltonian for firm i is given by $(H1)$:

$$H(q(\theta), V(\theta), \lambda(\theta), \theta) = \underbrace{[S(q(\theta), \theta) - V(\theta)]}_{\Omega(\theta)} \cdot D(\theta) f(\theta) + \lambda(\theta) \cdot v(q(\theta), \theta),$$

where the subindex i has been dropped for notational simplicity. As mentioned in the text, $q(\theta)$ represents the firm's control variable, $V(\theta)$ represents the associated state (governed by the law of motion $(IS\text{-}FS)$), and $\lambda(\theta)$ denotes the co-state for $V(\theta)$. In addition, since the initial and terminal values for $V(\cdot)$ are free, we obtain the transversality conditions $\lambda(\theta_L) = \lambda(\theta_H) = 0$.

From the Principle of the Maximum, the optimal menu solves, for all θ, the following system:

$$S_q(q(\theta), \theta) = -\frac{2\lambda(\theta)}{f(\theta)} v_q(q(\theta), \theta), \tag{C3}$$

$$\lambda'(\theta) = \frac{1}{2}\left[1 - \frac{\Omega(\theta)}{\tau}\right] f(\theta), \text{ and} \tag{C4}$$

$$\Omega'(\theta) = \theta \cdot v_s(q(\theta), \theta) + q'(\theta) \cdot S_q(q(\theta), \theta) \tag{C5}$$

Equation $(C3)$ corresponds to the first-order condition for $q(\theta)$ (i.e., $\frac{\partial H}{\partial q(\theta)} = 0$) after rearranging terms, and equation $(C4)$ is the co-state equation $\lambda'(\theta) = -\frac{\partial H}{\partial V(\theta)}$. From the symmetry between firms, both $(C3)$ and $(C4)$ have been evaluated at $\widehat{x}(\theta) = \frac{1}{2}$ and $D(\theta) = \frac{1}{2}$. Also recall that $\tau \equiv t/g'(\frac{1}{2})$. Equation $(C5)$, on the other hand, corresponds to the law of motion $(IC\text{-}FS)$. This law of motion has been expressed in terms of $\Omega'(\theta)$, rather than $V'(\theta)$, using remark 3 and rearranging terms.[10]

We proceed with three claims.

[10]Recall that remark XXX requires that $q(\theta)$ is differentiable. But this differentiability is guaranteed by $(A1)$ and the existence of $\lambda'(\theta)$. In particular, by totally differentiating $(A1)$ w.r.t. q and θ we obtain

$$\frac{\partial}{\partial q}\left(\frac{S_q}{v_q}\right) \cdot dq = -\frac{\partial}{\partial \theta}\left(\frac{S_q}{v_q} + \frac{2\lambda(\theta)}{f(\theta)}\right) \cdot d\theta.$$

It follows that $q'(\theta)$ exists whenever the coefficient on dq is nonzero. But thanks to the concavity of $v(q, \theta)$ in q and the convexity of production costs $c(q)$, this coefficient is in fact strictly negative.

Claim 1. *For any given type θ, if $\lambda(\theta) \leq 0$, then $\Omega'(\theta) > 0$.*

Proof. Suppose $\lambda(\theta) \leq 0$. From equation $(C3)$ it follows that $S_q(q(\theta), \theta) \geq 0$. Moreover, since the monotonicity constraint does not bind (by hypothesis), we have $q'(\theta) \geq 0$. It follows that the second term on the R.H.S. of equation $(C5)$, $q'(\theta) \cdot S_q(q(\theta), \theta)$, is non-negative. On the other hand, the first term on the R.H.S. of $(C5)$ is strictly positive by assumption, and therefore $\Omega'(\theta) > 0$. □

Claim 2. *For all interior types $\theta \in (\theta_L, \theta_H)$, we have $\lambda(\theta) \geq 0$.*

Proof. Suppose toward a contradiction that $\lambda(\widehat{\theta}) < 0$ for some $\widehat{\theta} \in (\theta_L, \theta_H)$. Since $\lambda(\cdot)$ is differentiable, and $\lambda(\theta_L) = \lambda(\theta_H) = 0$, there must exist a nonempty interval (θ_1, θ_2) containing $\widehat{\theta}$ such that:

$$\text{(i) } \lambda(\theta) < 0 \text{ for all } \theta \in (\theta_1, \theta_2) \text{ and (ii) } \lambda'(\theta_1) \leq 0 \leq \lambda'(\theta_2).$$

By combining the inequalities in (ii) with equation $(C4)$ we obtain, respectively,

$$\Omega(\theta_1) \geq \tau \text{ and } \Omega(\theta_2) \leq \tau. \tag{C6}$$

On the other hand, from (i) and Clam 1 it follows that $\Omega'(\theta) > 0$ for all $\theta \in (\theta_1, \theta_2)$. But this fact implies that $\Omega(\theta_1) < \Omega(\theta_2)$, which contradicts $(C6)$. □

Claim 3. *For all interior types $\theta \in (\theta_L, \theta_H)$, we have $\lambda(\theta) > 0$.*

Proof. Suppose toward a contradiction that $\lambda(\widehat{\theta}) < 0$ for some $\widehat{\theta} \in (\theta_L, \theta_H)$. From claim 2 it follows that $\lambda(\widehat{\theta}) = 0$ and, in addition, $\lambda'(\widehat{\theta}) = 0$ (otherwise $\lambda(\theta)$ would be negative in a neighborhood either to the right or left of $\widehat{\theta}$).

From $(C4)$, $\lambda'(\widehat{\theta}) = 0$ implies that $\Omega(\widehat{\theta}) = \tau$. Moreover, from claim 1, $\lambda(\widehat{\theta}) = 0$ implies that $\Omega'(\widehat{\theta}) > 0$, and therefore there exists a small $\varepsilon > 0$ such that $\Omega(\theta) > \tau$ for all $\theta \in (\widehat{\theta}, \widehat{\theta} + \varepsilon)$. This last fact, combined with $(C4)$, implies that $\lambda'(\widehat{\theta}) < 0$ for all $\theta \in (\widehat{\theta}, \widehat{\theta} + \varepsilon)$, which in turn implies that $\lambda'(\widehat{\theta} + \varepsilon) < \lambda'(\widehat{\theta})$. But since $\lambda'(\widehat{\theta}) = 0$ we must have $\lambda'(\widehat{\theta} + \varepsilon) < 0$, which contradicts claim 4. □

We are now ready to prove the theorem. We begin with the equilibrium quality levels $q^*(\cdot)$. That $q^*(\theta) > q^{FB}(\theta)$ for every interior θ follows from combining $(C5)$ (i.e., $S_q(q(\theta), \theta)$ has the opposite sign of $\lambda(\theta)$) with claim 3 (i.e., $\lambda(\theta) > 0$ for all such types). Similarly, the claim that quality is first-best for both extreme types follows from combining $(C5)$ with the transversality conditions $\lambda(\theta_L) = \lambda(\theta_H) = 0$.

We now turn to the equilibrium markups $\Omega^*(\cdot)$. Select an arbitrary interior type $\widehat{\theta}$. From the transversality conditions and $(C4)$ we obtain the follows relations:

$$\lambda(\widehat{\theta}) = \int_{\theta_L}^{\widehat{\theta}} \lambda'(\theta)d\theta = \frac{1}{2\tau}\int_{\theta_L}^{\widehat{\theta}} \left[\tau - \Omega^*(\theta)\right] f(\theta)d\theta, \text{ and} \tag{C7}$$

$$\lambda(\widehat{\theta}) = -\int_{\widehat{\theta}}^{\theta_H} \lambda'(\theta)d\theta = -\frac{1}{2\tau}\int_{\widehat{\theta}}^{\theta_H} \left[\tau - \Omega^*(\theta)\right] f(\theta)d\theta. \tag{C8}$$

But since $\lambda(\widehat{\theta}) > 0$, $(C7)$ implies that $\int_{\theta_L}^{\widehat{\theta}} \left[\tau - \Omega^*(\theta)\right] f(\theta)d\theta > 0$ and therefore $\tau > E\left[\Omega^*(\theta) \mid \theta \leq \widehat{\theta}\right]$. Similarly $(C8)$ implies that $\int_{\theta_L}^{\widehat{\theta}} \left[\tau - \Omega^*(\theta)\right] f(\theta)d\theta < 0$ and therefore $E\left[\Omega^*(\theta) \mid \theta \geq \widehat{\theta}\right] > \tau$.

Proof of Theorem 2

Available from the authors.

Proof of Corollary 1

Following the proof of Theorem 1 (which applies for any monotonic quality allocation), the equilibrium is characterized by the system $(C3)-(C5)$ with $\widetilde{c}(q)$ in the place of $c(q)$, and $\widetilde{\Omega}(\theta)$ in the place of $\widetilde{\Omega}(\theta)$. This system becomes

$$S_q(q(\theta), \theta) - \alpha'(q) = -\frac{2\lambda(\theta)}{f(\theta)} v_q(q(\theta), \theta), \tag{$\widetilde{C}3$}$$

$$\widetilde{\Omega}'(\theta) = \theta \cdot v_s(q(\theta), \theta) + q'(\theta) \cdot (S_q(q(\theta), \theta) - \alpha'(q)), \text{ and} \tag{$\widetilde{C}4$}$$

$$\lambda'(\theta) = \frac{1}{2}\left[1 - \frac{\widetilde{\Omega}(\theta)}{\tau}\right] f(\theta). \tag{$\widetilde{C}5$}$$

It therefore follows from the proof of Theorem 2 that $\lambda(\theta)$ is zero for both extreme types and positive for all interior ones.

Moreover we require that $S_q(q(\theta), \theta) = 0$ for all θ (so that quality is first best). Thus, from $(\widetilde{C}3)$ and $(\widetilde{C}5)$ we obtain

$$\begin{aligned} \alpha'(q^{FB}(\theta)) &= \frac{2\lambda(\theta)}{f(\theta)} v_q(q^{FB}(\theta), \theta) \\ &= \frac{1}{f(\theta)} v_q(q^{FB}(\theta), \theta) \int_0^{\theta} \left[1 - \frac{\widetilde{\Omega}(z)}{\tau}\right] f(z)dz, \end{aligned} \tag{C9}$$

where the second equality follows from integrating over $(\widetilde{C}5)$ to obtain $\lambda(\theta)$. Finally, the fact that $\alpha'(q^{FB}(\theta))$ is zero from the extreme types, and positive for the interior ones, follows from $(C9)$ and the fact that $\lambda(\theta)$ has this same properties.

Proof of Theorem 3

Once the price tax $\beta(p)$ is added, dropping the L subindex, firm L's problem becomes

$$\max_{V(\theta),q(\theta)} \int_{\theta_L}^{\theta_H} [S(q(\theta),\theta) - V(\theta) - \beta(p(\theta))]\, G(\widehat{x}(\theta))dF(\theta)$$

$$\max_{V(\theta),q(\theta)} \int_{\theta_L}^{\theta_H} [S(q(\theta),\theta) - V(\theta) - \beta(\theta v(q(\theta),\theta) - V(\theta))]\, G(\widehat{x}(\theta))dF(\theta)$$

$$s.t.$$

$$V'(\theta) = v(q(\theta),\theta) \text{ for all } \theta,$$

where $p(\theta)$ has been expressed, from the definition of $V(\theta)$, as $\theta v(q(\theta),\theta) - V(\theta)$.

As before, the above problem can be stated as an optimal control problem with state variable $V(\theta)$ and control variable $q(\theta)$. The corresponding Hamiltonian is given by

$$H(\theta) = [S(q(\theta),\theta) - V(\theta) - \beta(\theta v(q(\theta),\theta) - V(\theta))]\, G(\widehat{x}(\theta))f(\theta) + \lambda(\theta)v(q(\theta),\theta),$$

The solution is characterized by the transversality condition $\lambda(\theta_H) = 0$ combined with the following Hamiltonian system, which is an extension of the system $(C3)$ – $(C5)$. For all θ,

$$S_q(q(\theta),\theta) - \beta'(\theta v(q(\theta),\theta) - V(\theta))\theta v_q(q(\theta),\theta) = -\frac{2\lambda(\theta)}{f(\theta)} v_q(q(\theta),\theta), \qquad (C10)$$

$$\Omega'(\theta) = \theta \cdot v_s(q(\theta),\theta) + q'(\theta) \cdot S_q(q(\theta),\theta), \text{ and} \qquad (C11)$$

$$\lambda'(\theta) = \frac{1}{2}\left[1 - \frac{\widetilde{\Omega}(\theta)}{\tau}\right] f(\theta) - \frac{1}{2}\beta'(\theta v(q(\theta),\theta) - V(\theta))f(\theta), \qquad (C12)$$

As before, $\int_{\theta_L}^{\theta_H} \lambda'(\theta)f(\theta)d\theta = 0$, and therefore $\lambda(\theta_L) = 0$.

Moreover, since we require that $S_q(q(\theta),\theta) = 0$ for all θ, from $(C10)$ and $(C11)$ we obtain

$$\beta'(p^*(\theta)) = \beta'(\theta v(q(\theta),\theta) - V(\theta)) = \frac{2\lambda(\theta)}{\theta f(\theta)} \qquad (C13)$$

$$= \frac{1}{\theta f(\theta)} \int_0^{\theta} \left[1 - \frac{\widetilde{\Omega}(z)}{\tau}\right] f(z)dz - \frac{1}{\theta f(\theta)} \int_0^{\theta} \beta'(p^*(z))f(z)dz.$$

where the last equality follows from integrating over $(C12)$ to obtain $\lambda(\theta)$. This relation delivers part **b** of the theorem.

Finally, part **a** of the theorem follows from $(C13)$ and the fact that both $\lambda(\theta_L)$ and $\lambda(\theta_H)$ are zero.

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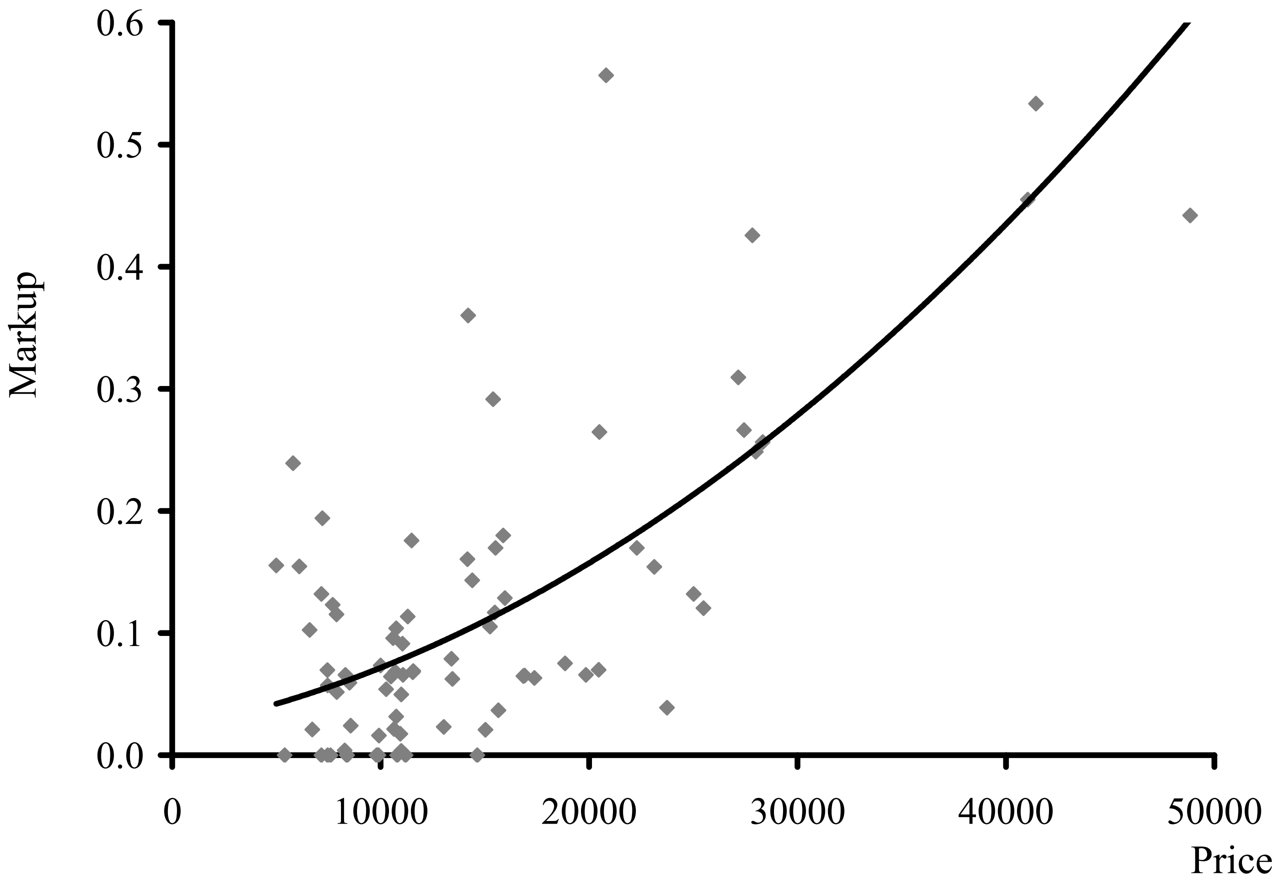
Figure 1

Markups as a percentage of Car Price



Markups as a percentage of price for 1987 car models (1987 dollars). Underlying market structure: Cournot for European cars, Bertrand for all others. Taken from Feenstra and Levinsohn (1995).

Figure 2

Quality, Markup, and Equilibrium Utility

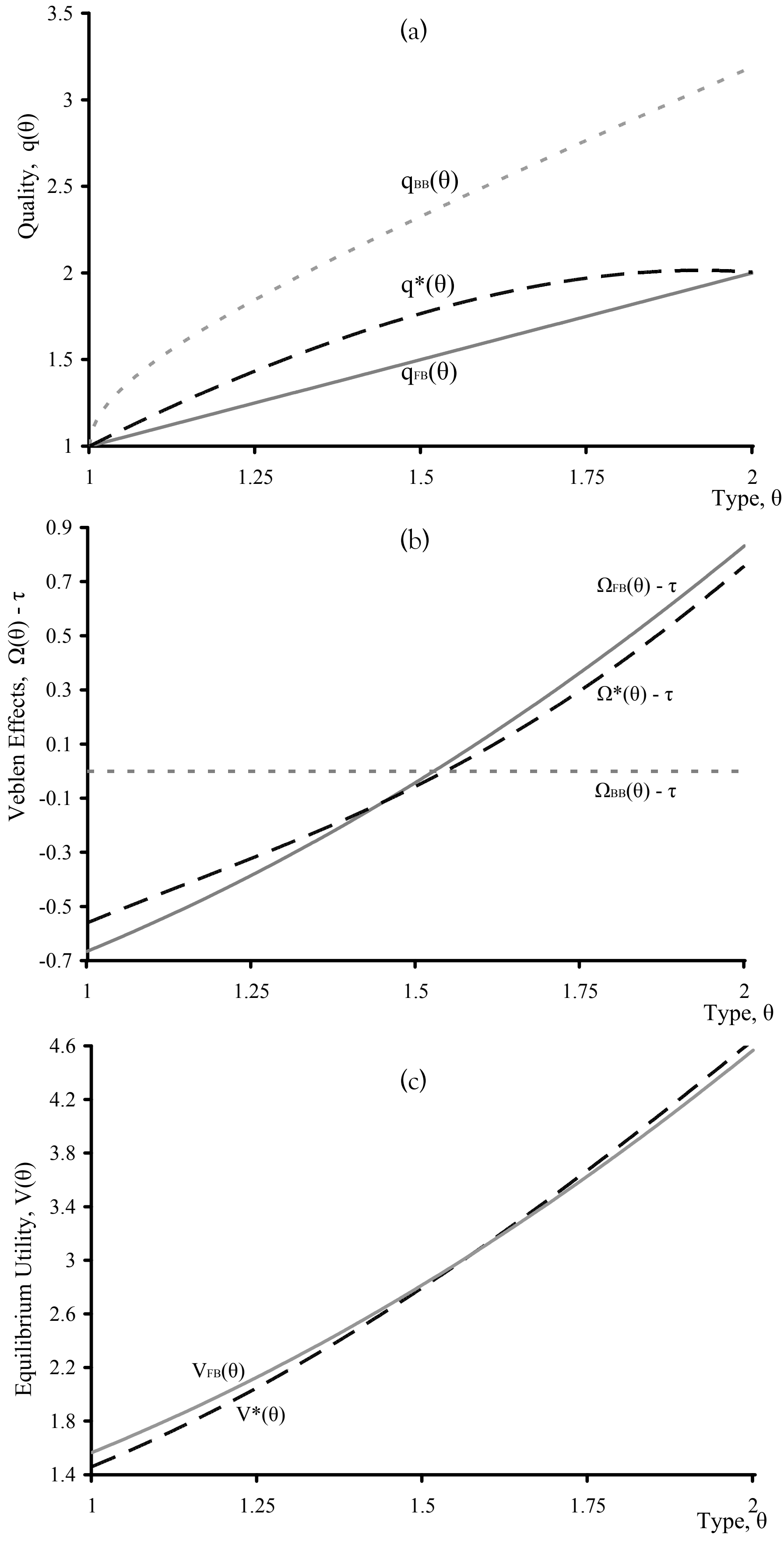


Figure 3

Tax Schedule on Quality