Adverse Selection in Team Formation under Discrimination

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Abstract

A model of adverse selection when teams form endogenously shows: (1) Discrimination can persist even when management allocates credit fairly, if worker beliefs about discrimination influence their decision to engage in teamwork. (2) Among those working individually, discrimination victims outperform beneficiaries. Thus, measures of discrimination calculated only in team settings overestimate the relative adversity faced by victims in the population at large.

1 Introduction

Teamwork facilitates discrimination because it obscures the individual contributions of team members – managers can impute unobservable individual contribution from observable characteristics like race or sex. Several empirical studies on occupational segregation indicate (1) minorities are more likely to choose (or be chosen for) occupations in which teamwork plays a minimal role and (2) that minorities excel in these individualistic positions.

For example, Clark and Drinkwater (2000) document minority over-representation in the British self-employment sector, even though most new firms fail within four years (Shane, 2008). Lempert, Chambers and Adams (2000) report that among law school graduates choosing private sector employment, minorities are far more likely to open their own practice or join very small practices. Strikingly, they also find that minority graduates enjoy higher income than their white peers,[1] despite the fact that salaries of small firm attorneys are significantly lower than those of large firms (NALP 2009).

Even when teamwork characterizes the industry, as in the case of team sports, minorities disproportionately occupy positions where contribution is individualistic and more measurable. Loy and Elvogue (1970) categorize the various positions in baseball and football as either central or peripheral based on their interaction level with other players. They observe that minorities occupy the majority of peripheral, or so-called "[measurable] skill," positions while whites dominate central ones. Similarly, in hockey, Lavoie, Grenier and Coulombie (1987) observe that Francophones are overrepresented at goalie and underrepresented on defense. By objective measures, like points scored, minority players outperform non-minorities in all of these sports (see Kahn 1991 for a review).

A minority worker should choose self-employment, or other occupation where his individual accomplishments cannot easily be attributed to others, *if doing so will make him better off*. And indeed, Clark and Drinkwater (2000) verify that self-employed minorities earn more than if they were traditionally employed. But clearly, individuals who are most talented relative to their employers' perceptions have the greatest incentive to eschew teamwork – if high ability minorities believe they will face discrimination on teams, they will avoid them. Could this self-selection sustain discrimination even if managers paid workers proportional to their expected ability – that is, according to their merit?

This paper shows that it can. In the model, discrimination arises because workers endogenously choose either to work as part of a team, where individual merit is hard to measure, or as an individual, where it is easy to measure. Workers differ in unobservable ability as well as *ex ante* uncorrelated, observable characteristics, like race. If high ability minority[2] workers mostly elect to work outside of teams, then meritocratic managers will rationally discriminate in allocating credit for team production. This effect reinforces itself, since high ability minorities will be better off working individually than face discrimination on a team.

Race becomes an *ex post* indicator of ability not just for team workers but for those

[1] Though Lempert et al (2000) find a positive correlation between non-white ethnicities or minority status and log income, it is not always significant.

[2] Minority here is synonymous with discrimination victim and non-minority with beneficiary. The attribute of discrimination can be completely arbitrary.

who work alone – among individuals working individually, minorities will outperform non-minorities on average. Thus, if the fraction of team credit allocated to minority team members quantifies the level of discrimination, then this measure always overestimates the relative disadvantage minorities face overall. In fact, under certain circumstances, minorities receive higher average compensation than non-minorities when both team and solo workers are considered.

Discrimination may induce teamwork that would not occur in an egalitarian setting. For example, a highly skilled white worker may be unwilling to collaborate with anyone less competent than himself if credit is to be split evenly, but under discrimination he may be willing to team up with a minority worker who is not as competent as he is, since he will get most of the credit anyway. If synergy between these teammates is high enough, discrimination is socially optimal in that total societal output is higher than under egalitarianism.

The paper's primary findings are summarized as follows:

1. Discrimination can persist even when management fairly allocates credit, if worker beliefs about discrimination influence their decision to engage in teamwork.

2. Among those working individually, victims will outperform beneficiaries. Thus, measures of discrimination calculated only in team settings overestimate the relative adversity faced by victims in the population at large.

Previous models of statistical discrimination require that victims differ from beneficiaries at the time of employment either (a) in discourse or (b) (endogenously) acquired human capital. But when teams form endogenously adverse selection can lead to statistical discrimination, which leaves victim and beneficiary characteristics in the population, beyond the attribute of discrimination, statistically identical at all times.

The next section highlights related literature. The third presents a basic model of endogenous team formation, production and credit allocation. The fourth section illustrates the basic intuitions when workers are only of two abilities. The fifth tests the robustness of these intuitions in more general settings, and the sixth concludes. Technical proofs are removed to the appendix for clarity.

2 Related Literature

In economics, workplace discrimination either stems from employer tastes (Becker 1957) or from conditioning expected worker contribution on observable characteristics, like race, to (partially) resolve imperfect observability of employee productivity. Phelps (1972)[3] and Arrow (1973) pioneered work on this latter type, known as statistical discrimination.

Arrow and later Coate and Loury (1993) showed that because minorities respond strategically to lower incentives to invest in unobservable skill sets prior to employment, employers' discriminatory beliefs can be confirmed ex post, even when minorities and non-minorities have identical ability ex ante. The discriminatory equilibrium resembles the one presented here, but "ex ante" in their models means "at birth." Since minorities and non-minorities acquire different human capital, they no longer possess identical ability when they take jobs.

My model's discrimination is classically statistical, arising from imperfect observability of worker productivity. Team synergy both incents workers to collaborate and obscures their individual contributions. Although minorities who select into teams differ from those who do not, the overall populations of minorities and non-minorities always have statistically identical ability. The signal in this sorting model is slightly more complex than in others. In Arrow's and Coate and Loury's models, race alone (eventually) correlates to ability, but in this model, it is the interaction of race and *observable* self-selection, which informs the manager.

The model relates in a secondary way to the literature on team productivity. Since Alchian and Demsetz' (1972) discussion of the free-rider problem inherent in teams, the team literature has focused on moral hazard in effort.[4] Hamilton, Nickerson and Owan (2003) empirically suggest that this focus on free-riding in teams is too narrow, because even when individual incentives are feasible, many firms implement group incentives to increase productivity.[5] Comprehending such synergies, my model examines the adverse selection in

[3] The body of literature after Phelps' tradition relies on employers being able to extract a less noisy signal of ability from employees they most closely resemble (e.g. in race or sex). It does not inform the model presented here.

[4] See Holmström 1982, McAfee and McMillan 1991, Legros and Matthews 1993 among many others.

[5] First, Hamilton et al (2003) capture an 18% productivity increase in a garment factory after a change from individual piece rate compensation to group piece rate pay. Second, they attribute about a fifth of

team formation that occurs before any moral hazard can; imperfect observability of individual contributions in teams drives both.

On the surface, this gap in the literature seems modest, because many employees cannot choose their coworkers. However, even when a manager hires all employees, the teamwork decision and, hence, adverse selection often remain. Presented with a menu of tasks, an employee may choose an individual task with high management visibility over another, in which his contribution may be lost because of the number and rank[6] of other contributors. The common managerial performance critique, "(not) a team player" indicates that even when tasks are completely assigned, how they are performed can be an expression of the teamwork decision. The model applies to any situation in which workers can trade off synergy for visibility.

3 Basic Model

Two workers, A and B, decide to produce individually or together. A and B have positive iid abilities, α and β respectively, known to both A and B but unobservable to management. Their name (i.e. A or B) is the only available attribute of discrimination.

A and B produce their respective abilities when working individually. Teams produce $g(\alpha, \beta)$. Assume team production is

$$\text{Symmetric:} \quad g(x, y) = g(y, x) \tag{1}$$

$$\text{Synergistic:} \quad g(\alpha, 0) \geq \alpha \text{ and } g(\alpha, \beta) > \alpha + \beta \ \forall \alpha, \beta > 0 \tag{2}$$

$$\frac{d}{d\alpha} g(\alpha, \beta) > 1 \tag{3}$$

A manager determines the portion of team credit (or production) due each worker. This

that increase to the fact that high ability workers were more likely to join teams but the remaining 14% is attributable to the synergistic team effect. High ability workers were no more likely to leave the company than low ability ones after joining a team. These findings counter the free-riding theories, which have dominated economic analysis of teams.

[6] Even such characteristics such as job title or seniority can be attributes of discrimination to the extent they are imperfectly correlated to private ability. A gifted junior employee may rather work alone than share a majority of credit with a mediocre senior one.

credit may be compensation or a manager identifying her next promotee. The fraction of team credit assigned to A is denoted by γ, where γ lies in the unit interval. B receives the remainder. Thus, γ represents the strength of discrimination: $\gamma = 1$ represents a world in which A gets all credit from group work and $\gamma = 0$ one in which he gets none. Importantly, a solo worker always receives all credit for his work. A and B have common beliefs on γ.[7]

The ratio of a worker's proportional (realized) ability to the fraction of team credit he believes he will receive (i.e. $\frac{\alpha}{\alpha+\beta} : \gamma$, $\frac{\beta}{\alpha+\beta} : 1-\gamma$) measures his teamwork incentive. If this *discriminatory alignment* is less than unity[8] then team production must be sufficiently synergistic for that worker to choose teamwork.

The law[9] constrains the manager to allocate credit according to employees' relative contributions as well as she can with available information. If work was completed as a team, the manager credits A equal to his expected proportion of total team ability, given that the work was done as a team. In the Bayesian Nash Equilibrium (BNE), if the manager is assigning credit according to merit, this fraction must, in expectation, equal the credit split, γ, that A and B believed when they decided to form a team:[10]

$$\gamma = \psi(\gamma) = E\left[\frac{\alpha}{\alpha+\beta} \mid team, \gamma\right] \tag{4}$$

Intuitively, the following steps take place:

1. Abilities, α and β, are realized and learned by both A and B.

2. A and B simultaneously[11] decide whether or not to form a team. A and B collaborate

[7] Assuming that victims and beneficiaries believe that discrimination exists to a common degree is strong, but the assuming common beliefs is standard in BNE analysis. Likewise, one may assume that the abilities of *previous* employees working on teams have been revealed to the manager to give some basis for beliefs, but this is not strictly necessary for the analyisis.

[8] If it is greater than unity for one worker then it is less than unity for the other.

[9] Statistical discrimination is illegal in the US; however, since the burden of proof that an employer is not paying equally for equal work belongs to the employee, this constraint of the model is practical.

[10] Although the denominator does not reflect the total output of the team in the above above definition of meritocracy, it is trivial to show that the alternative formulation, $\gamma = E\left[C\frac{\alpha}{g(\alpha,\beta)} \mid team, \gamma\right]$, where $C = \frac{g(\alpha,\beta)}{\alpha+\beta}$ is the unique normalizer such that $1-\gamma = E\left[C\frac{\beta}{g(\alpha,\beta)} \mid team, \gamma\right]$ is equivalent.

[11] The decision need not be simultaneous given the payoffs specified, because both work alone if either or both reject teamwork.

if and only if it is mutually advantageous under their beliefs.

3. The project is completed (either as a group or separately).

4. Credit is apportioned meritocratically.

Worker B

Worker A

	Team	Solo
Team	$g(\alpha,\beta)\gamma,\ g(\alpha,\beta)(1-\gamma)$	$\alpha,\ \beta$
Solo	$\alpha,\ \beta$	$\alpha,\ \beta$

Table 1. Normal form of the simultaneous game

$$\text{Worker } A \text{ chooses} \begin{Bmatrix} \textit{Team} & \text{if } g(\alpha,\beta)\gamma \geq \alpha \\ \textit{Solo} & \text{if } g(\alpha,\beta)\gamma < \alpha \end{Bmatrix} \tag{C_A}$$

$$\text{Worker } B \text{ chooses} \begin{Bmatrix} \textit{Team} & \text{if } g(\alpha,\beta)(1-\gamma) \geq \beta \\ \textit{Solo} & \text{if } g(\alpha,\beta)(1-\gamma) < \beta \end{Bmatrix} \tag{C_B}$$

Table 2. Dominant Strategies of Workers

Even when workers are of just two abilities, the basic insights of the model are clear.

4 Two Ability Types

For exposition assume the following team production function: $g(\alpha,\beta) = (\alpha+\beta)\kappa$ where κ measures synergy ($\kappa > 1$). Suppose also, α and β are distributed iid generalized Bernoulli:

$$\begin{aligned} \alpha,\beta &= \begin{Bmatrix} H \text{ with probability } p \\ L \text{ with probability } \overline{p} = 1-p \end{Bmatrix} \\ H &> L > 0 \end{aligned} \tag{5}$$

I first establish the intuitive result that if workers believe that no discrimination exists, then, in this completely symmetric world with fair management, none will. But such beliefs are not enough to guarantee talented individuals will cooperate with those less so – in fact, as we will see, egalitarianism may impede it.

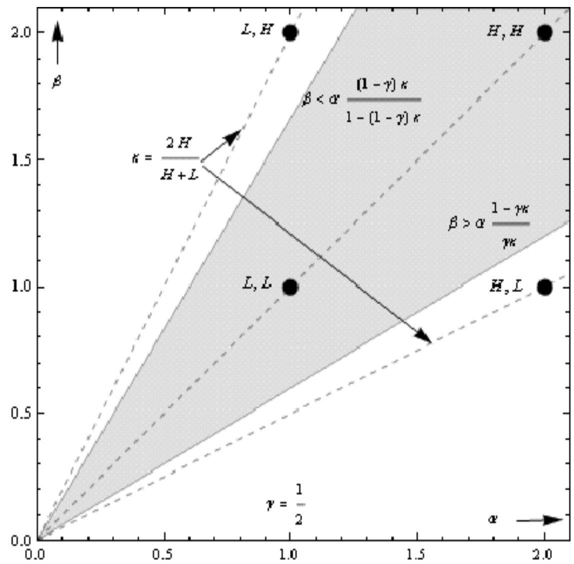

Figure 1: Team forming region of ability sample space under egalitarianism

Proposition 1 *(a) Egalitarianism (i.e. $\gamma = \frac{1}{2}$) is always an equilibrium. (b) Under egalitarianism, teams always form if synergy is strong enough (i.e. $\kappa \geq \kappa_4$, where $\kappa_4 = \frac{2H}{H+L}$), but low synergy (i.e. $\kappa < \kappa_4$) induces only homogeneous (i.e. $\alpha = \beta$) teams.*[12]

Proof. When $\frac{2H}{H+L} \leq \kappa$, (C_A) and (C_B) are satisfied for all α and β. When $\kappa < \frac{2H}{H+L}$, (C_A) and (C_B) are satisfied if and only if $\alpha = \beta$. In both cases $\frac{1}{2} = \psi\left(\frac{1}{2}\right)$ by symmetry. ■

Since workers have iid ability, if they believe management thinks observable characteristics are orthogonal to ability, workers will disregard observables in choosing teammates. Fair managers will then disregard observables too, and egalitarianism will be an equilibrium. Proposition 1 part (a) holds for all distributions and production functions (see the Appendix for a proof).

Even absent any discrimination, teams will not always form, though teamwork is efficient. Thus, egalitarianism does not alleviate adverse selection. Proposition 1 part (b) can be restated for general distributions and productions functions: *Under egalitarianism, a team*

[12] The notation for indexing key synergy levels can be thought of in the following way: κ_4 is the threshold above which all four possible ability realizations form a team under egalitarianism, κ_3 and $\kappa_{\bar{4}}$ represent the thresholds between which three but not four realizations will form teams if beliefs are those specified in Proposition 2. Finally $\kappa_{\bar{2}}$ represents the threshold below which one realization, but not two will form if beliefs are those specified in Proposition 3.

will form iff the synergy of production is greater than the difference in worker abilities (see the Appendix for a proof).

As synergy declines in an egalitarian world, only exact matches will team up. If synergy results from labor specialization or leadership, this homogeneity may be undesirable. In fact, Hamilton, *et al.* (2003) find that, with average ability held constant, heterogeneous teams are more productive.[13] Figure 1 graphically depicts the team forming region of the ability sample space when $\kappa < \kappa_4$; observe that heterogenous pairings $\langle H, L \rangle$ and $\langle L, H \rangle$ fall outside.

Each of the following two propositions identifies a discriminatory equilibrium. That is, if workers believe that discrimination (of a particular level) exists, they will form teams such that a fair manager will reinforce those beliefs. The first equilibrium is mild, causing both homogeneous and heterogeneous ability teams to form. The second equilibrium is severely discriminatory, and only heterogeneous teams form. Figures 2 and 3 illustrate the shift in team forming regions of the ability sample space as beliefs change.

For every discriminatory equilibrium belief, $\gamma > \frac{1}{2}$, favoring A, model symmetry induces another, $1 - \gamma < \frac{1}{2}$, favoring B. Analysis, though, is limited to $\gamma \geq \frac{1}{2}$.

Proposition 2 *For intermediate synergy levels there exists a moderately discriminatory equilibrium in which both homogeneous teams and heterogeneous teams form. Formally,*

$$\kappa_3 \leq \kappa < \kappa_{\overline{4}} \implies \gamma_M = \frac{1}{2} p_e + \frac{H}{H+L} (1 - p_e)$$

where

$$\kappa_3 = \frac{2H (1 - p\overline{p})}{H + L (1 - 2p\overline{p})}, \; \kappa_{\overline{4}} = \frac{2H (1 - p\overline{p})}{H (1 - 2p\overline{p}) + L}$$

and

$$p_e = \Pr \{\alpha = \beta \mid team\} = \frac{p^2 + \overline{p}^2}{p^2 + p\overline{p} + \overline{p}^2}$$

[13] The reader will observe that $g(\alpha, \beta) = (\alpha + \beta) \kappa$ does not increase with hetrogeneity of ability. Although a team production function with negative cross partials with respect to ability may reflect Hamilton et al's empirical findings better, the example function is conservative in that it overstates productivity under egalitarianism relative to discrimination. An in depth study of group production is beyond the scope of this paper.

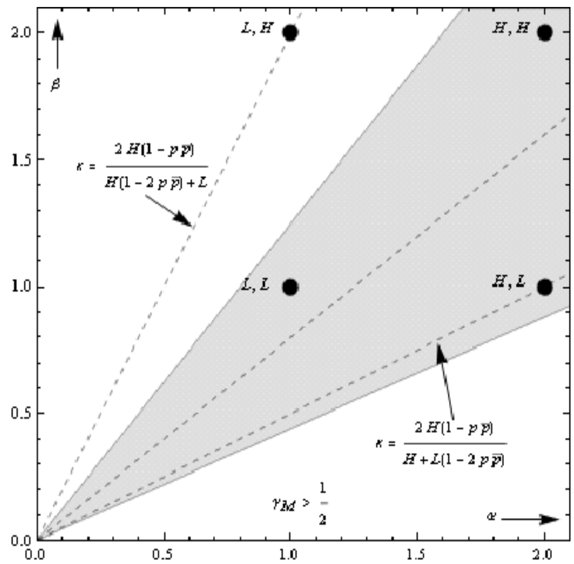

Figure 2: Team forming region of ability sample space under moderate discrimination

is a discriminatory equilibrium in which realizations $\langle \alpha, \beta \rangle \in \{\langle H, H \rangle, \langle H, L \rangle, \langle L, L \rangle\}$*form teams.*

Proof. Let $\gamma' = \frac{1}{2}p_e + \frac{H}{H+L}(1 - p_e) = E\left[\frac{\alpha}{\alpha+\beta} \mid \langle \alpha, \beta \rangle \in \{\langle H, H \rangle, \langle H, L \rangle, \langle L, L \rangle\}\right]$. Assume $\gamma = \gamma'$. (C_A) and (C_B) are satisfied for all $\langle \alpha, \beta \rangle \in \{\langle H, H \rangle, \langle H, L \rangle, \langle L, L \rangle\}$ if and only if $\frac{2H(1-p\overline{p})}{H+L(1-2p\overline{p})} \leq \kappa$. (C_B) is not satisfied for $\langle \alpha, \beta \rangle = \langle L, H \rangle$ if and only if $\kappa < \frac{2H(1-p\overline{p})}{H(1-2p\overline{p})+L}$. Thus, $\psi(\gamma') = \gamma'$. ■

Proposition 3 *If synergy is not extremely strong, then there exists a severely discriminatory equilibrium in which only heterogeneous teams form. Formally,*

$$\kappa < \kappa_{\overline{2}} \implies \gamma_s = \frac{H}{H+L}$$

where

$$\kappa_{\overline{2}} = \frac{H+L}{2L}$$

is a discriminatory equilibrium in which realizations $\langle \alpha, \beta \rangle \in \{\langle H, L \rangle\}$*form teams.*

Proof. Let $\gamma' = \frac{H}{H+L} = E\left[\frac{\alpha}{\alpha+\beta} \mid \langle \alpha, \beta \rangle = \langle H, L \rangle\right]$. Assume $\gamma = \gamma'$. (C_A) and (C_B) are satisfied for $\langle \alpha, \beta \rangle = \langle H, L \rangle$ if and only if $1 < \kappa$. (C_B) is not satisfied for $\langle \alpha, \beta \rangle = \langle L, H \rangle$

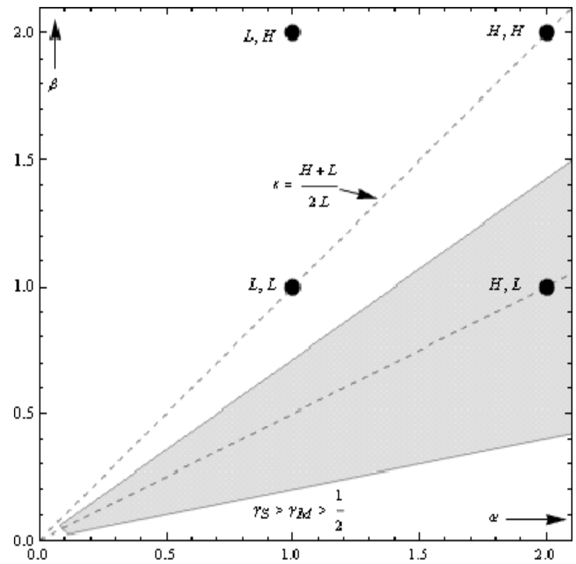

Figure 3: Team forming region of ability sample space under severe discrimination

if and only if $\kappa < \frac{H}{L}$. (C_B) is not satisfied for $\langle \alpha, \beta \rangle \in \{\langle H, H \rangle, \langle L, L \rangle\}$ if and only if $\kappa < \frac{H+L}{2L} < \frac{H}{L}$. Thus, $\psi(\gamma') = \gamma'$. ■

Propositions 1-3 say that if synergy is so strong that everyone chooses teamwork all the time, egalitarianism is the only equilibrium, but as it declines (1) discriminatory and egalitarian equilibria coexist (note that $\kappa_3 < \kappa_4 < \kappa_{\overline{4}}$ and $\kappa_4 < \kappa_{\overline{2}}$) and (2) adverse selection prevents efficient team formation under egalitarianism and discrimination alike.

Conventional wisdom holds that discriminating against anyone on an attribute that has no direct causal link to productivity cannot improve output and may very well be counter-productive. Indeed, since any pair working together produces more than the two working alone, in a first-best world all possible parings would cooperate and egalitarianism would be the *only equilibrium*. But adverse selection creates a tension between social optimality and egalitarianism, because the severity of this adverse selection is not equal under both regimes – as κ falls below κ_4 and until it falls below κ_3 (always an open interval), more teamwork occurs under the discriminatory equilibrium of Proposition 2. In fact, since every team that forms under egalitarianism also forms under discrimination, and teams always outproduce their members working individually, the following proposition is immediate:

Proposition 4 *If synergy is moderate ($\kappa_3 \leq \kappa < \kappa_4$) there exists a discriminatory equilib-*

rium ($\gamma = \frac{1}{2}p_e + \frac{H}{H+L}(1-p_e)$) *that is socially preferable to egalitarianism.*

Clearly discrimination harms individuals, and if it were true that it unequivocally harmed society as well, then debates about policy to eradicate it could be limited to how and not whether it is desirable to do so. Lamentably, Proposition 4 dispels such an unambiguous justification: *with respect to productivity, egalitarianism is not always socially preferable to discrimination.*[14]

To understand how synergy and discrimination influence relative productivity under discrimination and egalitarianism, consider the progression of the team forming region in the ability sample space depicted in Figures 1 through 3. The level of (believed) discrimination defines a locus of worker pairings that will cooperate even without synergy, namely all pairings with discriminatory alignment equal to unity: $\frac{\beta}{\alpha} = \frac{1-\gamma}{\gamma}$. Adding synergy expands the team forming region around this locus, increasing both the minimum B type A is willing to work with (i.e. $\beta > \alpha\frac{1-\gamma\kappa}{\gamma\kappa}$ when $g(\alpha,\beta) = \kappa(\alpha+\beta)$) and the minimum A type B is willing to work with (i.e. $\beta > \alpha\frac{(1-\gamma)\kappa}{1-(1-\gamma)\kappa}$ when $g(\alpha,\beta) = \kappa(\alpha+\beta)$). Changing discriminatory beliefs changes the slope of that central locus and shifts the entire team forming region with it – as discriminatory beliefs increase, every A type is willing to accept weaker teammates, but every B type is less willing to do so. Therefore, an increase in discrimination induces some new teams to form and some existing ones to break up. The new teams' increased output is a societal gain, but society loses output from those individuals formerly working on teams. Whether or not the gain exceeds the loss depends on the specific distribution of ability and team production function.

If synergy is very low only those teams near the central locus will form; the productivity difference between egalitarianism and discrimination (and individual work) is minimal. As synergy increases, the productivity difference can grow, but if synergy increases enough to induce all teams to form, a fair manager cannot discriminate; the discriminatory equilibrium disappears rendering the comparison irrelevant. Although beyond the scope of this model,

[14] Lundberg and Startz' (1983) conclude egalitarianism is always socially preferable. In their model, discrimination shifts training from individuals with lower costs for it to those with higher costs. In this one, discrimination may enable synergistic production when egalitarianism may not, and the increased productivity comes not from a change in the individual, but rather the production function.

in which the manager is constrained to be fair, these observations suggest that managers in highly synergistic industries may have incentive to be discriminatory and *unfair*, as it may be profitable.

Despite the fact that the manager here cannot strategically *choose* to be discriminatory, both egalitarianism and discrimination are equilibria in the moderate synergy range ($\kappa_3 \leq \kappa < \kappa_4$). The following evolutionary argument refines these equilibria. Suppose that the synergy of team production varies over a range of industries, each comprised of many competing firms. Some firms in each industry choose seniority based compensation (i.e. discriminatory) schemes while others compensate teammates equally. In industries with moderate ($\kappa_3 \leq \kappa < \kappa_4$) synergy levels only seniority based firms survive due to the efficiency gains of discrimination, and, of course, seniority both reflects the relative contribution of teammates and is the right competitive "strategy" ex post. In industries with synergy outside this range, egalitarian firms dominate.

The productivity of society is the productivity of its members. The equilibrium definition describes the relative productivity of minorities and non-minorities on teams, but what about those working alone? And minorities versus non-minorities overall? Victims and beneficiaries remain statistically identical with respect to ability across the population at large, but not among those working on teams – this is precisely why a fair manager can discriminate after observing the teamwork decision.

Proposition 5 *Among those working individually, discrimination victims outperform beneficiaries on average.*

Proof. The proposition can be formally written

$$E\left[\alpha \mid solo\right] < E\left[\beta \mid solo\right]$$

which can be verified under each discriminatory equilibrium.

Case $\gamma = \frac{H}{H+L}$:

$$\frac{Hp^2 + Lp\overline{p} + L\overline{p}^2}{p^2 + p\overline{p} + \overline{p}^2} < \frac{Hp^2 + Hp\overline{p} + L\overline{p}^2}{p^2 + p\overline{p} + \overline{p}^2}$$

Case $\gamma = \frac{1}{2}p_e + \frac{H}{H+L}\left(1 - p_e\right)$:

$$L < H$$

■

The model predicts greater success for self-selected minority entrepreneurs. Very able discrimination victims have the lowest discriminatory alignment, especially when potential partners are incompetent, and thus have the greatest incentive to work as individuals. Since a rejection of teamwork induces victim and beneficiary alike to work independently, victims have higher average ability in the solo working population. This intuition holds for general distributions and production functions (see the Appendix for a proof).

In the model presented here, a rejection by either teammate means both work alone. However, one of the teammates would have preferred teamwork. If either had another potential teamwork opportunity he might try again. Thus, in a market for teammates, the probability of rejection matters. The following two lemmas show that when teamwork is endogenous, adverse selection works causes two groups to eschew teamwork: (1) talented individuals and (2) discrimination victims.

Lemma 1 *Talented individuals reject teamwork more often.*

Proof. Let $\tilde{A}$ be the event that A rejects teamwork (i.e. $g(\alpha, \beta)\gamma < \alpha$) and $\tilde{B}$ be the event B rejects teamwork (i.e. $g(\alpha, \beta)(1-\gamma) < \beta$). Given that there is a rejection, the probability that the more able did it (recall that only one worker will reject due to synergy) can be written

$$\Pr\left\{\tilde{A} \mid \alpha > \beta\right\} + \Pr\left\{\tilde{B} \mid \alpha < \beta\right\} > \Pr\left\{\tilde{B} \mid \alpha > \beta\right\} + \Pr\left\{\tilde{A} \mid \alpha < \beta\right\}$$

where all probabilities are conditional on $\tilde{A} \cup \tilde{B}$. This holds iff

$$\begin{aligned}
& \Pr\left\{\tilde{A} \mid \alpha > \beta\right\} + 1 - \Pr\left\{\tilde{A} \mid \alpha < \beta\right\} > 1 - \Pr\left\{\tilde{A} \mid \alpha > \beta\right\} + \Pr\left\{\tilde{A} \mid \alpha < \beta\right\} \\
\iff \quad & \Pr\left\{\tilde{A} \mid \alpha > \beta\right\} > \Pr\left\{\tilde{A} \mid \alpha < \beta\right\} \\
\iff \quad & \Pr\left\{g(\alpha, \beta)\gamma < \alpha \mid \alpha > \beta\right\} > \Pr\left\{g(\alpha, \beta)\gamma < \alpha \mid \alpha < \beta\right\}
\end{aligned}$$

which is always true. ■

Lemma 2 *Discrimination victims reject teamwork (with beneficiaries) more often.*

Proof. By symmetry, $\gamma > \frac{1}{2} \Leftrightarrow \Pr\{g(\alpha,\beta)\gamma < \alpha\} < \Pr\{g(\alpha,\beta)(1-\gamma) < \beta\}$ ■

This suggests that talented individuals and discrimination victims reject teamwork more overall, choosing different work than discrimination beneficiaries and those of lesser ability. Consider the following set of empirical observations. Judge and Cable (2004) calculate that a worker, on average, earns $789 (1991 USD) more annually for each inch of physical height. In light of this apparent discrimination, a rational worker will weigh his privately known ability against his publicly observable height when deciding to work as part of a team or not. Djankov et al (2005) conducted a survey of Russian entrepreneurs. Among individual characteristics, two of the top three strongest predictors of entrepreneurship were found to be cognitive ability (positively correlated) and physical height (negatively correlated). Both proved more robust than such stereotypical characteristics such as risk-taking. Although Djankov et al do not explain why smart, short individuals choose entrepreneurship more often, the model presented here can. Likewise, as suggested by the occupational segregation in sports (highlighted in the introduction), even among those traditionally employed, discrimination victims and the gifted will choose individually measurable occupations over synergistic ones.

The natural context in which to empirically measure discrimination is when minorities and non-minorities are working side by side, such as within firms. And indeed, discrimination in these typically team settings is precisely what the model captures, but minority and non-minority workers in teams do not represent the populations as a whole, because occupational choice is endogenous. Thus, this measure of discrimination does not reflect the overall adversity faced by minorities in the population. Since, by Proposition 5, discrimination victims outperform beneficiaries when working individually, the following is immediate:

Proposition 6 *Discrimination victims receive more than* $1-\gamma$ *of total societal output on average.*

A high ability minority worker can opt out of discriminatory teams, say by choosing entrepreneurship or another occupation where individual contribution is more measurable. Thus, measuring discrimination as wage differences only within team settings overestimates the overall negative relative impact of discrimination on minorities. In fact, as the following example paradoxically shows, the total relative impact of discrimination on minorities need not be negative at all!

Example 1 *If High ability is rare (i.e.* $p = \frac{1}{8}$*), but very potent relative to Low ability (i.e.* $H = 20$ *and* $L = 1$*), and synergy is strong (i.e.* $\kappa = 175\%$*), then* $\gamma = \frac{5}{9}$ *is a discriminatory equilibrium – in other words, non-minorities receive* 125% *(i.e.* $\frac{\gamma}{1-\gamma}$*) what minorities do in teams. Despite the high synergy of teamwork, the presence of discrimination and the relative rarity of individual work (i.e.* $\frac{7}{64} \approx 11\%$ *of the time), minorities receive over* 127% *of the credit that non-minorities receive overall.*

Proposition 5, Lemmas 1 and 2 as well as Proposition 6 hold under any discriminatory beliefs, even if those beliefs are not confirmed by a fair manager – that is, γ need not be a self-reinforcing equilibrium. For example, even if the manager has a preference for employees of one race over another (i.e. she is not fair) as is assumed in some other models of discrimination (e.g. Becker, 1957), talented minorities will *still strategically respond* by opting out of teams. We conclude the basic analysis by considering some comparative statics of the self reinforcing equilibrium of the model.

In one industry all workers may produce at similar levels, but in another the gap between highly productive workers and low productivity ones may be large. How does this production sensitivity to ability impact discrimination? Can different industries support different levels of statistical discrimination?

Proposition 7 *As the ability gap between High and Low types increases, so does (a) discrimination (in any discriminatory equilibrium) and (b) the maximum synergy, for which discriminatory equilibria exist.*

Proof. (a) $\frac{d\gamma}{dH} > 0$ and $\frac{d\gamma}{dL} < 0$ for both discriminatory equilibria. (b) $\frac{d\kappa_{\overline{4}}}{dH} > 0, \frac{d\kappa_{\overline{4}}}{dL} < 0, \frac{d\kappa_{\overline{2}}}{dH} > 0$ and $\frac{d\kappa_{\overline{2}}}{dL} < 0$. ■

Proposition 7 predicts that discrimination will be stronger in occupations, in which the (relevant) ability of workers exhibits higher variance. To see the intuition behind part (a) recall that discriminatory beliefs are an equilibrium, because on teams the expected type of beneficiaries is higher than victims' – either beneficiaries (on teams) are more likely to be

High ability, less likely to be Low ability or both. Thus, if High increases or Low decreases, the expected ability gap increases and so does discrimination. The intuition behind part (b) is also simple. A High ability worker will work with a Low ability worker even if he believes he will be the victim of discrimination, *so long as synergy is high enough.* If this happens, a fair manager cannot discriminate (i.e. discrimination cannot be an equilibrium). But if the ability gap between these two increases even more, then these two may not work together, and now a fair manager must discriminate.

The model provides a way to analyze discriminatory settings in which existing theory says little. Although applications of the model are varied, one familiar to many readers is academic coauthoring. Einav and Yariv (2006) show that the probability of receiving tenure at a top economics department declines significantly with the alphabetic ordering of one's surname initial, even when accounting for country of origin, ethnicity and religion. This discrimination is difficult to analyze with existing economic models of discrimination – it is hard to imagine that a taste for individuals with last names beginning with A exists, that their discourse is somehow different from those with last names beginning with B, or that their childhood environment has been so different that they have endogenously acquired different human capital. It is not farfetched, though, that economics researchers with surname initials at the end of the alphabet may choose coauthors (teamwork) strategically, given the discriminatory convention of alphabetic surname ordering on economics publications. By casting the body of academic peers in the role of a merit-fair manager (credit allocator) the coauthoring (team-formation) decision may be analyzed with the predictions of the model. Proposition 5 predicts that the solo work of authors with surname initials near the end of the alphabet are of higher quality than the solo work of authors near the beginning of the alphabet. If the variance in ability at a department increases with its rank then Proposition 7 predicts discrimination will be strongest within the highest tiered departments.

5 Robustness

The simple model can be generally extended in several ways: (1) ability distributions and production functions can be made general, (2) a market for teammates can be added, and

(3) the credit split can be made contractible on the output. In this section, I show that discriminatory equilibria survive each of these extensions.

5.1 General Ability Distributions and Production Functions

One might worry that the existence of discriminatory equilibria is an artifact of the two type model or a very specific team production function. The next two propositions reassure that they are not. One additional definition is required:

Definition 1 *If an individual with no ability whatsoever joins a team and production is not improved (i.e.* $g(x,0) = x$*), then team production is* regular.

Example 2 $g(\alpha,\beta) = \alpha + \beta + \kappa\alpha\beta$ *is a* regular *production function.*

Proposition 8 *If team production is regular, and ability is continuously distributed with support from* 0*, then at least one discriminatory equilibrium exists in which teams form.*

Proof. The proof in the Appendix has the following steps: (1) $\lim_{\gamma\to 1}\psi(\gamma) = 1$, (2) $\lim_{\gamma\to 1}\psi'(\gamma) < 1$, (3) $\psi'\left(\frac{1}{2}\right) = 0 < 1$ and (4) this implies $\psi(\gamma)$ has a fixed point in $\left(\frac{1}{2},1\right)$. ■

Proposition 8 provides a sufficient but unnecessary condition for discriminatory equilibria to exist. There may be many such equilibria; Proposition 8 simply says that under reasonable conditions, *at least one* set of discriminatory beliefs exists such that workers will choose teamwork strategically such that a fair manager will confirm those beliefs. Regularity guarantees that the second step of the proof holds – many irregular production functions also satisfy the second step of the proof but must be handled on a case by case basis.

The concept of a self-enforcing equilibrium (SRE), considered so far, is precise – an equilibrium exists whenever $\gamma = \psi(\gamma)$. If private worker abilities can be uncovered, then empirical analysis can only tell us that γ is sufficiently close to $\psi(\gamma)$. This, as the following proposition highlights, is a much looser condition than SRE and may hold for a very wide set of beliefs that are not, strictly speaking, equilibria.

Proposition 9 *If ability is continuously distributed with support from* 0 *and synergy is low enough, the credit split of a fair manager will be arbitrarily close to confirming* any *beliefs.*

Formally, if synergy is measured by $\kappa(\alpha,\beta) = g(\alpha,\beta) - \alpha - \beta$*, then for all* $\varepsilon > 0$*,* $0 < \kappa(\alpha,\beta) \leq \varepsilon(\alpha+\beta) \implies |\gamma - \psi(\gamma)| < \varepsilon$*.*

Proof.

$$\begin{aligned}
|\gamma - \psi(\gamma)| &= \left|\gamma - E\left[\tfrac{\alpha}{\alpha+\beta} \mid g(\alpha,\beta)\gamma \geq \alpha, g(\alpha,\beta)(1-\gamma) \geq \beta\right]\right| \\
&= \left|\gamma - E\left[\tfrac{\alpha}{\alpha+\beta} \mid -\left(\frac{\kappa(\alpha,\beta)}{\alpha+\beta}+1\right)\gamma \leq -\frac{\alpha}{\alpha+\beta} \leq \left(\frac{\kappa(\alpha,\beta)}{\alpha+\beta}+1\right)(1-\gamma) - 1\right]\right| \\
&= \left|E\left[\gamma - \tfrac{\alpha}{\alpha+\beta} \mid -\gamma\tfrac{\kappa(\alpha,\beta)}{\alpha+\beta} \leq \gamma - \tfrac{\alpha}{\alpha+\beta} \leq (1-\gamma)\tfrac{\kappa(\alpha,\beta)}{\alpha+\beta}\right]\right|
\end{aligned}$$

Thus, for all $\kappa(\alpha,\beta) \leq \varepsilon(\alpha+\beta)$

$$|\gamma - \psi(\gamma)| = \left|E\left[\gamma - \tfrac{\alpha}{\alpha+\beta} \mid -\gamma\varepsilon \leq -\gamma\tfrac{\kappa(\alpha,\beta)}{\alpha+\beta} \leq \gamma - \tfrac{\alpha}{\alpha+\beta} \leq (1-\gamma)\tfrac{\kappa(\alpha,\beta)}{\alpha+\beta} \leq (1-\gamma)\varepsilon\right]\right| < \varepsilon$$

■

Low synergy produces not only few teams but ones in which discriminatory beliefs reflect workers' respective actual, not just expected, abilities quite precisely. Thus, when synergy is very low, any discrimination level is close to an equilibrium.

This observation presents an empirical challenge. For example, the 1963 Equal Pay Act says that US employers must pay employees equally for equal work; however, the burden of establishing a prima facie case that different wages are paid to employees of the opposite sex and that the employees perform substantially equal work belongs to the employee. While US law prohibits even statistical discrimination, this burden of proof amounts to showing that the proportion of wages paid to men γ is statistically different from their proportional (expected) work $\psi(\gamma)$ – an employee must show $|\gamma - \psi(\gamma)| > \varepsilon$, where ε is the measurement error. Proposition 9 implies that establishing that meritocracy is not functioning may be more difficult in low synergy industries, because the measurement error must be smaller.

5.2 Frictionless Market for Teammates

The basic model implicitly assumes no market for teammates; a rejection by either teammate means both work alone. Of course, in the real world, the outside option of an individual deciding whether or not to join a team, is not usually limited to working alone, but rather includes working on one of several different teams. Here I examine the extreme opposite situation, namely that the market for teammates is frictionless – every worker in the economy is a potential teammate. I will show that, even in this extreme case, discriminatory equilibria can still exist, and thus we should expect them in a more realistic market with frictions.

Suppose all workers who reject teamwork (or are rejected) randomly draw new potential teammates from the pool of individual workers until every worker (a) finds a teammate, (b) rejects or is rejected by all remaining individual workers. Without loss of generality, assume minorities and majorities each have an even number of individuals with each supported ability.

Lemma 3 *In a frictionless market for teammates, no one works individually.*

Proof. Suppose someone chose to work individually. By symmetry another identical worker also did. These two could team up without facing discrimination and, because team production is synergistic, both be better off, a contradiction. ■

As before egalitarianism is always an equilibrium, but since team production increases in ability, everyone will search until he finds the highest ability teammate who will accept him. The highest ability teammates will work together since there are none higher. And so on. Thus the following holds:

Proposition 10 *In a frictionless market for teammates, under egalitarianism, all workers are members of homogeneous teams.*

Can discrimination exist in a frictionless market for teammates? For simplicity, assume ability distributed generalized Bernoulli (see eqn. (5)) and that all workers assume discrimination exists. Clearly, High ability victims will always work together to avoid discrimination. Similarly Low ability beneficiaries will work together because no one else will work with them. Thus, if a heterogeneous team were to form, it could only be between a High ability beneficiary and a Low ability victim. A fair manager, seeing a team, would know this and divide credit accordingly: $\gamma = \frac{H}{H+L}$. These teams would form if and only if both parties prefer this heterogeneous team to working with their peers (i.e. those with identical discriminatory attribute and ability):

$$g(H,L)\frac{H}{H+L} \geq g(H,H)\frac{1}{2} \qquad (M_A)$$

$$g(H,L)\frac{L}{H+L} \geq g(L,L)\frac{1}{2} \qquad (M_B)$$

Thus, we have proved the following proposition.

Proposition 11 *In a frictionless market for teammates, (a) discrimination can exist if and only if heterogeneous teams are sufficiently more productive than homogeneous ones (i.e. (M_A) and (M_B) are satisfied); (b) otherwise discriminatory beliefs completely segregate society and all teams will be homogeneous.*

Example 3 $g(\alpha,\beta)=\alpha+\beta+|\alpha-\beta|\kappa$ *satisfies (M_A) and (M_B).*

As noted previously, Hamilton et al (2003) empirically find that heterogeneous teams produce more. So, one should not be surprised to find discriminatory compensation even if a perfect markets for teammates existed. Since, discriminatory equilibria exist both when the market for teammates does not exist and when it has no frictions, one can reasonably conclude that they exist in a more realistic imperfect market for teammates.

Lemma 2 suggested one cause of occupational segregation, namely that minorities will prefer occupations where individual contribution is more easily measured. Proposition 11 highlights a second possible cause of occupational segregation that exists even when there are no substantive differences between jobs – minorities may simply choose to team with other minorities to strategically eliminate the possibility of discrimination.

5.3 Equilibria with Contractible Output

Previous analysis assumes that the fractional credit split did not depend on the realized output. The system of offering a salary to join a team and a proportional bonus based on company profitability fits this setting. The total bonus amount is tied to team output, but the fraction relative to one's peers is not. Similarly, corporate shares and options are typically divvied up before their exercisable worth is ever known; an output contractible split is impossible. Some compensation schemes, though, do recognize team output when splitting the reward. For example, Senior team member bonuses may more closely tied to team output than Junior members'.

Could the existence of discriminatory equilibria stem from the manager's inability to contract on the realized output? After all, the manager learns a great deal about her employees' abilities from the team's output. This subsection shows that discriminatory equilibria can exist even when the manager can contract on team output.

This requires that the credit split be a function of team output, $\gamma(Q)$, where $Q = g(\alpha, \beta)$. Theoretically, very little changes. The definition of equilibrium beliefs changes to

$$\gamma(Q) = \psi(\gamma(Q)) = E\left[\tfrac{\alpha}{\alpha+\beta} \mid g(\alpha,\beta)\gamma(Q) \geq \alpha, g(\alpha,\beta)(1-\gamma(Q)) \geq \beta\right]$$

This means the belief set is much more complex; workers must have beliefs for each possible output level. Without additional restrictions imposed on equilibria, workers may rationally believe that the manager discriminates for some output levels and not others. Consider the following example:

Example 4 *Assume abilities are distributed* $H > M = \frac{H+L}{2} > L > 0$ *each arising with equal probability and constant synergy,* $g(\alpha,\beta) = \alpha + \beta + \kappa$[15]*. If the manager sees a team produce* $Q = H + L + \kappa = 2M + \kappa$ *then she does not know exactly what each worker contributed. It can be shown that if* $\frac{(H-L)(H+L)}{H+3L} \leq \kappa \leq 3\frac{(H-L)(H+L)}{H+3L}$*, then* $\gamma(2M+\kappa) = \frac{1}{2}\frac{H}{H+L} + \frac{1}{4}$ *is an a equilibrium belief, which forms a team if and only if* $\langle\alpha,\beta\rangle \in \{\langle H,L\rangle, \langle M,M\rangle\}$*.*

Thus, despite the increased complexity of calculating complete belief sets contingent on output, versions of the previous propositions still hold with similar proofs.

6 Conclusion

The model shows that talented discrimination victims should be expected to choose teamwork strategically, choosing entrepreneurship or other occupation where individual contribution is particularly measurable – although the doors to certain occupations are technically open to

[15] A new example production function here is chosen simply to illustrate the robustness of the phenomena. A similar example holds with $g(\alpha,\beta) = \kappa(\alpha+\beta)$ and a wide array of others.

talented minorities, they rationally choose not to enter. As a result,(1) victims outperform beneficiaries when working individually, and (2) measuring discrimination purely in team settings overstates the (relative) adverse impact of discrimination on its victims in society. In fact, in certain circumstances, they may be better off.

The model also reveals the pertinacious nature of discrimination. Senator Hubert Humphrey argued for the passage of The Civil Rights Act of 1964: "We seek to give people an opportunity to be hired on the basis of merit... rather than to keep their talents buried under prejudice or discrimination." Humphrey echoes the prevailing view, among both laymen and policy makers, that discrimination stems from unfair or unmeritocratic management practices, and that by eliminating them, egalitarianism will emerge. Unfortunately it may not, because manager and worker beliefs about discrimination can be self-reinforcing.

Furthermore, although this work fits clearly in the tradition of statistical discrimination, it is subtly different than the existing branches characterized by (1) cultural differences in discourse or (2) human capital acquisition. These require some hidden substantive difference between majority and minority members at the time of employment; either they (1) communicate differently or (2) possess statistically (endogenously acquired) different abilities. At least in theory, economic policy could level these playing fields through education. But In the model presented here, minority characteristics statistically mirror majority ones before, during and after employment. How could the playing field be further leveled? It is hard to conceive of economic instruments to affect the necessary change *in beliefs only* to move from a discriminatory equilibrium to an egalitarian one. Such manipulation is the domain of cultural rather than economic policy.

And although models of differences in discourse and human capital acquisition seem to explain some discrimination, they require forceful systems of individual transformation from early childhood, limiting the analyzable set of discrimination attributes, but discrimination can be over any attribute. The model here indicates that any attribute, even one *ex ante* orthogonal to ability, can become a valid *ex post* indicator of it.

Alchian and Demsetz (1972) observed that the fundamental incentive structure within teams changes, because individual actions are unobservable. That work spawned a substantial theoretical investigation of moral-hazard in exogenously formed teams. By endogenizing the teamwork decision, the model here uniquely exposes an adverse selection problem created by the same unobservability of individual contribution in teams.

This model captures one force among many potentially operating in teams. Its simplicity facilitates incorporation into models with richer institutional details that may better explain adverse team selection or discrimination in specific settings.

Like other discrimination models it is static. Future work will examine teamwork decisions in a dynamic setting – when future potential teammates may share the attribute of discrimination or not, and ability may be revealed over time.

7 Appendix

Proposition 1 (General) *(a) Egalitarianism (i.e. $\gamma = \frac{1}{2}$) is always a team forming equilibrium. (b) Under egalitarianism, a team will form iff the synergy of production is greater than the difference in worker abilities.*

Proof. (a) Since $g(\alpha, \beta) > \alpha + \beta$, a team forms whenever A and B have identical ability. Thus, equation (4) must hold:

$$\psi\left(\frac{1}{2}\right) = E\left[\tfrac{\alpha}{\alpha+\beta} \mid g(\alpha,\beta)\frac{1}{2} \geq \alpha, g(\alpha,\beta)(1-\frac{1}{2}) \geq \beta\right] = \frac{1}{2}$$

where the last equality results because α and β are iid and g is symmetric.

(b) Let synergy be measured by $\kappa(\alpha, \beta) = g(\alpha, \beta) - \alpha - \beta > 0$. Then under egalitarianism, a team forms iff $\alpha + \beta + \kappa(\alpha, \beta) > 2\alpha$ and $\alpha + \beta + \kappa(\alpha, \beta) > 2\beta$. This can be rewritten $\kappa(\alpha, \beta) > \alpha - \beta$ and $\kappa(\alpha, \beta) > \beta - \alpha$, which reduces to $\kappa(\alpha, \beta) > |\alpha - \beta|$. ■

Proposition 5 (General) *Discrimination victims produce better average solo work than beneficiaries.*

Proof. Abbreviate $g(\alpha, \beta)$ as g and $1 - \gamma$ as $\overline{\gamma}$ for notational simplicity. When no team forms, exactly one worker objects to teamwork because $g > \alpha + \beta$. Therefore, the expectation of a random variable x over the solo sample space can be partitioned as follows:

$$E[x \mid Solo] = E[x \mid g\gamma < \alpha] \Pr\{g\gamma < \alpha\} + E[x \mid g\overline{\gamma} < \beta] \Pr\{g\overline{\gamma} < \beta\}$$

The second term can be further partitioned

$$\begin{aligned} E[x|Solo] &= E[x \mid g\gamma < \alpha]\Pr\{g\gamma < \alpha\} + E[x \mid g\gamma < \beta, g\overline{\gamma} < \beta]\Pr\{g\gamma < \beta, g\overline{\gamma} < \beta\} \\ &\quad + E[x \mid g\gamma \geq \beta, g\overline{\gamma} \geq \alpha, g\overline{\gamma} < \beta]\Pr\{g\gamma \geq \beta, g\overline{\gamma} \geq \alpha, g\overline{\gamma} < \beta\} \\ &\quad + E[x \mid g\gamma \geq \beta, g\overline{\gamma} < \alpha, g\overline{\gamma} < \beta]\Pr\{g\gamma \geq \beta, g\overline{\gamma} < \alpha, g\overline{\gamma} < \beta\} \end{aligned}$$

Observe (1) $\gamma > \frac{1}{2}$ and $g\gamma < \beta$ imply $g\overline{\gamma} < \beta$ and (2) $g > \alpha + \beta$ and $g\overline{\gamma} < \alpha$ imply $g\gamma \geq \beta$. Thus, the conditional expected abilities can be simplified as follows:

$$\begin{aligned} E[x \mid Solo] &= E[x \mid g\gamma < \alpha]\Pr\{g\gamma < \alpha\} + E[x \mid g\gamma < \beta]\Pr\{g\gamma < \beta\} \\ &\quad + E[x \mid \alpha \leq g\overline{\gamma} < \beta \leq g\gamma]\Pr\{\alpha \leq g\overline{\gamma} < \beta \leq g\gamma\} \\ &\quad + E[x \mid g\overline{\gamma} < \alpha, g\overline{\gamma} < \beta]\Pr\{g\overline{\gamma} < \alpha, g\overline{\gamma} < \beta\} \end{aligned}$$

Because α and β are iid and g is symmetric with respect to α and β the difference in the expected ability of Bs who work alone from the expected ability of As who work alone is

$$E[\alpha \mid Solo] - E[\beta \mid Solo] = E[\alpha - \beta \mid \alpha \leq g\overline{\gamma} < \beta \leq g\gamma]\Pr\{\alpha \leq g\overline{\gamma} < \beta \leq g\gamma\} < 0$$

■

Definition 2 *Given α and γ, the lowest type B who will (be permitted by A to) join a team is defined by $L(\alpha,\gamma)$ satisfying $g(\alpha, L(\alpha,\gamma))\gamma = \alpha$. Similarly, the highest type B who will (willingly) join a team is defined by $H(\alpha,\gamma)$ satisfying $g(\alpha, H(\alpha,\gamma))(1-\gamma) = H(\alpha,\gamma)$.*

Remark 1 *Observe that as γ approaches 1, any A type will permit any B type to join the team, although none but the lowest B types will be willing to do so. Formally,*

$$\lim_{\gamma\to 1} H(\alpha,\gamma) = \lim_{\gamma\to 1} L(\alpha,\gamma) = 0 \tag{6}$$

Remark 2 *Applying the Implicit Function Theorem to the definitions of $H(\alpha,\gamma)$ and $L(\alpha,\gamma)$ yields*

$$\begin{aligned} H_\gamma(\alpha,\gamma) &= -\frac{\frac{\partial}{\partial\gamma}(g(\alpha,\beta)(1-\gamma)-\beta)}{\frac{\partial}{\partial\beta}(g(\alpha,\beta)(1-\gamma)-\beta)}\Big|_{\beta=H(\alpha,\gamma)} = \frac{g(\alpha,H(\alpha,\gamma))}{g_\beta(\alpha,H(\alpha,\gamma))(1-\gamma)-1} \\ L_\gamma(\alpha,\gamma) &= -\frac{\frac{\partial}{\partial\gamma}(g(\alpha,\beta)\gamma-\alpha)}{\frac{\partial}{\partial\beta}(g(\alpha,\beta)\gamma-\alpha)}\Big|_{\beta=L(\alpha,\gamma)} = -\frac{g(\alpha,L(\alpha,\gamma))}{g_\beta(\alpha,L(\alpha,\gamma))\gamma} \end{aligned}$$

Definition 3 *Define the expectation of a random variable* $\zeta(\alpha,\beta)$ *conditional on a team forming given A's ability* α *and beliefs about discrimination* γ

$$Z(\alpha,\gamma) = E_\beta\left[\zeta(\alpha,\beta) \mid L(\alpha,\gamma) \le \beta \le H(\alpha,\gamma)\right] = \frac{\int_{L(\alpha,\gamma)}^{H(\alpha,\gamma)} \zeta(\alpha,\beta)\, dF(\beta)}{\int_{L(\alpha,\gamma)}^{H(\alpha,\gamma)} dF(\beta)} \tag{7}$$

where F *is distribution of (*α *and)* β*. Then by the Quotient Rule*

$$Z_\gamma(\alpha,\gamma) = \frac{\left(\int_{L(\alpha,\gamma)}^{H(\alpha,\gamma)} dF(\beta)\right)\left(\frac{d}{d\gamma}\int_{L(\alpha,\gamma)}^{H(\alpha,\gamma)} \zeta(\alpha,\beta) dF(\beta)\right) - \left(\int_{L(\alpha,\gamma)}^{H(\alpha,\gamma)} \zeta(\alpha,\beta) dF(\beta)\right)\left(\frac{d}{d\gamma}\int_{L(\alpha,\gamma)}^{H(\alpha,\gamma)} dF(\beta)\right)}{\left(\int_{L(\alpha,\gamma)}^{H(\alpha,\gamma)} dF(\beta)\right)^2}$$

For fixed α *and* $0 < \gamma < 1$ *Leibniz Rule may be applied*

$$Z_\gamma(\alpha,\gamma) = \frac{(\zeta(\alpha,H(\alpha,\gamma)) - Z(\alpha,\gamma)) H_\gamma(\alpha,\gamma) F'(H(\alpha,\gamma)) - (\zeta(\alpha,L(\alpha,\gamma)) - Z(\alpha,\gamma)) L_\gamma(\alpha,\gamma) F'(L(\alpha,\gamma))}{F(H(\alpha,\gamma)) - F(L(\alpha,\gamma))} \tag{8}$$

Lemma 4 *If* $Z(\alpha,\gamma)$ *is defined as in (7) then as* γ *approaches unity* $Z(\alpha,\gamma)$ *converges pointwise*

$$\lim_{\gamma\to 1} Z(\alpha,\gamma) = \zeta(\alpha,0)$$

Proof. Since from (7) both numerator and denominator of $Z(\alpha,\gamma)$ approach 0, apply L'Hôpital's Rule once

$$\lim_{\gamma\to 1} Z(\alpha,\gamma) = \lim_{\gamma\to 1} \frac{\zeta(\alpha,H(\alpha,\gamma)) H_\gamma(\alpha,\gamma) F'(H(\alpha,\gamma)) - \zeta(\alpha,L(\alpha,\gamma)) L_\gamma(\alpha,\gamma) F'(L(\alpha,\gamma))}{H_\gamma(\alpha,\gamma) F'(H(\alpha,\gamma)) - L_\gamma(\alpha,\gamma) F'(L(\alpha,\gamma))}$$

From (6) $\lim_{\gamma\to 1} \zeta(\alpha, H(\alpha,\gamma)) = \lim_{\gamma\to 1} \zeta(\alpha, L(\alpha,\gamma)) = \zeta(\alpha,0)$:

$$\lim_{\gamma\to 1} Z(\alpha,\gamma) = \zeta(\alpha,0) \lim_{\gamma\to 1} \frac{H_\gamma(\alpha,\gamma) F'(H(\alpha,\gamma)) - L_\gamma(\alpha,\gamma) F'(L(\alpha,\gamma))}{H_\gamma(\alpha,\gamma) F'(H(\alpha,\gamma)) - L_\gamma(\alpha,\gamma) F'(L(\alpha,\gamma))} = \zeta(\alpha,0)$$

■

Lemma 5 *If* $Z(\alpha,\gamma)$ *is defined as in (7) then as* γ *approaches unity* $Z_\gamma(\alpha,\gamma)$ *converges pointwise*

$$\lim_{\gamma\to 1} Z_\gamma(\alpha,\gamma) = \frac{H_\gamma(\alpha,1) + L_\gamma(\alpha,1)}{2} \zeta_\gamma(\alpha,0)$$

Proof. Observe from (6) and Lemma 4 that the numerator and denominator of (8) both go to 0 as γ approaches 1. Apply L'Hôpital's Rule once. The derivative of the first term of the numerator with respect to γ

$$\begin{aligned}
&\frac{d}{d\gamma}\left(\zeta\left(\alpha,H\left(\alpha,\gamma\right)\right)-Z\left(\alpha,\gamma\right)H_{\gamma}\left(\alpha,\gamma\right)F'\left(H\left(\alpha,\gamma\right)\right)\right)\\
= \;& H_{\gamma}\left(\alpha,\gamma\right)F'\left(H\left(\alpha,\gamma\right)\right)\frac{d}{d\gamma}\left(\zeta\left(\alpha,H\left(\alpha,\gamma\right)\right)-Z\left(\alpha,\gamma\right)\right)\\
&+\left(\zeta\left(\alpha,H\left(\alpha,\gamma\right)\right)-Z\left(\alpha,\gamma\right)\right)\frac{d}{d\gamma}H_{\gamma}\left(\alpha,\gamma\right)F'\left(H\left(\alpha,\gamma\right)\right)\\
= \;& H_{\gamma}\left(\alpha,\gamma\right)F'\left(H\left(\alpha,\gamma\right)\right)\left(H_{\gamma}\left(\alpha,\gamma\right)\zeta_{\gamma}\left(\alpha,H\left(\alpha,\gamma\right)\right)-Z_{\gamma}\left(\alpha,\gamma\right)\right)\\
&+\left(\zeta\left(\alpha,H\left(\alpha,\gamma\right)\right)-Z\left(\alpha,\gamma\right)\right)\left(H_{\gamma\gamma}\left(\alpha,\gamma\right)F'\left(H\left(\alpha,\gamma\right)\right)+H_{\gamma}\left(\alpha,\gamma\right)^{2}F''\left(H\left(\alpha,\gamma\right)\right)\right)
\end{aligned}$$

Take the limit as γ approaches 1 and simplify using (6) and Lemma 4

$$\begin{aligned}
&\lim_{\gamma\to1}\frac{d}{d\gamma}\left(\zeta\left(\alpha,H\left(\alpha,\gamma\right)\right)-Z\left(\alpha,\gamma\right)H_{\gamma}\left(\alpha,\gamma\right)F'\left(H\left(\alpha,\gamma\right)\right)\right)\\
= \;& H_{\gamma}\left(\alpha,1\right)F'\left(0\right)\left(H_{\gamma}\left(\alpha,1\right)\zeta_{\gamma}\left(\alpha,0\right)-\lim_{\gamma\to1}Z_{\gamma}\left(\alpha,\gamma\right)\right)
\end{aligned}$$

Similarly, the derivative of the numerator's second term with respect to γ as γ approaches 1

$$\begin{aligned}
&\lim_{\gamma\to1}\frac{d}{d\gamma}\left(\zeta\left(\alpha,L\left(\alpha,\gamma\right)\right)-Z\left(\alpha,\gamma\right)\right)L_{\gamma}\left(\alpha,\gamma\right)F'\left(L\left(\alpha,\gamma\right)\right)\\
= \;& L_{\gamma}\left(\alpha,1\right)F'\left(0\right)\left(L_{\gamma}\left(\alpha,1\right)\zeta_{\gamma}\left(\alpha,0\right)-\lim_{\gamma\to1}Z_{\gamma}\left(\alpha,\gamma\right)\right)
\end{aligned}$$

The derivative of the denominator with respect to γ as γ approaches 1

$$\lim_{\gamma\to1}\frac{d}{d\gamma}\left(F\left(H\left(\alpha,\gamma\right)\right)-F\left(L\left(\alpha,\gamma\right)\right)\right)=H_{\gamma}\left(\alpha,1\right)F'\left(0\right)-L_{\gamma}\left(\alpha,1\right)F'\left(0\right)$$

Thus,

$$\begin{aligned}
\lim_{\gamma\to1}Z_{\gamma}\left(\alpha,\gamma\right) &= \frac{H_{\gamma}(\alpha,1)\left(H_{\gamma}(\alpha,1)\zeta_{\gamma}(\alpha,0)-\lim_{\gamma\to1}Z_{\gamma}(\alpha,\gamma)\right)-L_{\gamma}(\alpha,1)\left(L_{\gamma}(\alpha,1)\zeta_{\gamma}(\alpha,0)-\lim_{\gamma\to1}Z_{\gamma}(\alpha,\gamma)\right)}{H_{\gamma}(\alpha,1)-L_{\gamma}(\alpha,1)}\\
&= \frac{H_{\gamma}(\alpha,1)H_{\gamma}(\alpha,1)-L_{\gamma}(\alpha,1)L_{\gamma}(\alpha,1)}{H_{\gamma}(\alpha,1)-L_{\gamma}(\alpha,1)}\zeta_{\gamma}\left(\alpha,0\right)-\frac{H_{\gamma}(\alpha,1)-L_{\gamma}(\alpha,1)}{H_{\gamma}(\alpha,1)-L_{\gamma}(\alpha,1)}\lim_{\gamma\to1}Z_{\gamma}\left(\alpha,\gamma\right)\\
&= \frac{H_{\gamma}\left(\alpha,1\right)+L_{\gamma}\left(\alpha,1\right)}{2}\zeta_{\gamma}\left(\alpha,0\right)
\end{aligned}$$

■

Remark 3 *Since $g_{\beta}\left(\alpha,\beta\right)>1$ and $H_{\gamma}\left(\alpha,\gamma\right)<0$ for all γ satisfying $g_{\beta}\left(\alpha,H\left(\alpha,\gamma\right)\right)\left(1-\gamma\right)<1$, there exists some γ^{ε} for all $\varepsilon>0$ such that for all $\gamma^{\varepsilon}\leq\gamma\leq1$ the following holds: $g_{\beta}\left(\alpha,H\left(\alpha,\gamma\right)\right)\left(1-\gamma\right)<\varepsilon$.*

Remark 4 *Since F has finite variance, $H(\alpha,\gamma) > L(\alpha,\gamma) > 0$ and $\lim_{\gamma\to 1} H(\alpha,\gamma) = \lim_{\gamma\to 1} L(\alpha,\gamma) = 0$, there exists some γ^{ε} for all $\varepsilon > 0$ such that for all $\gamma^{\varepsilon} \le \gamma \le 1$ the following holds: $\overline{F}'(\gamma) - \underline{F}'(\gamma) < \varepsilon$, where*

$$\begin{aligned} \overline{F}(\gamma)' &= \max\{F'(x) : x \in (0, H(\alpha,\gamma))\} \\ \underline{F}'(\gamma) &= \min\{F'(x) : x \in (0, H(\alpha,\gamma))\} \end{aligned}$$

Definition 4 *Define $\hat{\gamma}$ such that for all $\gamma > \hat{\gamma}$ (1) $g_{\beta}(\alpha, H(\alpha,\gamma))(1-\gamma) < \frac{1}{2}$ and (2) $\frac{\overline{F}'(\hat{\gamma})}{\underline{F}'(\hat{\gamma})} < 2$. From Remarks 3 and 4 such a $\hat{\gamma}$ always exists.*

Lemma 6 *If $Z(\alpha,\gamma)$ is defined as in (7), $\zeta_{\alpha}(\alpha,\beta) > 0$ and $\zeta_{\alpha}(\alpha,\beta) > 0$ then for all $\gamma > \hat{\gamma}$, $Z_{\gamma}(\alpha,\gamma)$ is bounded as follows:*

$$0 < Z_{\gamma}(\alpha,\gamma) \le \frac{\zeta(\alpha, L(\alpha,\gamma)) - \zeta(\alpha, H(\alpha,\gamma))}{H(\alpha,\gamma) - L(\alpha,\gamma)} \left(-H_{\gamma}(\alpha,\gamma) - L_{\gamma}(\alpha,\gamma)\right) \frac{\overline{F}'(\hat{\gamma})}{\underline{F}'(\hat{\gamma})} \tag{9}$$

Proof. From (8)

$$Z_{\gamma}(\alpha,\gamma) = \frac{(Z(\alpha,\gamma)-\zeta(\alpha,H(\alpha,\gamma)))(-H_{\gamma}(\alpha,\gamma))F'(H(\alpha,\gamma))+(\zeta(\alpha,L(\alpha,\gamma))-Z(\alpha,\gamma))(-L_{\gamma}(\alpha,\gamma))F'(L(\alpha,\gamma))}{F(H(\alpha,\gamma))-F(L(\alpha,\gamma))} > 0$$

Observe that $\zeta(\alpha, H(\alpha,\gamma)) \le Z(\alpha,\gamma) \le \zeta(\alpha, L(\alpha,\gamma))$ and every factor in the numerator and the denominator are always positive for all $\gamma > \hat{\gamma}$. Thus,

$$Z_{\gamma}(\alpha,\gamma) \le \frac{\zeta(\alpha, L(\alpha,\gamma)) - \zeta(\alpha, H(\alpha,\gamma))}{F(H(\alpha,\gamma)) - F(L(\alpha,\gamma))} \left(-H_{\gamma}(\alpha,\gamma) F'(H(\alpha,\gamma)) - L_{\gamma}(\alpha,\gamma) F'(L(\alpha,\gamma))\right)$$

The form of the lemma results from applying the Mean Value Theorem to the denominator and bounding the F' terms in both numerator and denominator. ■

Corollary 1 *If $Z(\alpha,\gamma)$ is defined as in (7) and $\zeta(\alpha,\beta) = \frac{\alpha}{\alpha+\beta}$, then for all $\gamma > \hat{\gamma}$, $Z_{\gamma}(\alpha,\gamma)$ is bounded by the following Lebesgue integrable function:*

$$|Z_{\gamma}(\alpha,\gamma)| \le \frac{1}{\alpha}\left(2g(\alpha, H(\alpha,\hat{\gamma})) + \frac{g(\alpha, L(\alpha,\hat{\gamma}))}{\hat{\gamma}}\right) \frac{\overline{F}'(\hat{\gamma})}{\underline{F}'(\hat{\gamma})} = \Theta(\alpha) \tag{10}$$

Proof. Substitute $\zeta(\alpha,\beta) = \frac{\alpha}{\alpha+\beta}$, $H_{\gamma}(\alpha,\gamma)$ and $L_{\gamma}(\alpha,\gamma)$ from Lemma 2

$$Z_{\gamma}(\alpha,\gamma) \le \frac{\frac{\alpha}{\alpha+L(\alpha,\gamma)} - \frac{\alpha}{\alpha+H(\alpha,\gamma)}}{H(\alpha,\gamma) - L(\alpha,\gamma)} \left(\frac{g(\alpha, H(\alpha,\gamma))}{1 - g_{\beta}(\alpha, H(\alpha,\gamma))(1-\gamma)} + \frac{g(\alpha, L(\alpha,\gamma))}{g_{\beta}(\alpha, L(\alpha,\gamma))\gamma}\right) \frac{\overline{F}'(\hat{\gamma})}{\underline{F}'(\hat{\gamma})}$$

Simplifying the first factor and then using the facts that $H(\alpha,\gamma) > L(\alpha,\gamma) > 0$, $1 - g_{\beta}(\alpha, H(\alpha,\gamma))(1-\gamma) \le \frac{1}{2}$, $g_{\beta}(\alpha,\beta) > 1$ to bound each factor yields the form of the corollary. ■

Lemma 7 *If ability is continuously distributed with support from* 0*, then* γ

$$\lim_{\gamma\to1}\psi(\gamma)=1$$

Proof. Define $Z(\alpha,\gamma)$ as in (7) where $\zeta(\alpha,\beta)=\frac{\alpha}{\alpha+\beta}$. Then

$$\lim_{\gamma\to1}\psi(\gamma)=\lim_{\gamma\to1}\int_{-\infty}^{\infty}Z(\alpha,\gamma)\,dF(\alpha)$$

From Lemma 4 $Z(\alpha,\gamma)$ converges pointwise to $\frac{\alpha}{\alpha+0}=1$ for all α, and $Z(\alpha,\gamma)$ is dominated by 1 (i.e. $|Z(\alpha,\gamma)|\leq1$). Thus, by Lebesgue's Dominated Convergence Theorem

$$\lim_{\gamma\to1}\psi(\gamma)=\int_{-\infty}^{\infty}1dF(\alpha)=1$$

■

Lemma 8 *If team production is regular and ability is continuously distributed with support from* 0*, then*

$$\lim_{\gamma\to1}\psi'(\gamma)<1$$

Proof. Define $Z(\alpha,\gamma)$ as in (7) where $\zeta(\alpha,\beta)=\frac{\alpha}{\alpha+\beta}$. Then (1) $Z(\alpha,\gamma)$ is a Lebesgue integrable function for all $\gamma\in(\hat{\gamma},1)$, (2) for almost all α, $Z_\gamma(\alpha,\gamma)$ exists for all $\gamma\in(\hat{\gamma},1)$ and (3) by Lemma 1 $Z_\gamma(\alpha,\gamma)$ is dominated by $\Theta(\alpha)$ as defined in (10) for all $\gamma\in(\hat{\gamma},1)$. Thus, by Leibniz' Rule (see Folland 1999, Theorem 2.27.b for the measure theory version)

$$\lim_{\gamma\to1}\psi'(\gamma)=\lim_{\gamma\to1}\int_{-\infty}^{\infty}Z_\gamma(\alpha,\gamma)\,dF(\alpha)$$

From Lemma 5 $Z_\gamma(\alpha,\gamma)$ converges pointwise to

$$-\frac{H_\gamma(\alpha,1)+L_\gamma(\alpha,1)}{2\alpha}$$

for all α, and from Corollary 1 $Z_\gamma(\alpha,\gamma)$ is dominated by $\frac{1}{\alpha}\left(2g(\alpha,H(\alpha,\hat{\gamma}))+\frac{g(\alpha,L(\alpha,\hat{\gamma}))}{\hat{\gamma}}\right)\frac{\overline{F}'(\hat{\gamma})}{\underline{F}'(\hat{\gamma})}$ for all $\gamma>\hat{\gamma}$. Thus, by Lebesgue's Dominated Convergence Theorem

$$\lim_{\gamma\to1}\psi'(\gamma)=-\int_{-\infty}^{\infty}\frac{H_\gamma(\alpha,1)+L_\gamma(\alpha,1)}{2\alpha}dF(\alpha)\tag{11}$$

From Remark 2 $H_\gamma(\alpha,1)=-g(\alpha,0)$ whenever $H(\alpha,\gamma)$ is interior. Observe $H(\alpha,\gamma)$ is always interior when γ is near unity, since $g\left(\alpha,\alpha\frac{1-\gamma}{\gamma}\right)(1-\gamma)\geq\left(\alpha+\alpha\frac{1-\gamma}{\gamma}\right)(1-\gamma)=$

$\alpha\frac{1-\gamma}{\gamma}$. Similarly $L_\gamma(\alpha,1) = -\frac{g(\alpha,0)}{g_\beta(\alpha,0)}$ whenever $L(\alpha,\gamma)$ is interior.

Case $L(\alpha,1)$ is interior: Simplify (11)

$$\lim_{\gamma\to 1}\psi'(\gamma) = -\int_{-\infty}^{\infty}\frac{-g(\alpha,0)+-\frac{g(\alpha,0)}{g_\beta(\alpha,0)}}{2\alpha}dF(\alpha) = \frac{1}{2}\int_{-\infty}^{\infty}\left(1+\frac{1}{g_\beta(\alpha,0)}\right)dF(\alpha) < 1$$

where the second equality follows because $g(\alpha,0)=\alpha$ by the regularity assumption and the inequality follows because $g_\beta(\alpha,\beta) > 1$ for all α and β.

Case $L(\alpha,1)$ is not interior: Simplify (11) using $H_\gamma(\alpha,1) = -g(\alpha,0)$ and $L_\gamma(\alpha,1) = 0$ (because $L(\alpha,1)$ is not interior).

$$\lim_{\gamma\to 1}\psi'(\gamma) = -\int_{-\infty}^{\infty}\frac{-g(\alpha,0)}{2\alpha}dF(\alpha) = \frac{1}{2}\int_{-\infty}^{\infty}dF(\alpha) = \frac{1}{2} < 1$$

where the last equality follows because $g(\alpha,0)=\alpha$ by the regularity assumption. ■

Lemma 9

$$\psi'\left(\frac{1}{2}\right) = 0$$

Proof. Rotate the ability sample space Ω by angle $-\frac{\pi}{4}$:

$$\begin{aligned}\widetilde{\alpha} &= \alpha\cos\left(-\tfrac{\pi}{4}\right) - \beta\sin\left(-\tfrac{\pi}{4}\right) = \tfrac{\alpha+\beta}{\sqrt{2}}\\ \widetilde{\beta} &= \alpha\sin\left(-\tfrac{\pi}{4}\right) + \beta\cos\left(-\tfrac{\pi}{4}\right) = \tfrac{-\alpha+\beta}{\sqrt{2}}\end{aligned}$$

This yields $\alpha = \frac{\widetilde{\alpha}-\widetilde{\beta}}{\sqrt{2}}$, $\beta = \frac{\widetilde{\alpha}+\widetilde{\beta}}{\sqrt{2}}$ and $\widetilde{\zeta}\left(\widetilde{\alpha},\widetilde{\beta}\right) = \frac{\widetilde{\alpha}-\widetilde{\beta}}{2\widetilde{\alpha}} = \frac{\alpha}{\alpha+\beta}$. Thus $\widetilde{Z}\left(\alpha,\frac{1}{2}\right) = \frac{1}{2}$, $\widetilde{\zeta}\left(\widetilde{\alpha},\widetilde{\beta}\right) - \widetilde{Z}\left(\alpha,\frac{1}{2}\right) = \frac{-\widetilde{\beta}}{2\widetilde{\alpha}}$ and from (8)

$$\widetilde{Z}_\gamma\left(\alpha,\frac{1}{2}\right) = \frac{\frac{-\widetilde{H}\left(\widetilde{\alpha},\frac{1}{2}\right)}{2\widetilde{\alpha}}\widetilde{H}_\gamma\left(\widetilde{\alpha},\frac{1}{2}\right)\widetilde{F}'\left(\widetilde{H}\left(\widetilde{\alpha},\frac{1}{2}\right)\right) - \frac{-\widetilde{L}\left(\widetilde{\alpha},\frac{1}{2}\right)}{2\widetilde{\alpha}}\widetilde{L}_\gamma\left(\widetilde{\alpha},\frac{1}{2}\right)\widetilde{F}'\left(\widetilde{L}\left(\widetilde{\alpha},\frac{1}{2}\right)\right)}{\widetilde{F}\left(\widetilde{H}\left(\widetilde{\alpha},\frac{1}{2}\right)\right) - \widetilde{F}\left(\widetilde{L}\left(\widetilde{\alpha},\frac{1}{2}\right)\right)}$$

Observe that when $\gamma = \frac{1}{2}$ the team forming region (i.e. $\Omega \ni g\left(\widetilde{\alpha},\widetilde{\beta}\right)\gamma \geq \widetilde{\alpha}$ and $g\left(\widetilde{\alpha},\widetilde{\beta}\right)(1-\gamma) \geq \widetilde{\beta}$) is symmetric about the $\widetilde{\alpha}$-axis. Thus $-\widetilde{L}\left(\widetilde{\alpha},\frac{1}{2}\right) = \widetilde{H}\left(\widetilde{\alpha},\frac{1}{2}\right)$, $\widetilde{f}\left(\widetilde{\alpha},\widetilde{L}\left(\widetilde{\alpha},\frac{1}{2}\right)\right) = \widetilde{f}\left(\widetilde{\alpha},\widetilde{H}\left(\widetilde{\alpha},\frac{1}{2}\right)\right)$ and $-\widetilde{L}_\gamma\left(\widetilde{\alpha},\frac{1}{2}\right) = \widetilde{H}_\gamma\left(\widetilde{\alpha},\frac{1}{2}\right)$. Thus, $\widetilde{Z}_\gamma\left(\widetilde{\alpha},\frac{1}{2}\right) = 0$. By Leibniz' Rule, then

$$\psi'\left(\frac{1}{2}\right) = \int_{-\infty}^{\infty}\widetilde{Z}_\gamma\left(\widetilde{\alpha},\frac{1}{2}\right)dF\left(\widetilde{\alpha}\right) = 0$$

■

Theorem 1 (Fixed Point) *If ψ is continuous and differentiable at distinct fixed points a and c, and $sign(1-\psi'(a)) = sign(1-\psi'(c))$ then there exists another fixed point b strictly between a and c.*

Proof. Define $\chi(\gamma) = \gamma - \psi(\gamma)$. Then $\chi(\gamma) = 0 \iff \gamma = \psi(\gamma)$. $\exists\delta > 0 \ni \forall 0 < \varepsilon < \delta, sign(\chi(a+\varepsilon)) = sign(1-\psi'(c)), sign(\chi(c-\varepsilon)) = -sign(1-\psi'(c))$. Thus if $sign(1-\psi'(a)) = sign(1-\psi'(c))$, then 0 lies between $\chi(a+\varepsilon)$ and $\chi(c-\varepsilon)$. Then by the Intermediate Value Theorem there exists $b \in (a+\varepsilon, c-\varepsilon)$ such that $\chi(b) = 0$. ■

Proposition 8 *If team production is regular, continuous and ability is continuously distributed with support from 0, then at least one discriminatory equilibrium exists in which teams form.*

Proof. $\gamma = \frac{1}{2}$ is always an equilibrium by Proposition 1 (General). $\lim_{\gamma\to 1}\psi(\gamma) = 1$ by Lemma 7. $\lim_{\gamma\to 1}\psi'(\gamma) < 1$ by Lemma 8. $\psi'\left(\frac{1}{2}\right) = 0 < 1$ by Lemma 9. Thus, $sign\left(1-\psi'\left(\frac{1}{2}\right)\right) = " - " = sign(1-\psi'(1))$. Theorem 1 implies $\psi(\gamma)$ has a fixed point in $\left(\frac{1}{2}, 1\right)$. ■

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