POLICY SELECTION IN PRIMARIES AND GENERAL ELECTIONS WITH CANDIDATES OF HETEROGENEOUS QUALITY

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Abstract. I consider a model in which candidates of differing quality must win a primary election to compete in the general election. Candidates choose policies on a one-dimensional policy space, and there is uncertainty about the preferences of the median voter in the general election. I show that there is an equilibrium in which Democrats choose liberal policies and Republicans choose conservative policies, but higher quality candidates choose more moderate policies than lower quality candidates.

1. Introduction

A large body of empirical literature addresses how candidates in United States elections choose policies under different circumstances. This literature notes several regularities regarding how policy choices vary with candidate characteristics. Candidates from different parties typically choose divergent policies in the sense that the positions chosen by candidates running as Democrats are normally more liberal than the positions chosen by candidates running as Republicans (Enelow and Hinch, 1984; Erikson and Wright, 1997; Page, 1978; Poole and Rosenthal, 1997; Sullivan and Minns, 1976; Sullivan and O'Connor, 1972). And higher quality candidates typically choose more moderate policies than lower quality candidates, where quality can be measured by things such as incumbency advantages and the electoral strength of an incumbent (Ansolabehere *et al.*, 2001; Fiorina, 1973; Stone and Simas, 2007).

While there has been extensive empirical work on how candidates choose policies in different circumstances, to the best of my knowledge, there is no theoretical model that is consistent with empirical evidence on how candidates of heterogeneous quality in different parties choose their

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policies. In addition, there has been little theoretical work on policy selection in primaries and general elections when candidates differ in quality. This paper presents a theoretical model of policy choice in primaries and general elections that is consistent with empirical evidence regarding how candidates choose policies when they may differ in both their party labels and their quality.

I consider a model in which there are four candidates, two of whom compete in the liberal primary, and two of whom compete in the conservative primary. In each primary there is one high quality candidate and one low quality candidate. Candidates first commit to policies on a continuous one-dimensional policy space before the primaries. Voters observe these policy choices and vote in their party's primary. The candidates who receive the most votes in the primaries then compete in the general election. The candidate who receives the most votes in the general election is then elected and adopts the policy he or she committed to before the primary. Candidates are exclusively motivated by the possibility of winning elections, and voters take electability into account when they vote in the primary elections. Throughout I assume that there is uncertainty about the preferences of the median voter in the general election.

I show that there exists a pure strategy equilibrium for the candidates in the policy selection game. In this equilibrium, the candidates in the liberal primary run on liberal policies and the candidates in the conservative primary run on conservative policies. But the high quality candidate in a given primary chooses a more moderate policy than the low quality candidate in the sense that higher quality candidates choose policies closer to the estimated preferences of the median voter in the general election.

The intuition for why the high quality candidates choose more moderate policies is as follows. Regardless of whether a candidate is high quality or low quality, the candidate will need to make an effort to try to appeal to the voters in the primary and choose policies that deviate from the center of the policy space. However, voters in the primaries will prefer to nominate a higher quality candidate because voters have an intrinsic preference for high quality candidates and high quality candidates are also more electable.

Since the high quality candidates can be confident that voters in their primaries will still be willing to vote for them even if they do not choose policies that quite match the preferences of the primary voters, these candidates can afford to focus more on choosing policies that will be appealing in the general election without worrying so much about appealing to voters in the primary. Thus high quality candidates will choose more moderate policies that will be relatively more appealing to the median voter in the general election even though these policies are not as favorable in the primary.

The predictions of the model are consistent with empirical evidence on how candidates choose policies when they may differ in both their party labels and their quality. First, the result that Democrats run on more liberal policies than the typical voter and Republicans run on more conservative policies than the typical voter is consistent with evidence regarding how Democrats and Republicans choose policies. Page (1978) and Enelow and Hinch (1984) note that Democratic presidential candidates choose more liberal policies than Republican presidential candidates. Erikson and Wright (1997) note similar results for congressional elections, and Poole and Rosenthal (1997) indicate that Democratic members of Congress support more liberal policies than Republican members of Congress. Finally, Sullivan and Minns (1976) and Sullivan and O'Connor (1972) note similar variation in policies for congressional candidates at the district level.

Similarly, the fact that the model predicts that high quality candidates choose more moderate policies is significant because there is empirical evidence that higher quality candidates choose more moderate policies in United States Congressional elections. For example, Ansolabehere *et al.* (2001) compare the policy choices of incumbents, candidates running for an open seat, and challengers to incumbents. Ansolabehere *et al.* (2001) note that, on average, one would expect incumbents to be the highest quality candidates, candidates running for open seats would be the next highest quality, and challengers would be the lowest quality candidates. Ansolabehere *et al.* (2001) give empirical evidence from the 1996 United States House elections that indicates that incumbents choose the most moderate policies, candidates running for open seats choose the next most moderate policies, and challengers choose the most extreme policies. This suggests that higher quality candidates typically choose more moderate policies than lower quality candidates.

Similarly, Stone and Simas (2007) analyze candidate policy selection in the 2006 United States House elections. The authors indicate that incumbents with quality advantages tend to choose policies that are closer to the district median than the policy choices of disadvantaged challengers. This also suggests that higher quality candidates choose more moderate policies than lower quality candidates.

Fiorina (1973) presents a study of the marginality hypothesis that gives evidence that higher quality candidates choose more moderate policies. Fiorina (1973) argues that electorally strong incumbents tend to choose policies closer to the preferences of their constituents than electorally weak incumbents. Since electorally strong incumbents are typically higher quality candidates than electorally weak incumbents, this also indicates that higher quality candidates choose more moderate policies than lower quality candidates.

Finally, while not focusing on how policy choices vary for different kinds of candidates, Canes-Wrone *et al.* (2002) give empirical evidence that indicates that candidates with an incumbency advantage would have an incentive to try to moderate their policy choices. Canes-Wrone *et al.* (2002) analyze data from House elections from 1956-1996, and note that incumbents who moderate their policy choices tend to receive greater vote shares when they run for re-election. This suggests that once a candidate has the benefit of a quality advantage due to incumbency, the candidate can improve his or her re-election prospects by choosing more moderate policies. Other studies have also found similar conclusions for smaller data sets (Erickson, 1971; Johannes and McAdams, 1981).

Several theory papers have addressed how known quality differences between candidates affects the policy choices of the candidates. Groseclose (2001) considers a model of a single election between two policy-motivated candidates, one of whom has an advantage in quality. Groseclose (2001) derives properties of pure-strategy equilibria when an equilibrium exists. However, Groseclose (2001) does not give conditions under which an equilibrium exists, and pure strategy equilibria often fail to exist in his model.

Aragones and Palfrey (2002, 2005) and Hummel (2009b) present alternative models in which there is a single election between a high quality candidate and a low quality candidate. In these papers, the authors prove existence of an equilibrium in which the low quality candidate chooses both a more liberal policy and a more conservative policy than the high quality candidate with positive probability.

While there are some situations in which it is plausible that there could be a positive probability that a given candidate could lie either to the left or the right of another candidate, there are other situations in which this is not reasonable. In particular, in some elections there are party labels that indicate that one candidate falls to the left of the other candidate. For example, in the United States, a candidate running as a Democrat would be known to be more liberal than a candidate running as a Republican. In an election between a Democrat and a Republican, the type of policy selection in Aragones and Palfrey (2002, 2005) or Hummel (2009b) could not occur.

If one combines the information in party labels along with the empirical evidence regarding how quality of candidates affects policy choices, one would expect to find that Democrats run on more liberal policies than Republicans, but high quality candidates run on more moderate policies than low quality candidates. While such a prediction arises from the model in this paper with primaries and general elections, it cannot arise in the papers on candidate valence with a single election.

The most closely related paper that considers policy selection in a model with primaries and general elections with candidates of differing quality is Adams and Merrill (2008). My paper differs in several ways. I assume that there is uncertainty about the policy preferences of the voters in the general election, whereas Adams and Merrill (2008) do not. To be consistent with the views of applied scholars (e.g. Abramowitz, 1989; Stone and Abramowitz, 1983), I also assume that primary voters care about electability, but Adams and Merrill (2008) assume that such voters vote naively. Finally, Adams and Merrill (2008) do not present results for cases when there are known differences in quality between candidates in a given primary, but I do.

These differences in modeling assumptions lead to significant differences in the results. In the equilibrium in my paper, candidates of the same party choose different policies in the primary, but they choose the same policies in Adams and Merrill (2008). Also, the main results in Adams and Merrill (2008) imply that candidates will moderate their policy choices when the opposing candidates increase their quality.[1] By contrast, in my model, it is the high quality candidates that moderate, a result consistent with empirical evidence.

The only other paper I am aware of that analyzes a model with primaries and general elections when candidates may differ in quality is Kartik and McAfee (2007). This paper considers a model in which there are three possible policies and candidates with superior valence are exogenously committed to a policy. Kartik and McAfee (2007) focus on very different issues than this paper, since high quality candidates cannot choose policies in their model. Hummel (2009a) considers a model of primaries and general elections in which a candidate's actions may ultimately affect a voter's assessment of his or her valence, but candidates do not differ in quality in equilibrium. Other papers on policy selection in primaries and general elections (e.g. Alesina and Holden (2008), Aranson and Ordeshook (1972), Cadigan and Janeba (2002), Coleman (1971, 1972), Jackson *et al.*

[1]This is an immediate consequence of both Theorems 2 and 4 in Adams and Merrill (2008).

(2007), Meirowitz (2005), and Owen and Grofman (2006)) do not consider candidates who differ in quality.

2. The Model

There are four candidates, L_A, L_D, R_A, and R_D. The candidates L_A and L_D compete in a liberal primary while the candidates R_A and R_D simultaneously compete in a conservative primary. The winners of each primary election then compete in a general election, and the winner of the general election is elected. Throughout L_A and R_A are the advantaged or high quality candidates, and L_D and R_D are the disadvantaged or low quality candidates.[2]

If a candidate loses in the primary election, then the candidate obtains a utility of 0. If a candidate wins the liberal primary, then he or she obtains a utility of $u_l > 0$, and if a candidate wins the conservative primary, then he or she obtains a utility of $u_r > 0$. Finally, the winner of the general election receives an additional utility of $u > 0$. The candidates are thus motivated solely by the possibility of winning elections.

There is a set of voters $N = \{1, \ldots, n\}$, where n is odd. A subset $N_l \subset N$ of these voters vote in the liberal primary, and a subset $N_r \subset N$ of these voters vote in the conservative primary. I assume throughout that $N_l \cap N_r = \oslash$ so that no voter can vote in both the liberal and the conservative primary. I also assume that $N_l \neq \oslash$ and $N_r \neq \oslash$ so that at least one voter votes in each primary. However, I do not require that $N_l \cup N_r = N$, so there may be some voters who do not vote in either primary. After the primary elections are held, all n voters vote in the general election. Each election is decided by majority rule, and if the two candidates receive the same number of votes, they each win the election with probability $\frac{1}{2}$.

There is a one-dimensional policy space $X = (-\infty, \infty)$. Each voter i has an ideal point v_i in this policy space representing the voter's most preferred policy, but the ideal points of the voters are not known with certainty to the candidates. However, I do assume that all candidates and all voters in N_l and N_r agree on the following.

Each candidate and each voter in N_l or N_r knows that more than half of the voters in N_l have ideal points no greater than a fixed policy $x_l \in X$ and that more than half of the voters in N_r have ideal points greater than or equal to x_r, where $x_r > x_l$. Each of these actors also believes the

[2]Parts of this model are similar to the model in Hummel (2009a). As such, part of the description of the model is taken from Hummel (2009a) with only minor changes.

median ideal point of the voters in N is drawn from the continuous cumulative distribution function F with corresponding density f. I assume there are some a and b satisfying $x_l < a < b < x_r$ such that $f(x)$ is continuous and strictly positive for $x \in [a, b]$ and $f(x) = 0$ for $x \notin [a, b]$.

Throughout the paper I assume that the hazard rate $\frac{f(x)}{1-F(x)}$ is nondecreasing in x for all $x \in [a, b]$ and that $\frac{f(x)}{F(x)}$ is nonincreasing in x for all $x \in [a, b]$. This is a standard assumption in many theoretical models in economics and political science, and is satisfied by most commonly used distributions F.

The game proceeds as follows. All candidates simultaneously choose a policy in the policy space X before their respective primary elections. Voters observe these policy choices and then vote in the primary elections. After observing which candidates win their respective primaries, voters vote between these two candidates in the general election. The candidate that wins the general election is elected and implements the policy he or she chose before the primary election.

If either candidate L_A or candidate R_A wins the general election with the policy x, then voter i obtains utility $u_i = \delta - |v_i - x|$, where $\delta > 0$ is the additional utility a voter obtains by electing a high quality candidate. If either candidate L_D or R_D wins the general election with the policy x, then voter i obtains utility $u_i = -|v_i - x|$. Throughout I assume that if a voter obtains a strictly higher expected utility from the election of a given candidate, then the voter votes for the candidate he or she strictly prefers.

3. Preliminaries

This section derives optimal policy choices for each of the candidates in terms of the policies chosen by the other candidates. I first illustrate how the disadvantaged candidates would want to choose their policies given the policy choices of the other candidates, and then illustrate how the advantaged candidates would want to choose their policies given the policy choices of the other candidates. Throughout this paper I let l_A denote the policy chosen by L_A, l_D denote the policy chosen by L_D, r_A denote the policy chosen by R_A, and r_D denote the policy chosen by R_D. First I identify the policy voters in the liberal primary would want L_D to run on if L_D wins the liberal primary:

Lemma 1. *Suppose that L_D wins the liberal primary with certainty and R_A wins the conservative primary with certainty by running on a policy $r_A > a + \delta$. Then there is at most one $l_D \in (2a -$*

$r_A+\delta, r_A-\delta)$ *satisfying* $F(\frac{l_D+r_A-\delta}{2}) = \frac{r_A-l_D-\delta}{2}f(\frac{l_D+r_A-\delta}{2})$. *If* l_D^* *satisfies this condition, then a voter with ideal point* $v_i < l_D^*$ *strictly prefers that* L_D *run on the policy* l_D^* *instead of any other policy.*

All proofs are in the appendix. To understand the intuition behind this result, note that voters in the liberal primary with sufficiently liberal preferences face a tradeoff in deciding what policies they would like L_D to run on if L_D were to win the liberal primary. On one hand, these voters would like L_D to choose a more liberal policy because this means that L_D will be running on a policy closer to their own preferences. On the other hand, these voters do not want L_D to run on a policy that is too liberal because this will make L_D significantly less electable in the general election. The ideal policy l_D^* given in Lemma 1 reflects the tradeoff between these two competing motivations.

To understand how this tradeoff is resolved, note that if L_D runs on a policy $l_D < r_A - \delta$ and both L_D and R_A win their respective primaries, then voters with ideal points $v_i < \frac{l_D+r_A-\delta}{2}$ vote for L_D in the general election and voters with ideal points $v_i > \frac{l_D+r_A-\delta}{2}$ vote for R_A in the general election. Thus L_D wins the general election with probability $F(\frac{l_D+r_A-\delta}{2})$.

If L_D changes policies to $l_D - \epsilon$ from l_D for some small $\epsilon > 0$, then voters with sufficiently liberal ideal points gain ϵ utility when L_D wins the general election. Thus since L_D wins the general election with probability $F(\frac{l_D+r_A-\delta}{2})$, voters with sufficiently liberal ideal points would gain approximately $\epsilon F(\frac{l_D+r_A-\delta}{2})$ expected utility from this change.

But this change in L_D's policy also decreases the probability that L_D wins the general election by approximately $\frac{\epsilon}{2}f(\frac{l_D+r_A-\delta}{2})$. And if L_D runs on a policy $l_D < r_A - \delta$, then voters with sufficiently liberal ideal points obtain approximately $r_A - l_D - \delta$ more utility from the election of L_D than they do from the election of R_A. Thus this change would also cost voters with sufficiently liberal ideal points approximately $\frac{\epsilon(r_A-l_D-\delta)}{2}f(\frac{l_D+r_A-\delta}{2})$ in expected utility.

Putting this together, it must be the case that $\epsilon F(\frac{l_D+r_A-\delta}{2}) = \frac{\epsilon(r_A-l_D-\delta)}{2}f(\frac{l_D+r_A-\delta}{2})$ or $F(\frac{l_D+r_A-\delta}{2}) = \frac{r_A-l_D-\delta}{2}f(\frac{l_D+r_A-\delta}{2})$ for the gains from choosing a more liberal policy to equal the costs from incurring a lower probability of winning the election. Thus the optimal policy for voters with sufficiently liberal ideal points is given by the solution to $F(\frac{l_D+r_A-\delta}{2}) = \frac{r_A-l_D-\delta}{2}f(\frac{l_D+r_A-\delta}{2})$. This gives the result in Lemma 1.

By similar reasoning, voters in the conservative primary would want R_D to run on a policy r_D^* if R_D wins the conservative primary with certainty and L_A wins the liberal primary with certainty.

Lemma 2. *Suppose that R_D wins the conservative primary with certainty and L_A wins the liberal primary with certainty by running on a policy $l_A < b - \delta$. Then there is at most one $r_D \in (l_A + \delta, 2b - l_A - \delta)$ satisfying $1 - F(\frac{l_A+r_D+\delta}{2}) = \frac{r_D-l_A-\delta}{2} f(\frac{l_A+r_D+\delta}{2})$. If r_D^* satisfies this condition, then a voter with ideal point $v_i > r_D^*$ strictly prefers that R_D run on the policy r_D^* instead of any other policy.*

The proof of Lemma 2 is virtually identical to the proof Lemma 1 and is omitted. Next consider the advantaged candidates. Note that L_A would like to win the liberal primary by running on a policy as moderate as possible because running on a more moderate policy gives L_A a better chance of winning the general election. However, L_A does not want to run on a policy that is too moderate because then voters in the liberal primary will prefer to vote for L_D. Instead L_A wants to run on a policy that is just liberal enough that voters in the liberal primary will be willing to vote for L_A, while still choosing a policy that is as moderate as possible. Such a policy is characterized in Lemma 3.

Lemma 3. *Suppose that (1) R_A wins the conservative primary with certainty by running on a policy $r_A > a + \delta$, (2) there is some policy $l_D \in (2a - r_A + \delta, r_A - \delta)$ that satisfies $F(\frac{l_D+r_A-\delta}{2}) = \frac{r_A-l_D-\delta}{2} f(\frac{l_D+r_A-\delta}{2})$, and (3) L_D runs on the unique policy $l_D \in (2a - r_A + \delta, r_A - \delta)$ that satisfies $F(\frac{l_D+r_A-\delta}{2}) = \frac{r_A-l_D-\delta}{2} f(\frac{l_D+r_A-\delta}{2})$. Then there is a unique $l_A \in [l_D + \delta, r_A)$ satisfying $(r_A - l_A)F(\frac{l_A+r_A}{2}) = (r_A - l_D - \delta)F(\frac{l_D+r_A-\delta}{2})$. If l_A^* satisfies this condition, then l_A^* is the most conservative policy that L_A can choose such that voters with ideal points $v_i < l_D$ will weakly prefer voting for L_A in the liberal primary.*

The policy l_A^* given in Lemma 3 reflects the optimal policy given the strategic considerations faced by L_A. This policy is derived by comparing the expected utility a voter in the liberal primary obtains from electing L_D with the expected utility a voter in the liberal primary obtains from electing L_A. To understand why $l_A^* \in [l_D + \delta, r_A)$, note that voters in the liberal primary will always be willing to vote for L_A if L_A runs on the policy $l_D + \delta$. If L_A runs on the policy $l_D + \delta$, then voters in the liberal primary would obtain the same utility if L_A won the general election as they would if L_D won the general election, and L_A would be at least as electable as L_D, so voters in the liberal primary would vote for L_A. But if L_A runs on a policy $l_A \geq r_A$, then voters in the liberal primary will prefer R_A to L_A and hence prefer to vote for L_D over L_A in the liberal primary. Thus $l_A^* \in [l_D + \delta, r_A)$.

The incentives faced by R_A are similar to the incentives faced by L_A. R_A wishes to choose the most moderate policy such that conservative voters will vote for R_A instead of R_D in the conservative primary. The policy R_A chooses given these incentives is characterized in Lemma 4.

Lemma 4. *Suppose that (1) L_A wins the conservative primary with certainty by running on a policy $l_A < b - \delta$, (2) there is some policy $r_D \in (l_A + \delta, 2b - l_A - \delta)$ that satisfies $1 - F(\frac{l_A+r_D+\delta}{2}) = \frac{r_D-l_A-\delta}{2} f(\frac{l_A+r_D+\delta}{2})$, and (3) R_D runs on the unique policy $r_D \in (l_A + \delta, 2b - l_A - \delta)$ that satisfies $1 - F(\frac{l_A+r_D+\delta}{2}) = \frac{r_D-l_A-\delta}{2} f(\frac{l_A+r_D+\delta}{2})$. Then there is a unique $r_A \in (l_A, r_D - \delta]$ satisfying $(r_A - l_A)(1 - F(\frac{l_A+r_A}{2})) = (r_D - l_A - \delta)(1 - F(\frac{l_A+r_D+\delta}{2}))$. If r_A^* satisfies this condition, then r_A^* is the most liberal policy that R_A can choose such that voters with ideal points $v_i > r_D$ will weakly prefer voting for R_A in the conservative primary.*

The proof of this result is substantively identical to the proof of Lemma 3 and is omitted from the appendix. I now use these preliminary results to derive equilibrium candidate policy selection in Section 4.

4. Main Results

This section presents analysis of the equilibrium policy selection for the candidates. I first use the results in Lemmas 1-4 to give general conditions under which candidate policy choices are an equilibrium. I then use these conditions to find specific circumstances under which there exists an equilibrium. The general conditions under which candidate policy choices are an equilibrium are given in Proposition 1:

Proposition 1. *Suppose that l_A, l_D, r_A, and r_D satisfy the following properties:*

(a). $F(\frac{l_D+r_A-\delta}{2}) = \frac{r_A-l_D-\delta}{2} f(\frac{l_D+r_A-\delta}{2})$.

(b). $1 - F(\frac{l_A+r_D+\delta}{2}) = \frac{r_D-l_A-\delta}{2} f(\frac{l_A+r_D+\delta}{2})$.

(c). $(r_A - l_A)F(\frac{l_A+r_A}{2}) = (r_A - l_D - \delta)F(\frac{l_D+r_A-\delta}{2})$.

(d). $(r_A - l_A)(1 - F(\frac{l_A+r_A}{2})) = (r_D - l_A - \delta)(1 - F(\frac{l_A+r_D+\delta}{2}))$.

(e). $2a - r_A + 2\delta < l_D + \delta \leq l_A < r_A \leq r_D - \delta < 2b - l_A - 2\delta$.

(f). $x_l < l_D$ *and* $x_r > r_D$.

Then there is an equilibrium in which L_A chooses the policy l_A, L_D chooses the policy l_D, R_A chooses the policy r_A, and R_D chooses the policy r_D.

Proposition 1 follows by combining the conditions in Lemmas 1-4. If L_A and R_A win their respective primaries with certainty, then the most favorable policies that L_D and R_D can choose in an effort to win their respective primaries are the policies given by Lemmas 1 and 2, or the policies given in properties (a) and (b) of Proposition 1. Thus L_D and R_D cannot profitably deviate from the policies given by this proposition.

And if L_D and R_D are running on these policies, then the most favorable way for L_A and R_A to win their primaries is by choosing the policies given by Lemmas 3 and 4, or the policies given in properties (c) and (d) of Proposition 1. Thus L_A and R_A will also not be able to profitably deviate from the policies given by this proposition.

Thus as long as the policies given by properties (a)-(d) also satisfy the bounds in Lemmas 1-4, it will be an equilibrium for the candidates to run on these policies when the voters in the primaries have sufficiently extreme preferences. Properties (e) and (f) present the inequalities that must be satisfied for these bounds to hold. From this it follows that if l_A, l_D, r_A, and r_D satisfy the properties in Proposition 1, then it is an equilibrium for the candidates to run on these policies.

With this result in mind, I now derive conditions under which an equilibrium exists. Proving equilibrium existence in full generality is a difficult task, so I make an assumption about the distribution of the median voter's ideal point in the general election to simplify the analysis. In particular, I assume that the density from which the median voter's ideal point is drawn is symmetric about 0 and weakly single-peaked at 0. Given this assumption, there exists an equilibrium to the candidate policy-selection game. This result is stated formally in Proposition 2.

Proposition 2. *Suppose that $b > \delta$, $a = -b$, $x_l \leq -2b + \delta$, $x_r \geq 2b - \delta$, $f(x) = f(-x)$ for all x, and $f(x)$ is nonincreasing in x for all $x \in [0, b]$. Then there is an equilibrium in which the policies chosen by L_A, L_D, R_A, and R_D satisfy $l_D = -r_D$, $l_A = -r_A$, $r_D \in (\delta, 2b - \delta)$, and $r_A \in (0, r_D - \delta)$.*

Here the assumptions about $f(x)$ guarantee that the density from which the median voter's ideal point is drawn is symmetric about 0 and weakly single-peaked at 0. The conditions that $b > \delta$ and $a = -b$ indicate that there is enough uncertainty about the policy preferences of the median voter compared to the difference in quality that a disadvantaged candidate would have a chance of winning the general election.[3] And the conditions that $x_l \leq -2b + \delta$ and $x_r \geq 2b - \delta$ mean that

[3]If $\delta > b > 0$, $x_l \leq -b$, $x_r \geq b$, and the other conditions in Proposition 2 hold, then there is an equilibrium in which L_A and R_A both choose the policy 0, L_D and R_D choose any policy in X, and L_A and R_A win their respective primaries with certainty.

most voters in the primaries have extreme preferences compared to the general population. Given these assumptions, Proposition 2 indicates that there is an equilibrium to this game.

When the density $f(x)$ is symmetric about 0, as in Proposition 2, the most moderate policy compared to the likely position of the median voter in the general election is 0. And the type of equilibrium in Proposition 2 has the advantaged candidate in a given primary running on policies that are at least δ units closer to 0 than the disadvantaged candidate. Thus this result indicates that the high quality candidates run on more moderate policies than the low quality candidates. This is consistent with empirical evidence on how quality affects policy choices (Ansolabehere *et al.*, 2001; Fiorina, 1973; Stone and Simas, 2007).

At the same time, candidates in the liberal primary choose policies to the left of 0 and candidates in the conservative primary choose policies to the right of 0. Thus candidates in the liberal primary choose liberal policies and candidates in the conservative primary choose conservative policies. This indicates the equilibrium given in Proposition 2 is consistent with empirical evidence on how the policies chosen by candidates in different parties vary with party and candidate quality.

5. Conclusion

This paper has analyzed a model in which candidates of different quality strategically choose policies when they must win a primary election to compete in the general election. The main result indicates that candidates in the liberal primary choose liberal policies, candidates in the conservative primary choose conservative policies, but higher quality candidates choose more moderate policies. Higher quality candidates moderate their policy choices because their quality advantage enables them to be viable candidates in the primary election even if they choose policies that do not match the preferences of the primary voters. Because of this, these candidates have an incentive to focus more on choosing moderate policies that will be appealing in the general election. I now discuss the robustness of the results to modeling assumptions.

One natural extension to the model is to allow for the possibility that there are more than two candidates in the primaries, all of whom differ in their quality, and elections are decided by plurality rule. In this case, one obtains similar policy selection to that in Proposition 2. Suppose, for example, the difference in quality between the best candidate in each primary and the second best candidate in each primary is δ, but there are also some lower quality candidates in each primary. Then there is an equilibrium in which the highest quality candidates in the primaries choose the policies l_A and r_A in Proposition 2, the second highest quality candidates choose the policies l_D and r_D in Proposition 2, and the other candidates choose any policies in X. If the candidates choose these policies, then the majority of voters in each primary would still have an incentive to vote for the highest quality candidate in the primary, and the other candidates would have no chance of winning the primaries. Thus in mulitcandidate elections, there exists an equilibrium in which the two best candidates in each primary choose the same policies as those in Proposition 2.

Another natural extension is to consider what happens when candidates can choose different policies in the primary and the general election but changing positions results in accusations of flip-flopping that hurt a candidate, as in Hummel (2009a). If one assumes that flip-flopping affects a voter's utility in the manner given in Hummel (2009a), then there is an equilibrium in which candidates in the primaries choose policies more extreme than those in Proposition 2 by some fixed amount x^*, and general election candidates choose the same policies as those given in Proposition 2. Thus the type of equilibrium in Proposition 2 is robust to the assumption that candidates must use the same policies in the primary and the general election.

Finally, I address what happens when candidates have the option of being ambiguous about their policy selection in the primary, as in Meirowitz (2005). Meirowitz (2005) considers a model in which candidates can either commit to a policy before the primary or remain ambigious. If a candidate is ambiguous in the primary, then the candidate commits to a policy before the general election. If one introduces this possibility to the model, the policy choices in Proposition 2 would still be an equilibrium because if a candidate deviates by being ambiguous before the primary, the candidate loses the primary. Thus the results in this paper are robust to a variety of natural extensions to the model.

Appendix

Lemma 1. *Suppose that L_D wins the liberal primary with certainty and R_A wins the conservative primary with certainty by running on a policy $r_A > a + \delta$. Then there is at most one $l_D \in (2a - r_A + \delta, r_A - \delta)$ satisfying $F(\frac{l_D + r_A - \delta}{2}) = \frac{r_A - l_D - \delta}{2} f(\frac{l_D + r_A - \delta}{2})$. If l_D^* satisfies this condition, then a voter with ideal point $v_i < l_D^*$ strictly prefers that L_D run on the policy l_D^* instead of any other policy.*

Proof. First I show that if $r_A > a + \delta$, then there is at most one $l_D \in (2a - r_A + \delta, r_A - \delta)$ satisfying $F(\frac{l_D + r_A - \delta}{2}) = \frac{r_A - l_D - \delta}{2} f(\frac{l_D + r_A - \delta}{2})$. To see this, first note that any such l_D must satisfy $a < \frac{l_D + r_A - \delta}{2} \leq b$ because $l_D > 2a - r_A + \delta$ implies $a < \frac{l_D + r_A - \delta}{2}$ and $\frac{l_D + r_A - \delta}{2} > b$ implies $F(\frac{l_D + r_A - \delta}{2}) = 1 > 0 = \frac{r_A - l_D - \delta}{2} f(\frac{l_D + r_A - \delta}{2})$. Also note that $\frac{F(\frac{l_D + r_A - \delta}{2})}{f(\frac{l_D + r_A - \delta}{2})} + \frac{l_D - r_A + \delta}{2}$ is strictly increasing in l_D when $a < \frac{l_D + r_A - \delta}{2} \leq b$ since $\frac{F(\frac{l_D + r_A - \delta}{2})}{f(\frac{l_D + r_A - \delta}{2})}$ is nondecreasing in l_D when $a < \frac{l_D + r_A - \delta}{2} \leq b$. Thus any $l_D \in (2a - r_A + \delta, r_A - \delta)$ satisfying $\frac{F(\frac{l_D + r_A - \delta}{2})}{f(\frac{l_D + r_A - \delta}{2})} + \frac{l_D - r_A + \delta}{2} = 0$ must be unique, and there is at most one $l_D \in (2a - r_A + \delta, r_A - \delta)$ satisfying $F(\frac{l_D + r_A - \delta}{2}) = \frac{r_A - l_D - \delta}{2} f(\frac{l_D + r_A - \delta}{2})$.

Now I show that if such an l_D exists and l_D^* denotes the unique $l_D \in (2a - r_A + \delta, r_A - \delta)$ satisfying $F(\frac{l_D + r_A - \delta}{2}) = \frac{r_A - l_D - \delta}{2} f(\frac{l_D + r_A - \delta}{2})$, then a voter with ideal point $v_i < l_D^*$ strictly prefers that L_D run on the policy l_D^* instead of any other policy.

To see this, first note that a voter with ideal point $v_i \leq r_A - \delta$ weakly prefers that L_D run on the policy $l_D = r_A - \delta$ instead of any other policy $l_D \geq r_A - \delta$. If R_A wins the general election, then a voter with ideal point $v_i \leq r_A - \delta$ obtains utility $\delta - |r_A - v_i| = \delta - r_A + v_i$. But if L_D wins the general election by running on a policy $l_D \geq r_A - \delta$, then a voter with ideal point $v_i \leq r_A - \delta$ obtains utility $-|l_D - v_i| = v_i - l_D \leq \delta - r_A + v_i$.

Thus if L_D runs on a policy $l_D \geq r_A - \delta$, then a voter with ideal point $v_i \leq r_A - \delta$ obtains utility no greater than $\delta - r_A + v_i$. But if L_D runs on the policy $l_D = r_A - \delta$, then a voter with ideal point $v_i \leq r_A - \delta$ is assured of obtaining utility $\delta - r_A + v_i$ regardless of whether R_A or L_D wins the general election. Thus a voter with ideal point $v_i \leq r_A - \delta$ weakly prefers that L_D run on the policy $l_D = r_A - \delta$ instead of any other policy $l_D \geq r_A - \delta$.

Now note that if L_D runs on a policy $l_D < r_A - \delta$, then voters with ideal points $v_i < \frac{l_D + r_A - \delta}{2}$ strictly prefer L_D over R_A and voters with ideal points $v_i > \frac{l_D + r_A - \delta}{2}$ strictly prefer L_D over R_A. Thus if $l_D < r_A - \delta$, then L_D wins the general election with probability $F(\frac{l_D + r_A - \delta}{2})$, and R_A wins the general election with probability $1 - F(\frac{l_D + r_A - \delta}{2})$. So if L_D runs on a policy $l_D < r_A - \delta$, then a voter with ideal point $v_i \leq l_D$ obtains expected utility $-F(\frac{l_D + r_A - \delta}{2})|v_i - l_D| + (1 - F(\frac{l_D + r_A - \delta}{2}))(\delta - |v_i - r_A|) = F(\frac{l_D + r_A - \delta}{2})(v_i - l_D) + (1 - F(\frac{l_D + r_A - \delta}{2}))(\delta - r_A + v_i) = v_i - l_D F(\frac{l_D + r_A - \delta}{2}) + (\delta - r_A)(1 - F(\frac{l_D + r_A - \delta}{2}))$. And if L_D runs on the policy $l_D = r_A - \delta$, then a voter with ideal point $v_i \leq l_D$ obtains utility $\delta - r_A + v_i = v_i - l_D F(\frac{l_D + r_A - \delta}{2}) + (\delta - r_A)(1 - F(\frac{l_D + r_A - \delta}{2}))$.

From this it follows that if L_D runs on a policy $l_D \leq r_A - \delta$, then a voter with ideal point $v_i \leq l_D$ obtains utility $U(v_i; l_D) \equiv v_i - l_D F(\frac{l_D + r_A - \delta}{2}) + (\delta - r_A)(1 - F(\frac{l_D + r_A - \delta}{2}))$. Now $\frac{d}{dl_D} U(v_i; l_D) =$

$-F(\frac{l_D+r_A-\delta}{2}) - \frac{l_D}{2} f(\frac{l_D+r_A-\delta}{2}) + \frac{r_A-\delta}{2} f(\frac{l_D+r_A-\delta}{2}) = \frac{r_A-l_D-\delta}{2} f(\frac{l_D+r_A-\delta}{2}) - F(\frac{l_D+r_A-\delta}{2})$, so $\frac{d}{dl_D}U(v_i; l_D^*) = 0$, and $\frac{d}{dl_D}U(v_i; l_D) < 0$ if $\frac{l_D+r_A-\delta}{2} > b$ since $\frac{l_D+r_A-\delta}{2} > b$ implies $f(\frac{l_D+r_A-\delta}{2}) = 0$ and $F(\frac{l_D+r_A-\delta}{2}) = 1$. And since $\frac{F(\frac{l_D+r_A-\delta}{2})}{f(\frac{l_D+r_A-\delta}{2})}$ is nondecreasing in l_D when $\frac{l_D+r_A-\delta}{2} \in [a, b]$, it follows that $\frac{r_A-l_D-\delta}{2} - \frac{F(\frac{l_D+r_A-\delta}{2})}{f(\frac{l_D+r_A-\delta}{2})}$ is strictly decreasing in l_D for $l_D \in [2a - r_A + \delta, 2b - r_A + \delta]$. Thus since $\frac{r_A-l_D^*-\delta}{2} - \frac{F(\frac{l_D^*+r_A-\delta}{2})}{f(\frac{l_D^*+r_A-\delta}{2})} = 0$, it follows that $\frac{r_A-l_D-\delta}{2} - \frac{F(\frac{l_D+r_A-\delta}{2})}{f(\frac{l_D+r_A-\delta}{2})} < 0$ for $l_D \in (l_D^*, 2b - r_A + \delta]$ and $\frac{r_A-l_D-\delta}{2} f(\frac{l_D+r_A-\delta}{2}) - F(\frac{l_D+r_A-\delta}{2}) < 0$ for $l_D \in (l_D^*, 2b - r_A + \delta]$. Thus $\frac{d}{dl_D}U(v_i; l_D) < 0$ for $l_D > l_D^*$ and $\frac{d}{dl_D}U(v_i; l_D^*) = 0$. From this it follows that a voter with ideal point $v_i < l_D^*$ strictly prefers that L_D run on the policy $l_D = l_D^*$ instead of any other policy $l_D \in [l_D^*, r_A - \delta]$.

Now I show that a voter with ideal point $v_i < l_D^*$ strictly prefers that L_D run on the policy $l_D = l_D^*$ instead of any other policy $l_D \in [\max\{v_i, 2a - r_A + \delta\}, l_D^*]$. Since $\frac{r_A-l_D-\delta}{2} - \frac{F(\frac{l_D+r_A-\delta}{2})}{f(\frac{l_D+r_A-\delta}{2})}$ is strictly decreasing in l_D for $l_D \in [2a - r_A + \delta, 2b - r_A + \delta]$ and $\frac{r_A-l_D^*-\delta}{2} - \frac{F(\frac{l_D^*+r_A-\delta}{2})}{f(\frac{l_D^*+r_A-\delta}{2})} = 0$, we have $\frac{r_A-l_D-\delta}{2} - \frac{F(\frac{l_D+r_A-\delta}{2})}{f(\frac{l_D+r_A-\delta}{2})} > 0$ for $l_D \in [2a - r_A + \delta, l_D^*)$ and $\frac{r_A-l_D-\delta}{2} f(\frac{l_D+r_A-\delta}{2}) - F(\frac{l_D+r_A-\delta}{2}) > 0$ for $l_D \in [2a - r_A + \delta, l_D^*)$. Thus if $l_D \in [2a - r_A + \delta, l_D^*)$ and $v_i \leq l_D$, then $\frac{d}{dl_D}U(v_i; l_D) > 0$. Combining this with the fact that $\frac{d}{dl_D}U(v_i; l_D^*) = 0$ if $v_i < l_D^*$ shows that a voter with ideal point $v_i < l_D^*$ strictly prefers that L_D run on the policy $l_D = l_D^*$ instead of any other policy $l_D \in [\max\{v_i, 2a - r_A + \delta\}, l_D^*]$.

Now I show that a voter with ideal point $v_i < l_D^*$ weakly prefers that L_D run on the policy $l_D = v_i$ instead of any other policy $l_D \leq v_i$. To see this, note that if L_D runs on the policy $l_D = v_i$, then a voter with ideal point $v_i < l_D^*$ obtains expected utility $(\delta + v_i - r_A)(1 - F(\frac{l_D+r_A-\delta}{2})) = (\delta + v_i - r_A)(1 - F(\frac{v_i+r_A-\delta}{2}))$. But if L_D runs on a policy $l_D < v_i$, then a voter with ideal point $v_i < l_D^*$ obtains expected utility $-F(\frac{l_D+r_A-\delta}{2})|v_i - l_D| + (1 - F(\frac{l_D+r_A-\delta}{2}))(\delta + v_i - r_A) \leq (\delta + v_i - r_A)(1 - F(\frac{l_D+r_A-\delta}{2})) \leq (\delta + v_i - r_A)(1 - F(\frac{v_i+r_A-\delta}{2}))$. Thus a voter with ideal point $v_i < l_D^*$ weakly prefers that L_D run on the policy $l_D = v_i$ instead of any other policy $l_D \leq v_i$.

Finally I show that a voter with ideal point $v_i < l_D^*$ weakly prefers that L_D run on the policy $l_D = r_A - \delta$ instead of any other policy $l_D < 2a - r_A + \delta$. To see this, recall that if L_D runs on the policy $l_D = r_A - \delta$, then a voter with ideal point $v_i < l_D^*$ obtains utility $\delta - r_A + v_i$. And if L_D runs on a policy $l_D < 2a - r_A + \delta$, then a voter with ideal point $v_i < l_D^*$ obtains expected utility $-F(\frac{l_D+r_A-\delta}{2})|v_i - l_D| + (1 - F(\frac{l_D+r_A-\delta}{2}))(\delta + v_i - r_A) = \delta - r_A + v_i$. Thus a voter with ideal point $v_i < l_D^*$ weakly prefers that L_D run on the policy $l_D = r_A - \delta$ instead of any policy $l_D < 2a - r_A + \delta$.

Putting this all together, we see that a voter with ideal point $v_i < l_D^*$ strictly prefers that L_D run on the policy $l_D = l_D^*$ instead of any other policy $l_D \geq l_D^*$, because this voter strictly prefers that

L_D run on the policy $l_D = l_D^*$ instead of any other policy $l_D \in [l_D^*, r_A - \delta]$, and weakly prefers that L_D run on the policy $l_D = r_A - \delta$ to any other policy $l_D \geq r_A - \delta$.

A voter with ideal point $v_i < l_D^*$ also strictly prefers that L_D run on the policy $l_D = l_D^*$ instead of any other policy $l_D \leq l_D^*$. A voter with ideal point $v_i < l_D^*$ strictly prefers that L_D run on the policy $l_D = l_D^*$ instead of any policy $l_D < 2a - r_A + \delta$ because this voter strictly prefers that L_D run on the policy $l_D = l_D^*$ instead of the policy $l_D = r_A - \delta$ and weakly prefers that L_D run on the policy $l_D = r_A - \delta$ instead of any policy $l_D < 2a - r_A + \delta$. A voter with ideal point $v_i < l_D^*$ also strictly prefers that L_D run on the policy $l_D = l_D^*$ instead of any other policy $l_D \in [\max\{v_i, 2a - r_A + \delta\}, l_D^*]$. And a voter with ideal point $v_i < l_D^*$ strictly prefers that L_D run on the policy $l_D = l_D^*$ instead of any policy $l_D \leq v_i$ because a voter with ideal point $v_i < l_D^*$ strictly prefers that L_D run on the policy $l_D = l_D^*$ instead of the policy $l_D = v_i$, and a voter with ideal point $v_i < l_D^*$ weakly prefers that L_D run on the policy $l_D = v_i$ instead of any other policy $l_D \leq v_i$. Thus a voter with ideal point $v_i < l_D^*$ also strictly prefers that L_D run on the policy $l_D = l_D^*$ instead of any other policy $l_D \leq l_D^*$.

Thus a voter with ideal point $v_i < l_D^*$ strictly prefers that L_D run on the policy $l_D = l_D^*$ instead of any other policy $l_D \in X$. The result then follows.

□

Lemma 3. *Suppose that (1) R_A wins the conservative primary with certainty by running on a policy $r_A > a + \delta$, (2) there is some policy $l_D \in (2a - r_A + \delta, r_A - \delta)$ that satisfies $F(\frac{l_D + r_A - \delta}{2}) = \frac{r_A - l_D - \delta}{2} f(\frac{l_D + r_A - \delta}{2})$, and (3) L_D runs on the unique policy $l_D \in (2a - r_A + \delta, r_A - \delta)$ that satisfies $F(\frac{l_D + r_A - \delta}{2}) = \frac{r_A - l_D - \delta}{2} f(\frac{l_D + r_A - \delta}{2})$. Then there is a unique $l_A \in [l_D + \delta, r_A)$ satisfying $(r_A - l_A)F(\frac{l_A + r_A}{2}) = (r_A - l_D - \delta)F(\frac{l_D + r_A - \delta}{2})$. If l_A^* satisfies this condition, then l_A^* is the most conservative policy that L_A can choose such that voters with ideal points $v_i < l_D$ will weakly prefer voting for L_A in the liberal primary.*

Proof. First I show that there is a unique $l_A \in [l_D + \delta, r_A)$ satisfying $(r_A - l_A)F(\frac{l_A + r_A}{2}) = (r_A - l_D - \delta)F(\frac{l_D + r_A - \delta}{2})$. To see that there exists some $l_A \in [l_D + \delta, r_A)$ satisfying $(r_A - l_A)F(\frac{l_A + r_A}{2}) = (r_A - l_D - \delta)F(\frac{l_D + r_A - \delta}{2})$, first note that if $l_A = l_D + \delta$, then $(r_A - l_A)F(\frac{l_A + r_A}{2}) = (r_A - l_D - \delta)F(\frac{l_D + r_A + \delta}{2}) \geq (r_A - l_D - \delta)F(\frac{l_D + r_A - \delta}{2})$. Also note that $\lim_{l_A \to r_A}(r_A - l_A)F(\frac{l_A + r_A}{2}) = 0 < (r_A - l_D - \delta)F(\frac{l_D + r_A - \delta}{2})$ since $l_D \in (2a - r_A + \delta, r_A - \delta)$ implies $r_A - l_D - \delta > 0$ and $F(\frac{l_D + r_A - \delta}{2}) > F(a) = 0$. Thus since $(r_A - l_A)F(\frac{l_A + r_A}{2})$ is continuous in l_A, it follows from the intermediate value theorem that there exists some $l_A \in [l_D + \delta, r_A)$ that satisfies $(r_A - l_A)F(\frac{l_A + r_A}{2}) = (r_A - l_D - \delta)F(\frac{l_D + r_A - \delta}{2})$.

Now I show that the $l_A \in [l_D + \delta, r_A)$ that satisfies $(r_A - l_A)F(\frac{l_A+r_A}{2}) = (r_A - l_D - \delta)F(\frac{l_D+r_A-\delta}{2})$ must be unique. To prove this, it suffices to show that $(r_A - l_A)F(\frac{l_A+r_A}{2})$ is strictly decreasing in l_A for $l_A \in [l_D + \delta, r_A)$. To see that $g(l_A; r_A) \equiv (r_A - l_A)F(\frac{l_A+r_A}{2})$ is strictly decreasing in l_A, note that $\frac{d}{dl_A} g(l_A; r_A) = \frac{r_A-l_A}{2} f(\frac{l_A+r_A}{2}) - F(\frac{l_A+r_A}{2})$. Thus $\frac{d}{dl_A} g(l_A; r_A) < 0$ if and only if $\frac{r_A-l_A}{2} \frac{f(\frac{l_A+r_A}{2})}{F(\frac{l_A+r_A}{2})} - 1 < 0$ or $\frac{r_A-l_A}{2} \frac{f(\frac{l_A+r_A}{2})}{F(\frac{l_A+r_A}{2})} < 1$ and $\frac{d}{dl_A} g(l_A; r_A) = 0$ if and only if $\frac{r_A-l_A}{2} \frac{f(\frac{l_A+r_A}{2})}{F(\frac{l_A+r_A}{2})} = 1$.

Now l_D is chosen to satisfy $\frac{r_A-l_D-\delta}{2} \frac{f(\frac{l_D+r_A-\delta}{2})}{F(\frac{l_D+r_A-\delta}{2})} = 1$. And since $\frac{f(\frac{l_D+r_A-\delta}{2})}{F(\frac{l_D+r_A-\delta}{2})} \geq \frac{f(\frac{l_D+r_A+\delta}{2})}{F(\frac{l_D+r_A+\delta}{2})}$, it follows that $\frac{r_A-l_D-\delta}{2} \frac{f(\frac{l_D+r_A+\delta}{2})}{F(\frac{l_D+r_A+\delta}{2})} \leq 1$. Moreover, if $l_A > l_D + \delta$, then $\frac{f(\frac{l_A+r_A}{2})}{F(\frac{l_A+r_A}{2})} \leq \frac{f(\frac{l_D+r_A+\delta}{2})}{F(\frac{l_D+r_A+\delta}{2})}$, and we also have $\frac{r_A-l_A}{2} < \frac{r_A-l_D-\delta}{2}$. Thus $l_A > l_D + \delta$ implies either $\frac{r_A-l_A}{2} \frac{f(\frac{l_A+r_A}{2})}{F(\frac{l_A+r_A}{2})} = 0$ or $\frac{r_A-l_A}{2} \frac{f(\frac{l_A+r_A}{2})}{F(\frac{l_A+r_A}{2})} < \frac{r_A-l_D-\delta}{2} \frac{f(\frac{l_D+r_A+\delta}{2})}{F(\frac{l_D+r_A+\delta}{2})}$. In either case, we have $\frac{r_A-l_A}{2} \frac{f(\frac{l_A+r_A}{2})}{F(\frac{l_A+r_A}{2})} < 1$ if $l_A > l_D + \delta$. Combining this with the fact that $\frac{r_A-l_D-\delta}{2} \frac{f(\frac{l_D+r_A+\delta}{2})}{F(\frac{l_D+r_A+\delta}{2})} \leq 1$ shows that $\frac{d}{dl_A} g(l_A; r_A) \leq 0$ for $l_A = l_D + \delta$ and $\frac{d}{dl_A} g(l_A; r_A) < 0$ for $l_A > l_D + \delta$. Thus $g(l_A; r_A) = (r_A - l_A)F(\frac{l_A+r_A}{2})$ is strictly decreasing in l_A for $l_A \in [l_D + \delta, r_A)$, and the $l_A \in [l_D + \delta, r_A)$ that satisfies $(r_A - l_A)F(\frac{l_A+r_A}{2}) = (r_A - l_D - \delta)F(\frac{l_D+r_A-\delta}{2})$ must be unique.

Now I show that if l_A^* denotes the unique $l_A \in [l_D + \delta, r_A)$ satisfying $(r_A - l_A)F(\frac{l_A+r_A}{2}) = (r_A - l_D - \delta)F(\frac{l_D+r_A-\delta}{2})$, then l_A^* is the most conservative policy that L_A can choose such that voters with ideal points $v_i < l_D$ will weakly prefer voting for L_A in the liberal primary. To see this, note that if L_A wins the liberal primary by running on some policy $l_A \in [l_D + \delta, r_A)$, then L_A wins the general election with probability $F(\frac{l_A+r_A}{2})$ and voters with ideal points $v_i < l_D$ obtain expected utility $F(\frac{l_A+r_A}{2})(\delta - |v_i - l_A|) + (1 - F(\frac{l_A+r_A}{2}))(\delta - |v_i - r_A|) = F(\frac{l_A+r_A}{2})(\delta + v_i - l_A) + (1 - F(\frac{l_A+r_A}{2}))(\delta + v_i - r_A) = v_i + F(\frac{l_A+r_A}{2})(\delta - l_A) + (1 - F(\frac{l_A+r_A}{2}))(\delta - r_A)$. By contrast, as noted in the proof of Lemma 1, if L_D wins the liberal primary by running on a policy $l_D > v_i$, then a voter with ideal point v_i obtains expected utility $v_i - l_D F(\frac{l_D+r_A-\delta}{2}) + (\delta - r_A)(1 - F(\frac{l_D+r_A-\delta}{2}))$.

Thus if L_A runs on some policy $l_A \in [l_D + \delta, r_A)$, then voters with ideal points $v_i < l_D$ weakly prefer that L_A win the liberal primary if and only if $v_i + F(\frac{l_A+r_A}{2})(\delta - l_A) + (1 - F(\frac{l_A+r_A}{2}))(\delta - r_A) \geq v_i - l_D F(\frac{l_D+r_A-\delta}{2}) + (\delta - r_A)(1 - F(\frac{l_D+r_A-\delta}{2}))$, which holds if and only if $(r_A - l_A)F(\frac{l_A+r_A}{2}) \geq (r_A - \delta - l_D)F(\frac{l_D+r_A-\delta}{2})$. Since we have seen that $(r_A - l_A)F(\frac{l_A+r_A}{2})$ is strictly decreasing in l_A, it follows that if l_A^* denotes the unique value of $l_A \in [l_D + \delta, r_A)$ satisfying $(r_A - l_A)F(\frac{l_A+r_A}{2}) = (r_A - l_D - \delta)F(\frac{l_D+r_A-\delta}{2})$, then l_A^* is the most conservative policy in $[l_D + \delta, r_A)$ that L_A can choose such that voters with ideal points $v_i < l_D$ will weakly prefer voting for L_A in the liberal primary.

Since $l_A^* \in [l_D + \delta, r_A)$, it follows that l_A^* is a more conservative policy than all policies $l_A < l_D + \delta$. Thus l_A^* is also the most conservative policy in $(-\infty, r_A)$ that L_A can choose such that voters with ideal points $v_i < l_D$ will weakly prefer voting for L_A in the liberal primary. Thus to prove that l_A^* is the most conservative policy that L_A can choose such that voters with ideal points $v_i < l_D$ will weakly prefer voting for L_A in the liberal primary, it suffices to show that if L_A runs on some policy $l_A \geq r_A$, then voters with ideal points $v_i < l_D$ strictly prefer that L_D wins the liberal primary instead of L_A.

To see this, note that if L_A wins the liberal primary by running on some policy $l_A \geq r_A$, then voters with ideal points $v_i < l_D$ weakly prefer that R_A wins the general election, and voters with ideal points $v_i < l_D$ obtain expected utility no greater than the utility they obtain if R_A wins the general election. However, if L_D wins the liberal primary by running on the unique policy $l_D \in (2a - r_A + \delta, r_A - \delta)$ that satisfies $F(\frac{l_D + r_A - \delta}{2}) = \frac{r_A - l_D - \delta}{2} f(\frac{l_D + r_A - \delta}{2})$, then we know from Lemma 1 that voters with ideal points $v_i < l_D$ obtain greater expected utility than the utility they obtain if R_A wins the general election. Thus if L_A runs on some policy $l_A \geq r_A$, then voters with ideal points $v_i < l_D$ strictly prefer that L_D wins the liberal primary instead of L_A. From this it follows that l_A^* is the most conservative policy that L_A can choose such that voters with ideal points $v_i < l_D$ will weakly prefer voting for L_A in the liberal primary.

□

Proposition 1. *Suppose that l_A, l_D, r_A, and r_D satisfy the following properties:*

(a). $F(\frac{l_D + r_A - \delta}{2}) = \frac{r_A - l_D - \delta}{2} f(\frac{l_D + r_A - \delta}{2})$.

(b). $1 - F(\frac{l_A + r_D + \delta}{2}) = \frac{r_D - l_A - \delta}{2} f(\frac{l_A + r_D + \delta}{2})$.

(c). $(r_A - l_A) F(\frac{l_A + r_A}{2}) = (r_A - l_D - \delta) F(\frac{l_D + r_A - \delta}{2})$.

(d). $(r_A - l_A)(1 - F(\frac{l_A + r_A}{2})) = (r_D - l_A - \delta)(1 - F(\frac{l_A + r_D + \delta}{2}))$.

(e). $2a - r_A + 2\delta < l_D + \delta \leq l_A < r_A \leq r_D - \delta < 2b - l_A - 2\delta$.

(f). $x_l < l_D$ *and* $x_r > r_D$.

Then there is an equilibrium in which L_A chooses the policy l_A, L_D chooses the policy l_D, R_A chooses the policy r_A, and R_D chooses the policy r_D.

Proof. First I show that if l_A, l_D, r_A, and r_D satisfy the properties given in the proposition, then there is an equilibrium in which L_A wins the liberal primary with certainty and R_A wins the conservative primary with certainty.

To see this, note from Lemma 3 that if R_A wins the conservative primary with certainty, $r_A > a + \delta$, $l_D \in (2a - r_A + \delta, r_A - \delta)$, and properties (a) and (c) hold, then voters with ideal points $v_i < l_D$

will weakly prefer voting for L_A in the liberal primary. Now $2a - r_A + 2\delta < r_A$ implies $r_A > a + \delta$, $2a - r_A + 2\delta < l_D + \delta$ implies $l_D > 2a - r_A + \delta$, and $l_D + \delta < r_A$ implies $l_D < r_A - \delta$. Thus property (e) implies $r_A > a + \delta$ and $l_D \in (2a - r_A + \delta, r_A - \delta)$. From this it follows that if R_A wins the conservative primary with certainty and properties (a), (c), and (e) hold, then voters with ideal points $v_i < l_D$ will weakly prefer voting for L_A in the liberal primary.

Now since $x_l < l_D$, the majority of voters in N_l have ideal points $v_i < l_D$. Thus if R_A wins the conservative primary with certainty and properties (a), (c), (e), and (f) hold, the majority of voters in N_l weakly prefer voting for L_A in the liberal primary, and there is an equilibrium in which L_A wins the liberal primary with certainty.

Similar reasoning shows that if L_A wins the liberal primary with certainty and properties (b), (d), (e), and (f) hold, then there is an equilibrium in which R_A wins the conservative primary with certainty. Combining the results in these two paragraphs shows that if l_A, l_D, r_A, and r_D satisfy the properties given in the proposition, then there is an equilibrium in which L_A wins the liberal primary with certainty and R_A wins the conservative primary with certainty.

Now I show that if l_A, l_D, r_A, and r_D satisfy the properties given in the proposition, L_A wins the liberal primary with certainty, and R_A wins the conservative primary with certainty, then neither L_D nor R_D can profitably deviate from their selections of policies.

To see this, note from Lemma 1 that if R_A wins the conservative primary with certainty, $r_A > a+\delta$, $l_D \in (2a - r_A + \delta, r_A - \delta)$, and property (a) holds, then any voter with ideal point $v_i < l_D$ obtains a higher expected utility from the election of L_D when L_D runs on the policy l_D than when L_D runs on any other policy. Since property (e) implies $r_A > a + \delta$ and $l_D \in (2a - r_A + \delta, r_A - \delta)$, it follows that if R_A wins the conservative primary with certainty and properties (a) and (e) hold, then any voter with ideal point $v_i < l_D$ obtains a higher expected utility from the election of L_D when L_D runs on the policy l_D than when L_D runs on any other policy.

We have seen that if R_A wins the conservative primary with certainty and properties (a), (c), and (e) hold, then voters with ideal points $v_i < l_D$ will weakly prefer voting for L_A in the liberal primary. Combining this with the result in the previous paragraph shows that if L_D deviates to some other policy, l'_D, then voters with ideal points $v_i < l_D$ will strictly prefer voting for L_A in the liberal primary. And since we have seen that the majority of voters in N_l have ideal points $v_i < l_D$, it follows that if L_D deviates to some other policy, then the majority of voters in N_l strictly prefer voting for L_A in the liberal primary, and L_A wins the liberal primary. Thus if R_A wins the conservative primary with certainty and properties (a), (c), (e), and (f) hold, then L_D cannot win the liberal primary by deviating to some other policy.

Similar reasoning shows that if L_A wins the liberal primary with certainty and properties (b), (d), (e), and (f) hold, then R_D cannot win the conservative primary by deviating to some other policy. Thus if l_A, l_D, r_A, and r_D satisfy the properties given in the proposition, L_A wins the liberal primary with certainty, and R_A wins the conservative primary with certainty, then neither L_D nor R_D can profitably deviate from their selections of policies.

Now I show that if l_A, l_D, r_A, and r_D satisfy the properties given in the proposition, L_A wins the liberal primary with certainty, and R_A wins the conservative primary with certainty, then neither L_A nor R_A can profitably deviate from their selections of policies.

To see this, note that if l_A, l_D, r_A, and r_D satisfy the properties given in the proposition, L_A wins the liberal primary with certainty, and R_A wins the conservative primary with certainty, then L_A wins the general election with probability $F(\frac{l_A+r_A}{2})$ and obtains expected utility $u_l + F(\frac{l_A+r_A}{2})u$ for the game. But if L_A chooses a more liberal policy than l_A and L_A wins the liberal primary, then L_A wins the general election with probability no greater than $F(\frac{l_A+r_A}{2})$ and L_A obtains expected utility no greater than $u_l + F(\frac{l_A+r_A}{2})u$ for the game. And if L_A chooses a more liberal policy than l_A and L_A loses the liberal primary, then L_A obtains expected utility 0.

Finally, if L_A chooses a more conservative policy than l_A, then we know from Lemma 3 that voters with ideal points $v_i < l_D$ will strictly prefer voting for L_D instead of L_A. Since we have seen that the majority of voters in N_l have ideal points $v_i < l_D$, it follows that if L_A chooses a more conservative policy than l_A, then the majority of voters in N_l will vote for L_D and L_D will win the conservative primary. Thus if L_A chooses a more conservative policy than l_A, then L_A obtains expected utility 0. Combining this with the results in the previous paragraph shows that if l_A, l_D, r_A, and r_D satisfy the properties given in the proposition, L_A wins the liberal primary with certainty, and R_A wins the conservative primary with certainty, then L_A cannot profitably deviate by choosing a different policy than l_A.

Similar reasoning shows that if l_A, l_D, r_A, and r_D satisfy the properties given in the proposition, L_A wins the liberal primary with certainty, and R_A wins the conservative primary with certainty, then R_A cannot profitably deviate by choosing a different policy than r_A. Thus if l_A, l_D, r_A, and r_D

satisfy the properties given in the proposition, L_A wins the liberal primary with certainty, and R_A wins the conservative primary with certainty, then neither L_A nor R_A can profitably deviate from their selections of policies. From this it follows that if l_A, l_D, r_A, and r_D satisfy the properties given in the proposition, then there is an equilibrium in which L_A chooses the policy l_A, L_D chooses the policy l_D, R_A chooses the policy r_A, and R_D chooses the policy r_D.

□

Proposition 2. *Suppose that $b > \delta$, $a = -b$, $x_l \leq -2b + \delta$, $x_r \geq 2b - \delta$, $f(x) = f(-x)$ for all x, and $f(x)$ is nonincreasing in x for all $x \in [0, b]$. Then there is an equilibrium in which the policies chosen by L_A, L_D, R_A, and R_D satisfy $l_D = -r_D$, $l_A = -r_A$, $r_D \in (\delta, 2b - \delta)$, and $r_A \in (0, r_D - \delta)$.*

Proof. First I show that there exists some solution (r_A, r_D) to the equations $(r_A + r_D - \delta)F(\frac{r_D - r_A + \delta}{2}) - r_D + \delta = 0$ and $(r_A + r_D - \delta)^2 f(\frac{r_A - r_D - \delta}{2}) - 2r_A = 0$ such that $r_D \in (\delta, 2b - \delta)$ and $r_A \in (0, r_D - \delta)$. To prove this, I first demonstrate that for any $r_D \in (\delta, 2b - \delta)$, there exists some $r_A \in (0, r_D - \delta)$ such that $(r_A + r_D - \delta)F(\frac{r_D - r_A + \delta}{2}) - r_D + \delta = 0$.

To see this, note that $\lim_{r_A \to 0}(r_A + r_D - \delta)F(\frac{r_D - r_A + \delta}{2}) - r_D + \delta = (r_D - \delta)(F(\frac{r_D + \delta}{2}) - 1) < 0$ for $r_D \in (\delta, 2b - \delta)$ and $\lim_{r_A \to r_D - \delta}(r_A + r_D - \delta)F(\frac{r_D - r_A + \delta}{2}) - r_D + \delta = (r_D - \delta)(2F(\delta) - 1) > 0$. Thus since $(r_A + r_D - \delta)F(\frac{r_D - r_A + \delta}{2}) - r_D + \delta$ is continuous in r_A, it follows from the intermediate value theorem that for any $r_D \in (\delta, 2b - \delta)$, there exists some $r_A \in (0, r_D - \delta)$ such that $(r_A + r_D - \delta)F(\frac{r_D - r_A + \delta}{2}) - r_D + \delta = 0$.

Further note that for any $r_D \in (\delta, 2b - \delta)$, there is a unique $r_A \in (0, r_D - \delta)$ such that $(r_A + r_D - \delta)F(\frac{r_D - r_A + \delta}{2}) - r_D + \delta = 0$. To see this, note that if $g(r_A; r_D) \equiv (r_A + r_D - \delta)F(\frac{r_D - r_A + \delta}{2}) - r_D + \delta$, then $\frac{d}{dr_A} g(r_A; r_D) = F(\frac{r_D - r_A + \delta}{2}) - \frac{r_A + r_D - \delta}{2} f(\frac{r_D - r_A + \delta}{2})$ and $\text{sgn}[\frac{d}{dr_A} g(r_A; r_D)] = \text{sgn}[\frac{F(\frac{r_D - r_A + \delta}{2})}{f(\frac{r_D - r_A + \delta}{2})} - \frac{r_A + r_D - \delta}{2}]$. Now $\frac{F(\frac{r_D - r_A + \delta}{2})}{f(\frac{r_D - r_A + \delta}{2})} - \frac{r_A + r_D - \delta}{2}$ is strictly decreasing in r_A for $r_A \in (0, r_D - \delta)$ since $\frac{r_A + r_D - \delta}{2}$ is strictly increasing in r_A and $\frac{F(\frac{r_D - r_A + \delta}{2})}{f(\frac{r_D - r_A + \delta}{2})}$ is nonincreasing in r_A for $r_A \in (0, r_D - \delta)$. Thus either $\frac{d}{dr_A} g(r_A; r_D) < 0$ for all $r_A \in (0, r_D - \delta)$, $\frac{d}{dr_A} g(r_A; r_D) > 0$ for all $r_A \in (0, r_D - \delta)$, or there exists some $\hat{r}_A \in (0, r_D - \delta)$ such that $\frac{d}{dr_A} g(r_A; r_D) > 0$ for all $r_A \in (0, \hat{r}_A)$, $\frac{d}{dr_A} g(\hat{r}_A; r_D) = 0$, and $\frac{d}{dr_A} g(r_A; r_D) < 0$ for all $r_A \in (\hat{r}_A, r_D - \delta)$.

Now $\lim_{r_A \to 0} g(r_A; r_D) < \lim_{r_A \to r_D - \delta} g(r_A; r_D)$, so $\frac{d}{dr_A} g(r_A; r_D) < 0$ cannot hold for all $r_A \in (0, r_D - \delta)$. And if $\frac{d}{dr_A} g(r_A; r_D) > 0$ for all $r_A \in (0, r_D - \delta)$, it follows that there can only be one value of $r_A \in (0, r_D - \delta)$ for which $g(r_A; r_D) = 0$. Finally, if there is some $\hat{r}_A \in (0, r_D - \delta)$ such that $\frac{d}{dr_A} g(r_A; r_D) > 0$ for all $r_A \in (0, \hat{r}_A)$, $\frac{d}{dr_A} g(\hat{r}_A; r_D) = 0$, and $\frac{d}{dr_A} g(r_A; r_D) < 0$ for all

$r_A \in (\hat{r}_A, r_D - \delta)$, it must be the case that $g(r_A; r_D) > 0$ for all $r_A \in [\hat{r}_A, r_D - \delta)$ because the fact that $\lim_{r_A \to r_D - \delta} g(r_A; r_D) > 0$ and $\frac{d}{dr_A} g(r_A; r_D) \leq 0$ for all $r_A \in [\hat{r}_A, r_D - \delta)$ implies that $g(r_A; r_D) > 0$ for all $r_A \in [\hat{r}_A, r_D - \delta)$. Thus if there is some $\hat{r}_A \in (0, r_D - \delta)$ such that $\frac{d}{dr_A} g(r_A; r_D) > 0$ for all $r_A \in (0, \hat{r}_A)$, $\frac{d}{dr_A} g(\hat{r}_A; r_D) = 0$, and $\frac{d}{dr_A} g(r_A; r_D) < 0$ for all $r_A \in (\hat{r}_A, r_D - \delta)$, any r_A satsfying $g(r_A; r_D) = 0$ must be in the interval $(0, \hat{r}_A)$. But since $\frac{d}{dr_A} g(r_A; r_D) > 0$ for all $r_A \in (0, \hat{r}_A)$, there can only be one value of $r_A \in (0, \hat{r}_A)$ for which $g(r_A; r_D) = 0$. Thus the value of $r_A \in (0, r_D - \delta)$ for which $g(r_A; r_D) = (r_A + r_D - \delta) F(\frac{r_D - r_A + \delta}{2}) - r_D + \delta = 0$ is unique.

Now let $r_A^*(r_D)$ denote the set of $r_A \in (0, r_D - \delta)$ such that $(r_A + r_D - \delta) F(\frac{r_D - r_A + \delta}{2}) - r_D + \delta = 0$ for a given $r_D \in (\delta, 2b - \delta)$. Note that $r_A^*(r_D)$ is upper semicontinuous in r_D. If $\{(r_A^n, r_D^n)\}_{n=1}^{\infty}$ denotes an infinite sequence of (r_A, r_D) such that $r_A^n \in r_A^*(r_D^n)$, $r_A^n \in (0, r_D^n - \delta)$, and $r_D^n \in (\delta, 2b - \delta)$ for all n, then $(r_A^n + r_D^n - \delta) F(\frac{r_D^n - r_A^n + \delta}{2}) - r_D^n + \delta = 0$ for all n and $\lim_{n \to \infty} (r_A^n, r_D^n) = (r_A, r_D)$ implies $(r_A + r_D - \delta) F(\frac{r_D - r_A + \delta}{2}) - r_D + \delta = 0$ and $r_A \in r_A^*(r_D)$. Thus $r_A^*(r_D)$ is upper semicontinuous in r_D.

But we also know that $r_A^*(r_D)$ is a singleton for all $r_D \in (\delta, 2b - \delta)$. Thus since $r_A^*(r_D)$ is upper semicontinuous in r_D, it follows that $r_A^*(r_D)$ is also continuous in r_D. So if $r_A^{**}(r_D)$ denotes the unique $r_A \in (0, r_D - \delta)$ such that $(r_A + r_D - \delta) F(\frac{r_D - r_A + \delta}{2}) - r_D + \delta = 0$ for a given $r_D \in (\delta, 2b - \delta)$, it follows that $r_A^{**}(r_D)$ is continuous in r_D for all $r_D \in (\delta, 2b - \delta)$.

Now I show that there is some $r_D \in (\delta, 2b - \delta)$ such that if $r_A = r_A^{**}(r_D)$, then $(r_A + r_D - \delta)^2 f(\frac{r_A - r_D - \delta}{2}) - 2r_A = 0$. To see this, first note that if $r_D = \delta + \epsilon$ for some small $\epsilon > 0$, then $r_A^{**}(r_D) \in ((1 - F(\delta))\epsilon, \epsilon)$. We have $r_A^{**}(r_D) \in (0, \epsilon)$ because $r_A^{**}(r_D) \in (0, r_D - \delta)$ and $r_D = \delta + \epsilon$. And $r_A^{**}(r_D) \notin (0, (1 - F(\delta))\epsilon]$ for sufficiently small $\epsilon > 0$ because if $r_D = \delta + \epsilon$, then $(r_A + r_D - \delta) F(\frac{r_D - r_A + \delta}{2}) - r_D + \delta = (r_A + \epsilon) F(\frac{2\delta + \epsilon - r_A}{2}) - \epsilon$ and $r_A \in (0, (1 - F(\delta))\epsilon]$ implies $(r_A + \epsilon) F(\frac{2\delta + \epsilon - r_A}{2}) - \epsilon < (2 - F(\delta))\epsilon F(\delta + \epsilon) - \epsilon = \epsilon[(2 - F(\delta))F(\delta + \epsilon) - 1] = \epsilon[(2 - F(\delta))F(\delta) - 1 + (2 - F(\delta))(F(\delta + \epsilon) - F(\delta))] = \epsilon[(2 - F(\delta))(F(\delta + \epsilon) - F(\delta)) - (1 - F(\delta))^2] < 0$ for sufficiently small $\epsilon > 0$. Thus $(r_A + r_D - \delta) F(\frac{r_D - r_A + \delta}{2}) - r_D + \delta < 0$ if $r_D = \delta + \epsilon$ and $r_A \in (0, (1 - F(\delta))\epsilon]$ for sufficiently small $\epsilon > 0$, and $r_D = \delta + \epsilon$ implies $r_A^{**}(r_D) \in ((1 - F(\delta))\epsilon, \epsilon)$ for sufficiently small $\epsilon > 0$.

But if $r_D = \delta + \epsilon$ and $r_A \in ((1 - F(\delta))\epsilon, \epsilon)$, then $(r_A + r_D - \delta)^2 f(\frac{r_A - r_D - \delta}{2}) - 2r_A < 2\epsilon^2 f(0) - 2(1 - F(\delta))\epsilon < 0$ for sufficiently small $\epsilon > 0$. Thus if r_D is sufficiently close to δ, $r_A = r_A^{**}(r_D)$ implies $(r_A + r_D - \delta)^2 f(\frac{r_A - r_D - \delta}{2}) - 2r_A < 0$.

Now I show that if $r_D = 2b - \delta - \epsilon$, then $r_A^{**}(r_D) \in (0, \epsilon)$ for sufficiently small $\epsilon > 0$. Note that if $r_D = 2b - \delta - \epsilon$, then $r_A^{**}(r_D)$ is the unique $r_A \in (0, r_D - \delta)$ that is a solution to $(r_A + 2b - 2\delta - \epsilon)F(\frac{2b - r_A - \epsilon}{2}) - 2b + 2\delta + \epsilon = 0$. Thus if $h(r_A, \epsilon) \equiv (r_A + 2b - 2\delta - \epsilon)F(\frac{2b - r_A - \epsilon}{2}) - 2b + 2\delta + \epsilon$, then $\lim_{r_A \to 0} h(r_A, \epsilon) = (2b - 2\delta - \epsilon)(F(\frac{2b - \epsilon}{2}) - 1) < 0$ for sufficiently small $\epsilon > 0$. And $h(\epsilon, \epsilon) = 2(b - \delta)[F(b - \epsilon) - 1] + \epsilon > 0$ for sufficiently small $\epsilon > 0$ since $h(0, 0) = 0$ and $\frac{dh(\epsilon, \epsilon)}{d\epsilon} = 1 - 2(b - \delta)f(b - \epsilon) \geq 1 - \frac{b - \delta}{b} > 0$ for sufficiently small $\epsilon > 0$ because the fact that $f(x)$ is nondecreasing in x for all $x \in [0, b]$ implies $f(b - \epsilon) \leq \frac{1}{2b}$ for sufficiently small $\epsilon > 0$. But since $\lim_{r_A \to 0} h(r_A, \epsilon) < 0$ and $h(\epsilon, \epsilon) > 0$ for sufficiently small $\epsilon > 0$, it follows that there is some $r_A \in (0, \epsilon)$ such that $h(r_A, \epsilon) = 0$ for sufficiently small $\epsilon > 0$. Thus if $r_D = 2b - \delta - \epsilon$, then $r_A^{**}(r_D) \in (0, \epsilon)$ for sufficiently small $\epsilon > 0$.

But if $r_D = 2b - \delta - \epsilon$ and $r_A \in (0, \epsilon)$, then $(r_A + r_D - \delta)^2 f(\frac{r_A - r_D - \delta}{2}) - 2r_A > (2b - 2\delta - \epsilon)^2 f(b) - 2\epsilon > 0$ for sufficiently small $\epsilon > 0$. Thus if r_D is sufficiently close to $2b - \delta$, $r_A = r_A^{**}(r_D)$ implies $(r_A + r_D - \delta)^2 f(\frac{r_A - r_D - \delta}{2}) - 2r_A > 0$.

Now we have seen that $\lim_{r_D \to \delta^+} (r_A^{**}(r_D) + r_D - \delta)^2 f(\frac{r_A^{**}(r_D) - r_D - \delta}{2}) - 2r_A^{**}(r_D) < 0$ and $\lim_{r_D \to 2b - \delta^-} (r_A^{**}(r_D) + r_D - \delta)^2 f(\frac{r_A^{**}(r_D) - r_D - \delta}{2}) - 2r_A^{**}(r_D) > 0$. And since $r_A^{**}(r_D)$ is continuous in r_D, it follows that $(r_A^{**}(r_D) + r_D - \delta)^2 f(\frac{r_A^{**}(r_D) - r_D - \delta}{2}) - 2r_A^{**}(r_D)$ is continuous in r_D. Combining these results with the intermediate value theorem shows that there is some $r_D \in (\delta, 2b - \delta)$ such that $(r_A^{**}(r_D) + r_D - \delta)^2 f(\frac{r_A^{**}(r_D) - r_D - \delta}{2}) - 2r_A^{**}(r_D) = 0$. From this it follows that there is some $r_A \in (0, r_D - \delta)$ and $r_D \in (\delta, 2b - \delta)$ such that $(r_A + r_D - \delta)F(\frac{r_D - r_A + \delta}{2}) - r_D + \delta = 0$ and $(r_A + r_D - \delta)^2 f(\frac{r_A - r_D - \delta}{2}) - 2r_A = 0$.

Now consider some $r_A \in (0, r_D - \delta)$ and some $r_D \in (\delta, 2b - \delta)$ such that $(r_A + r_D - \delta)F(\frac{r_D - r_A + \delta}{2}) - r_D + \delta = 0$ and $(r_A + r_D - \delta)^2 f(\frac{r_A - r_D - \delta}{2}) - 2r_A = 0$. Also suppose $l_A = -r_A$ and $l_D = -r_D$. I seek to show that l_A, l_D, r_A, and r_D satisfy properties (a)-(f) in Proposition 1.

To see that property (a) is satisfied, note that $(r_A + r_D - \delta)F(\frac{r_D - r_A + \delta}{2}) - r_D + \delta = 0$ implies $(r_A + r_D - \delta)(1 - F(\frac{r_A - r_D - \delta}{2})) - r_D + \delta = 0$ and $r_A = (r_A + r_D - \delta)F(\frac{r_A - r_D - \delta}{2})$. And $(r_A + r_D - \delta)^2 f(\frac{r_A - r_D - \delta}{2}) - 2r_A = 0$ implies $r_A = \frac{(r_A + r_D - \delta)^2}{2} f(\frac{r_A - r_D - \delta}{2})$. Combining these two results shows that $F(\frac{r_A - r_D - \delta}{2}) = \frac{(r_A + r_D - \delta)}{2} f(\frac{r_A - r_D - \delta}{2})$, meaning $F(\frac{l_D + r_A - \delta}{2}) = F(\frac{r_A - r_D - \delta}{2}) = \frac{(r_A + r_D - \delta)}{2} f(\frac{r_A - r_D - \delta}{2}) = \frac{r_A - l_D - \delta}{2} f(\frac{l_D + r_A - \delta}{2})$. Thus property (a) holds.

Also note that $1 - F(\frac{l_A + r_D + \delta}{2}) = F(\frac{-l_A - r_D - \delta}{2}) = F(\frac{l_D + r_A - \delta}{2}) = \frac{r_A - l_D - \delta}{2} f(\frac{l_D + r_A - \delta}{2}) = \frac{r_D - l_A - \delta}{2} f(\frac{l_A + r_D + \delta}{2})$. Thus property (b) is satisfied as well. And since $(r_A - l_A)F(\frac{l_A + r_A}{2}) = r_A = (r_A + r_D - \delta)F(\frac{r_A - r_D - \delta}{2}) = (r_A - l_D - \delta)F(\frac{l_D + r_A - \delta}{2})$, property (c) is also satisfied. And $(r_A - l_A)(1 - F(\frac{l_A + r_A}{2})) = r_A =$

$(r_A + r_D - \delta)F(\frac{r_A - r_D - \delta}{2}) = (r_A + r_D - \delta)(1 - F(\frac{r_D - r_A + \delta}{2})) = (r_D - l_A - \delta)(1 - F(\frac{l_A + r_D + \delta}{2}))$, so property (d) holds as well.

Now $r_A \in (0, r_D - \delta)$ implies $r_A \leq r_D - \delta$, $l_A = -r_A$ implies $l_A < 0 < r_A$, $r_D < 2b - \delta$ implies $r_D - \delta < 2b - 2\delta < 2b - l_A - 2\delta$, and $l_A = -r_A$, $l_D = -r_D$, and $r_A \leq r_D - \delta < 2b - l_A - 2\delta$ imply $2a - r_A + 2\delta < l_D + \delta \leq l_A$. Thus property (e) holds. And property (f) holds if $x_l < l_D$ and $x_r > r_D$, so property (f) holds if $x_l \leq -2b + \delta$ and $x_r \geq 2b - \delta$.

Thus we see that, given the conditions in the problem, if $r_A \in (0, r_D - \delta)$ and $r_D \in (\delta, 2b - \delta)$ satisfy $(r_A + r_D - \delta)F(\frac{r_D - r_A + \delta}{2}) - r_D + \delta = 0$ and $(r_A + r_D - \delta)^2 f(\frac{r_A - r_D - \delta}{2}) - 2r_A = 0$, then there is an equilibrium in which L_A runs on the policy $l_A = -r_A$, L_D runs on the policy $l_D = -r_D$, R_A runs on the policy r_A, and R_D runs on the policy r_D. The result then follows.

$\square$

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