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# FIRM VALUE AND CORPORATE GOVERNANCE: DOES THE FORMER DETERMINE THE LATTER?\*

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## ABSTRACT

A common view, seemingly supported by empirical findings, is that better corporate governance leads to better corporate performance. But, if true, why do firms then leave money on the table by having poor governance? This paper builds a model that explains the empirical findings, but which doesn't suffer from this money-on-the-table critique. The paper argues that the common view essentially gets causality backward. Firms that have the best potential to perform well are the ones that have the most to lose from poor governance, so they are the ones that have strong governance. Strong governance and performance are positively correlated, but the former does not drive the latter. This perspective can explain a number of real-world phenomena, such as the correlation between firm size and executive compensation, the growth in executive compensation, and why measured incentives for executives often seem too low, among others.

*Keywords:* corporate governance, executive compensation, firm heterogeneity, trends in governance.

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## 1 INTRODUCTION

As a rule, people choose more security or protection the more they have at stake. Observations of homeowners, merchants, and institutions support this view: those with more at stake typically have greater security. The same rule should apply to firm owners when they must rely on others to manage their firms. A concern, dating at least as far back as Adam Smith, is that managers will abuse, misuse, or even misappropriate the resources of the firm.<sup>1</sup> Corporate governance is the security that owners put in place to protect their interests against such agency problems. It is well-documented that the strength of governance varies across firms; and one might ask which firms will have stronger governance than others? The answer, I will argue, are those in which more is at stake.

Which firms have more at stake? To an extent, those with the most resources (*e.g.*, capital, assets, etc.). But also those that possess the greatest potential return to those resources. Specifically, consider two firms, A and B, with B having the greater marginal return to resources. The marginal cost to B's owners of an abused, misused, or misappropriated dollar of resources is, thus, greater than it is for A's. Consequently, the marginal return to B's owners of investing in greater security—that is, stronger corporate governance—is greater than it is for A's. In equilibrium, firm B will have stronger governance than firm A. Furthermore, if the factors that give firm B greater marginal returns also directly increase its returns, then firm B will be more profitable than firm A in equilibrium. Profits and strength of governance will, therefore, be positively correlated. But note carefully the direction of causality: firms are not more profitable because they have better governance; rather, they have better governance because they will be more profitable.

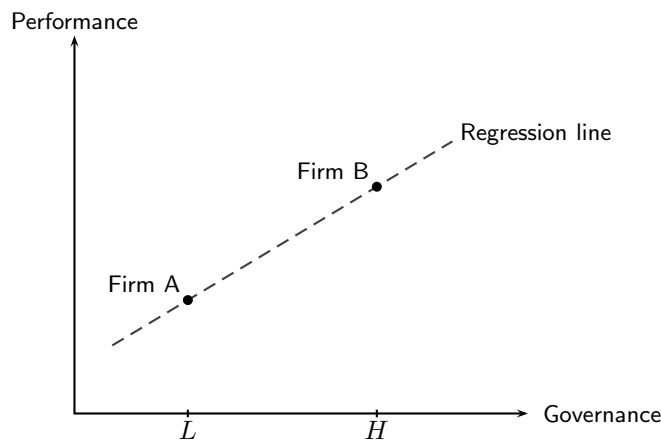
This observation about the direction of causality is one of the main points of this paper. Causality is an important, but vexing, issue in the study of corporate governance. This is especially true of empirical work. Figure 1 shows a stylized example of a regression in this area. Firm performance (*e.g.*, profits, firm value, Tobin's Q) is regressed on a measure of governance (*e.g.*, percentage of outside directors on the board, strength of managerial incentives, or a score on an index of governance measures).<sup>2</sup> It is tempting, when considering Figure 1, to see it as evidence of a causal link between governance and firm performance, with the former driving the latter. Without naming names, quite a few scholars in this area would seem to have given into that temptation. However, as just suggested, the truth could be, in large measure, the reverse.

Being less confrontational, could *both* causality stories contain elements of truth? Firms that have greater profit potential put in place stronger governance systems, generally; but, for some reason, there is additional variation in

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<sup>1</sup>“The directors of companies, however, being the managers rather of other people's money than of their own, it cannot well be expected, that they should watch over it with the same anxious vigilance . . . negligence and profusion, therefore, must always prevail, more or less, in the management of the affairs of such a company.” — Smith (1776, p. 700).

<sup>2</sup>See Bhagat and Jefferis (2002), Becht et al. (2003), and Hermalin and Weisbach (2003) for surveys and discussions of empirical work in the area of corporate governance.

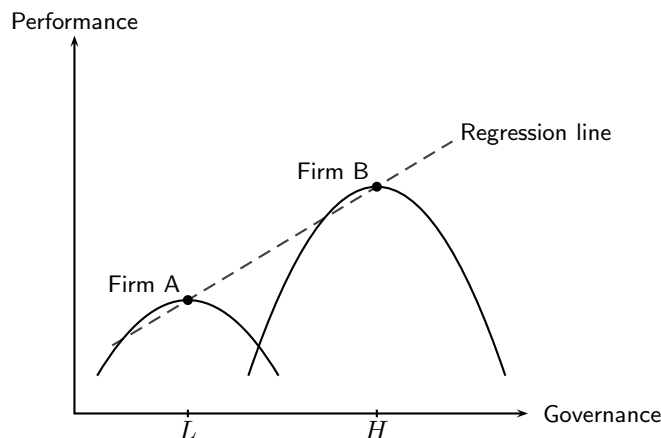


**Figure 1:** A typical regression of a measure of firm performance against a measure of corporate governance.

governance, and that variation explains some portion of the variation in firm performance. A serious concern with that view, however, is the “some reason” for additional variation in governance. Suppose, contrary to the earlier discussion, firms A and B have precisely the same profit potential, but for some reason A adopts weaker governance than B. But how could A’s reason be reasonable? Were the owners of A to simply emulate B’s and increase their firm’s governance from  $L$  to  $H$ , it would appear they would improve their firm’s performance. The “straightforward” causal interpretation of Figure 1 suggests firm A’s owners are leaving money on the table.

An alternative to such a problematic causal interpretation is shown in Figure 2. Each firm faces a different optimization program with respect to its choice of governance. These are represented by the upside-down parabolas with maxima at  $L$  and  $H$  for A and B, respectively. Were firm A’s owners to choose a level of governance other than  $L$ , then they be worse off than they are at  $L$ . In particular, the “policy prescription” that Figure 1 might seem to suggest—emulate B—would, in fact, be disastrous for firm A’s owners. In other words, both firm A’s owners and firm B’s owners are behaving optimally in equilibrium, but A’s owners simply face a worse situation.

But simply recognizing that firms are heterogeneous is not sufficient. Heterogeneity in itself cannot explain the slope of the regression line; that is, a heterogeneity story alone cannot explain why the data resemble Figure 2 and not, say, Figure 3 given that both are consistent with an assumption of underlying firm heterogeneity.



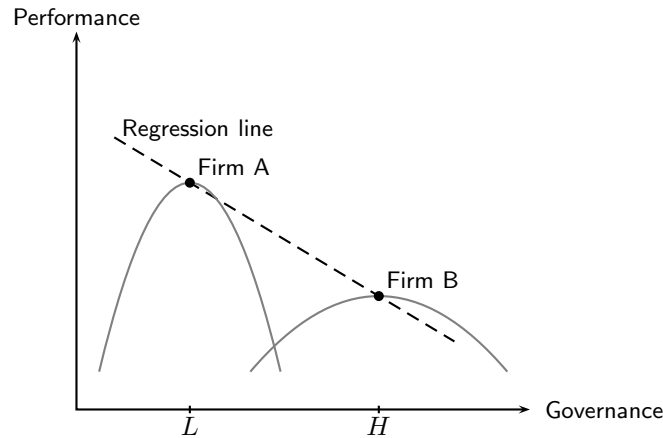
**Figure 2:** An equilibrium interpretation of the regression in Figure 1.

What is required, therefore, of a theory is threefold: (i) it must explain why governance matters; (ii) it must explain why there is *variation* in governance across firms; and (iii) it must also explain why we observe the *correlations* (slopes) that we do. The idea that A's profit potential is worse than B's is particularly attractive with respect to the last criteria because it readily explains both why A's owners choose weaker governance than B's and why level of governance and firm performance are positively correlated.

Section 2 formalizes these concepts. The key assumption is that firms vary in their marginal returns from resources utilized for the owners' benefit (*i.e.*, total resources less those diverted by the manager). It is shown that firms with higher marginal returns—*higher-type firms*—will have stronger governance than those with lower marginal returns—*lower-type firms*. If type also directly and positively affects profits, then there will be a positive correlation between profits and the level of governance (*i.e.*, the data will resemble Figure 1); however, the correlation is essentially spurious in that both variables are positively correlated with firm type (profit potential).

Section 2 also considers the situation in which firms are homogeneous in type, but vary with respect to total resources. The more-at-stake rule is shown to apply here too: firms with more resources will tend to have stronger governance and greater profits. Again, governance and profits are positively correlated, but also again the correlation is spurious—both variables are a function of total resources.

In Section 2, a firm's total resources are determined exogenously. In many contexts, it is better to think of them as endogenously determined, with the



**Figure 3:** Why heterogeneity alone is not sufficient to explain the data.

necessary funds coming from the capital markets. Section 3 extends the basic model to allow the firm's owners to also determine how much capital is to be raised. Higher-type firms will raise more capital, have stronger governance, and generate greater profits than will lower-type firms. To the extent the amount of capital raised or profits are indicators of firm size or correlated with other measures of size, the results from this section predict a positive correlation between firm size and the strength of governance. It is also shown that the level of governance could well be independent of a firm's capital structure.

A particularly important form of governance is incentive compensation. Section 4 explores the implications of the model for compensation. It is shown there that higher-type firms pay more in expectation than lower-type firms; that is, executives are paid more not only as a function of how their firms *actually* do, but also as a function of how they are *expected* to do. Given that profits are correlated with standard measures of firm size, this insight offers an explanation for the positive correlation between firm size and executive pay commonly found in the data.

Much of Section 4 concerns the standard cross-sectional regression of pay on performance. It is shown that equation is almost always misspecified. Importantly, this misspecification leads to the coefficient on performance (profit) being biased downward so that it understates the true strength of the incentives executives have. This finding represents a critical insight with regard to the debate over whether real-life executives are given sufficiently strong incentives. This discussion underscores the point made earlier that an understanding of the source of heterogeneity among firms is essential to properly interpreting data

on corporate governance, in this case compensation schemes.

Section 5 considers two alternative formulations of the model. The first, considered in Section 5.1, addresses the degree to which the analysis of the earlier sections continues to hold if governance is a multi-dimensional variable. That is, in that section, the fact the governance can vary simultaneously across firms on many dimensions, such as board structure, incentive compensation, shareholder activism, and so forth, is explicitly considered. It is readily shown that firms with better profit potentials will spend more on governance than firms with weaker profit potentials. This does not, however, mean that higher-type firms have stronger governance on all dimensions. Such a result follows, however, if it is assumed that there is complementarity in governance.

As discussed above, Figure 2 follows if firms vary in their profit potential so that firms with greater potential *ceteris paribus* have a higher marginal return to governance. Figure 2 would also follow if the heterogeneity were on the cost side: If firms with a lower cost of governance *ceteris paribus* also have lower marginal costs of governance, then lower-cost firms will both invest in more governance and have greater profits than higher-cost firms. Section 5.2 shows how this is just the same logic utilized in Sections 2 and 3. Whether it is heterogeneity on the benefit-side or the cost-side that is more important for explaining Figure 2 is an empirical question. As discussed in Sections 5.2 and 6, the trend toward stronger governance over the past quarter century or so would suggest it is primarily the benefit-side that is critical given that the empirical evidence suggests that the marginal cost of governance has, if anything, risen over this period.

Section 6 contains a brief discussion of how the analysis in this paper sheds light on trends in corporate governance over the past twenty to thirty years. Section 7 is a brief conclusion, which focuses on the implications of the analysis for future empirical work in the area.

Because much of the analysis involves a reduced-form model, it is important to show that the model in question can be derived from first principles. Appendices B and C develop two models from first principles that correspond to the reduced-form model used in the main part of the paper. Appendices C and D provide evidence on the robustness of the conclusions concerning executive compensation. Proofs not given in the text can be found in Appendix A. Rather than a separate literature review, the relations between this paper and previous work are explored in the context of the following analysis.

## 2 THE BASIC MODEL

Consider a firm's manager, who has utility

$$u = S + v(Y - S, g),$$

where  $Y$  is a source of funds or pool of assets from which the manager can divert  $S$  to his private use and  $g$  is a measure of the strength of corporate governance. Private use is meant to encompass a wide range of possible behaviors such as allocating funds or assets to pet projects not in the owners' interest, using

funds for empire building, acquiring perks, misusing assets for private benefit, or actually pocketing funds. The variable  $g$  could represent the percentage of independent directors on the board or on key board committees, a measure of the directors' diligence, a measure of the effectiveness of the monitoring and auditing systems in place, some measure of the strength of the incentives given the manager, or perhaps some index of governance strength (such as, *e.g.*, the index of Gompers et al., 2003).

One interpretation in particular is worth considering: given the many dimensions of governance, think of  $g$  as the firm's total expenditure on governance. Provided the owners set the dimensions of governance optimally, spending more on governance must correspond to better governance. Section 5.1 explores this interpretation in greater depth.

The function  $v : \mathbb{R}^2 \rightarrow \mathbb{R}$  represents the monetary equivalent of the benefit the manager derives from behaving in a manner desired by the firm's owners (*i.e.*, not diverting funds or assets for private benefit). Equivalently, it could be reexpressed—with a suitable change in sign—as the cost, in monetary units, the manager incurs from his efforts to divert  $S$ .<sup>3</sup> The function  $v(\cdot, \cdot)$ , like all functions in this paper, is assumed to be at least twice differentiable in each of its arguments. Throughout, the convention  $f_n$  is used to denote the derivative with respect to the  $n$ th argument of function  $f$  and  $f_{nm}$  to denote the second derivative with respect to the  $n$ th and  $m$ th arguments. Assume that

$$v_1(\cdot, g) > 0 \quad \forall g, \quad (1)$$

$$v_{11}(\cdot, g) < 0 \quad \forall g, \quad (2)$$

$$v_1(0, g) > 1 > \lim_{x \rightarrow \infty} v_1(x, g) \quad \forall g > 0, \quad (3)$$

$$v_{12}(\cdot, \cdot) > 0, \quad (4)$$

and

$$\forall x \exists g < \infty \text{ such that } v_1(x, g) \geq 1. \quad (5)$$

Expression (1) reflects that, due to governance, a component of the manager's utility is increasing in the amount of undiverted resources (alternatively, decreasing in the amount of diverted resources). Expression (2) implies that there is a unique value of  $S$  that maximizes the manager's utility. Expression (3) implies that the manager never finds it personally optimal to divert all resources (all  $Y$ ) to himself, but will divert some if the amount of resources is great enough. Expression (4) implies that the manager's marginal utility from not diverting resources increases with the level of governance,  $g$ . Finally, (5) implies that for

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<sup>3</sup>That is, the manager's utility could be expressed as  $S - c(S, g)$ . In this second interpretation, the manager's marginal cost of diverting funds falls with the total amount,  $Y$ , potentially available. This is consistent with the idea that diverting the marginal dollar is easier, less subject to detection, or less penalized when taken from a large pool than a small pool. For example, it could be easier for a manager to get away with trips on the company jet when the firm has lots of resources than when it is strapped for cash.



any level of resources, there exists a sufficiently tough level of governance such that the manager's optimal response is to divert no resources.<sup>4</sup>

Although expressed in reduced form, these assumptions are meant to capture the idea that, through the governance system, the manager benefits the better managed the firm is. This could reflect the direct impact on his compensation, the benefit of keeping his job, the utility from less interference from the directors and owners, etc. Equivalently, the manager's cost of diverting resources to his own use depends on the level of governance. The concavity-of-benefits assumption, expression (2)—equivalently, a convexity assumption about the cost of diverting resources—could reflect assumptions that the marginal increase in the probability of being retained or other benefits rise at a slower rate the better the manager performs. It could also reflect an assumption about the technology of diverting resources, namely that it shows decreasing returns to scale—the first dollar is likely easier to divert than the second. Alternatively, the risk of detection or the penalty if detected or both accelerate in the amount diverted. Assumption (3) assumes it is never in the manager's interest to divert all resources, at least given positive levels of governance. Finally, assumptions (4) and (5) simply say that governance matters—the better governed the firm, the less the marginal benefit (the greater the marginal cost) to the manager from diverting resources; and, in fact, the marginal benefit can be made so low that the manager prefers to divert no resources. Appendix B offers an example of an agency model satisfying these assumptions.

In this paper, the focus is on  $Y$  and  $S$ 's being monetary amounts; that is, the manager diverts  $S$  dollars from a total pool of  $Y$  dollars. The analysis, in this section at least, also applies, however, if  $Y$  is the total amount of some asset measured in non-monetary terms (*e.g.*, managerial time; so  $S$  is, *e.g.*, on-the-job leisure or time devoted to activities that benefit the manager but not the company, etc.). In this sense, this basic model encompasses the standard principal-agent model. Appendix C develops such a model in detail.

A consequence of assumptions (1)–(4) is

**Lemma 1.** *For all governance levels,  $g \in \mathbb{R}_+$ , there exists an amount  $Y(g)$ , such that, in equilibrium, the manager diverts a positive amount if and only if total resources exceed  $Y(g)$  (*i.e.*, iff  $Y > Y(g)$ ). The equilibrium amount of diversion is  $S = \max\{Y - Y(g), 0\}$ . Moreover,  $Y(g)$  is strictly increasing and differentiable in  $g$ .*

To avoid dealing with corner solutions in the level of governance, assume  $v_1(0, 0) = 1$ ; this implies  $Y(0) = 0$ —in the absence of governance, the manager will divert all available funds to his private use. Because the function  $Y(\cdot)$  is monotone, it is invertible. Let  $G(\cdot)$  denote its inverse.

Much of the analysis in this paper relies on the following well-known revealed-preference result, which is worth stating once, at a general level, for the sake of completeness and to avoid unnecessary repetition.

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<sup>4</sup>Observe, as one of many examples, that the function  $v(Y - S, g) = 2g\sqrt{Y - S}$  satisfies all these assumptions.

**Lemma 2.** *Let  $f(\cdot, \cdot) : \mathbb{R}^2 \rightarrow \mathbb{R}$  be a function at least twice differentiable in its arguments. Suppose that  $f_{12}(\cdot, \cdot) > 0$ . Let  $\hat{x}$  maximize  $f(x, z)$  and let  $\hat{x}'$  maximize  $f(x, z')$ , where  $z > z'$ . Then  $\hat{x} \geq \hat{x}'$ . Moreover, if  $\hat{x}'$  is an interior maximum, then  $\hat{x} > \hat{x}'$ .*

The owners of the firm have a payoff given by

$$B(Y - S, \tau) - C(g),$$

where  $\tau \in \mathcal{T} \subset \mathbb{R}$  is an index of the firm's type. The amount  $C(g)$  is the cost of implementing governance level  $g$ ; it is, for instance, the cost of establishing and maintaining auditing and monitoring procedures, the cost of incentive pay, etc. It could also include the cost owners incur overcoming managerial resistance to more oversight. The amount  $B(Y - S, \tau)$  is the benefit a type- $\tau$  firm's owners realize when the net resources utilized are  $Y - S$ . The function  $C(\cdot)$  is increasing. To avoid corner solutions at zero governance, assume  $C'(0) = 0$ .<sup>5</sup> Assume that  $B_1(\cdot, \tau) > 0$ ; that is, the more net resources utilized, the more the owners' benefit. As a definition of type, assume:

$$B_{12}(\cdot, \cdot) > 0; \tag{6}$$

that is, the marginal benefit of more net resources is greater for higher-index types than for lower-index types.

There are numerous possible underlying assumptions for  $B(\cdot, \cdot)$ . For instance,  $B(x, \tau) = \tau\psi(x)$ , where  $\psi(\cdot)$  is an increasing function that relates the net amount invested to expected production and  $\tau$  is the average price margin. Alternatively,  $\psi(\cdot)$  could be the probability of a successful R&D innovation and  $\tau$  the profit from such an innovation. As yet one more example,  $\psi(\cdot)$  is realized cash flow and  $\tau$  is the owners' claim on that cash flow (*i.e.*, excluding (i) the shares held by management and (ii) after taxes).<sup>6</sup>

The timing of the model is that the owners choose the level of governance,  $g$ , then the manager chooses how much to divert,  $S$ . Assume for the moment that  $Y$  is determined exogenously. From Lemma 1, net resources will be  $Y(g)$  if  $Y(g) < Y$  (the manager diverts  $Y - Y(g)$ ); or  $Y$  if  $Y(g) \geq Y$ . Given that raising  $g$  is costly and  $Y(\cdot)$  is strictly increasing, the owners will never choose a level of  $g$  such that  $Y(g) > Y$ . Define  $\bar{g}$  as the solution to  $Y(g) = Y$ ; that is,  $\bar{g} = G(Y)$ .<sup>7</sup> The owners' problem is, thus,

$$\max_{g \in [0, \bar{g}]} B(Y(g), \tau) - C(g).$$

<sup>5</sup>This assumption is not critical. There are other assumptions that could be made to avoid corner solutions at  $g = 0$ . Moreover, corner solutions only mean that some of the strict comparative static results below become weak comparative static results.

<sup>6</sup>Alternatively, if because of free-rider issues, only some shareholders act to shape governance (*e.g.*, large blockholders such as institutional investors take an active role), then  $\tau$  could be the proportion held by these active shareholders.

<sup>7</sup>The equation  $Y(g) = Y$  has a solution; this follows from the assumption that  $v_1(0, 0) \geq 1$  (*i.e.*,  $Y(0) = 0$ ), condition (5), and the continuity of  $Y(\cdot)$  as established by Lemma 1.

Because  $[0, \bar{g}]$  is compact and all functions are continuous, at least one solution must exist. Let  $g(\tau)$  be the solution adopted by a type- $\tau$  firm.

There is variance in the level of governance across firms. Specifically,

**Proposition 1.** *Higher-type firms adopt at least as great a level of governance as lower-type firms (i.e., if  $\tau > \tau'$ , then  $g(\tau) \geq g(\tau')$ ). Moreover, if a lower-type firm has not adopted the maximum level of governance (i.e.,  $g(\tau') < \bar{g}$ ), then a higher-type firm will have a strictly greater level of governance (i.e.,  $g(\tau) > g(\tau')$ ).*

**Proof:** Given Lemma 2, the proposition follows if the cross-partial derivative of

$$B(Y(g), \tau) - C(g)$$

with respect to  $\tau$  and  $g$  is positive everywhere. That cross-partial derivative is  $B_{12}(Y(g), \tau)Y'(g)$ , which is positive by Lemma 1 and definition of type, expression (6). ■

If we assume that total benefits—and not just the marginal benefit of resources—is increasing in type—that is,<sup>8</sup>

$$B_2(y, \tau) > 0 \text{ for all } y > 0 \quad (7)$$

—then we get the following relationship between firm profits and governance.

**Proposition 2.** *Under the assumptions of the basic model and assuming a common level of resources,  $Y$ , a firm that will be more profitable in equilibrium than another has at least as high a level of governance as the other firm.*

**Proof:** Equilibrium profits are

$$\pi(\tau) \equiv B\left(Y(g(\tau)), \tau\right) - C(g(\tau)).$$

By the envelope theorem,

$$\pi'(\tau) = B_2\left(Y(g(\tau)), \tau\right) > 0.$$

So  $\tau > \tau'$  implies  $\pi(\tau) > \pi(\tau')$ . From Proposition 1,  $\tau > \tau'$  implies  $g(\tau) \geq g(\tau')$ . ■

Observe the direction of causality: Profitability potential (type) causes better governance and, of course, it also directly causes better profits. Consequently, profits and governance end up positively correlated (consistent with Figure 1). The correlation is, however, spurious, not causal (both are a function of type). That is, the situation is as illustrated in Figure 2, where firm B is the higher- $\tau$  firm.

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<sup>8</sup>Note, in light of (6), assumption (7) is equivalent to assuming  $B_2(0, \tau) \geq 0$  because  $B_2(y, \tau) = B_2(0, \tau) + \int_0^y B_{12}(z, \tau) dz$ .

Suppose that  $\tau$  were invariant across firms. Suppose, however, that firms differed in terms of the gross resources,  $Y$ , available to them. Each firm's owners would solve

$$\max_{g \in [0, G(Y)]} B(Y(g)) - C(g), \quad (8)$$

where  $\tau$  has been suppressed because it is assumed constant across firms. Let  $\hat{g}(Y)$  denote the solution to (8). Because  $G(\cdot)$  is an increasing function,  $\hat{g}(\cdot)$  is non-decreasing. Moreover, a higher- $Y$  firm has more options than a lower- $Y$  firm (*i.e.*,  $[0, G(Y')] \subset [0, G(Y)]$  if  $Y' < Y$ ), which means its profits are weakly greater as well.<sup>9</sup> Hence,

**Proposition 3.** *Assume all firms are the same type, but they differ as to the gross resources,  $Y$ , available to them. Then a firm with more resources will have at least as great a level of governance as a firm with fewer resources. It will also have at least as much profits as a firm with fewer resources. That is, under these assumptions, gross resources and the level of governance are non-negatively correlated and profits and the level of governance are non-negatively correlated. The correlation between profits and the level of governance will, therefore, be non-negative.*

Adopting any of a myriad of possible assumptions that guarantee interior solutions would allow one to rewrite Proposition 3 so “non-negative” is replaced with “positive.”

### 3 ENDOGENOUS INVESTMENT

To this point, gross resources,  $Y$ , were fixed exogenously. Now consider a model in which resources must be funded from the capital market. Let  $I$  denote funds raised from the market. The resources potentially available for productive investment are  $Y = I - C(g)$ . Of these,  $S$  will be diverted, leaving  $N = I - C(g) - S$  available to be actually utilized. Normalize the model so the marginal opportunity cost of funds is 1.

Denote financial returns by  $r$ . Assume  $r \sim F(\cdot|N, \tau)$ . Assume the expectation

$$B(N, \tau) \equiv \int_{-\infty}^{\infty} r dF(r|N, \tau)$$

exists. Maintain the previously made assumptions about  $B(\cdot, \cdot)$ .<sup>10</sup> To these, add the assumption that, for all  $\tau$ ,

$$B_1(0, \tau) > 1 > B_1(n, \tau) \quad (9)$$

<sup>9</sup>One has to be careful about how these resources are accounted for. If, as assumed here, they are sunk, then they are not directly a component of (economic) profits and the statement in Proposition 3 is valid. If the accounting system, nevertheless, treats them as an expense, then the correlation between governance and accounting profits could prove ambiguous.

<sup>10</sup>Those assumptions would hold, for instance, if  $r = \tau\sqrt{N} + \eta$ , where  $\eta$  is a random variable drawn independently of  $N$  and  $\tau$ .

for  $n > \bar{n}$ , where  $\bar{n}$  is finite. The first inequality in (9) implies it is profitable to invest in firms; the second inequality rules out infinite investment as being optimal. The second inequality implies that we are free to act as if the set of possible investment levels is bounded; this will insure interior maxima for the optimization programs below.

Initially, assume that the owners self finance. Because every dollar provided over  $Y(g) + C(g)$  will be diverted by the manager, the owners will never provide funding in excess of  $Y(g) + C(g)$ . The owners problem can, thus, be stated as

$$\max_Y \int_{-\infty}^{\infty} r dF(r|Y, \tau) - C(G(Y)) - Y, \quad (10)$$

where, recall,  $G(\cdot)$  is the inverse function of  $Y(\cdot)$ .

**Proposition 4.** *Under the above assumptions, there will be a strictly positive correlation between the amount the owners invest in a firm and its level of governance. Furthermore, if financial return is increasing in firm type (i.e.,  $B_2(N, \tau) > 0$ ), then there will be a strictly positive, but non-causal, correlation between firm profit and level of governance.*

**Proof:** Let  $y^*(\tau)$  denote a solution to (10). By the assumptions above,  $0 < y^*(\tau) < \infty$  for all  $\tau$ ; that is, it is an interior solution. The first part of the proposition follows immediately from Lemma 2 provided the cross-partial derivative of

$$B(Y, \tau) - C(G(Y)) - Y$$

with respect to  $Y$  and  $\tau$  is positive everywhere. That it is follows from assumption (6).

The “furthermore” part follows from the envelope theorem, which establishes that equilibrium profits are increasing in type, and from the first part of the proposition, which established that investment is increasing in type. ■

One imagines that firm size is positively correlated with the amount invested in it. Indeed, the amount invested—firm capitalization—could be a definition of size. Hence,

**Corollary 1.** *If investment in a firm is positively correlated with its size, then firm size and level of governance are positively, but spuriously, correlated.*

What if the firm owners must raise capital? Consider two timing possibilities: first, the owners can wait to set  $g$  until they have received outside capital; second, they must set it and expend the money to do so prior to seeking capital. In the latter case, it follows that  $g \leq C^{-1}(I_0)$ , where  $I_0$  is the owners’ available capital. A firm’s type is taken to be common knowledge; in particular, it is known to would-be investors.

Consider the first possibility. Because every dollar of capital over  $Y(g) + C(g)$  will be diverted by the manager, total capital invested will be  $Y + C(G(Y))$ . Let  $I \in [0, I_0]$  be the amount of capital the owners self finance and  $Y + C(G(Y)) - I$ ,

therefore, the amount they must raise from outside investors. Let  $Z(\cdot)$  denote the financial contract; that is, the owners repay the outside investors  $Z(r)$  when the firm's gross profit is  $r$ .<sup>11</sup> Observe, this encompasses standard forms of financing such as debt, equity, a combination of debt and equity, or more exotic securities. Given the owners are the residual claimants, they get the cash left in the firm at the end,  $r$ , less what the outside investors are due. Hence, their problem is

$$\max_{\{Y, I, Z(\cdot)\}} \int_{-\infty}^{\infty} (r - Z(r)) dF(r|Y, \tau) - I \quad (11)$$

subject to

$$\int_{-\infty}^{\infty} Z(r) dF(r|Y, \tau) = Y + C(G(Y)) - I, \quad (12)$$

where (12) is the condition for investors to be willing to provide the required capital. Using (12) to substitute out  $I$  in (11) and, then, simplifying yields (10); hence, the total amount invested is unaffected by the need to raise funds and the financial structure of the firm (*i.e.*, the  $Z(\cdot)$  function) is indeterminate. This establishes

**Proposition 5.** *If a firm's owners are not obligated to fund the level of governance before raising capital from the market, then the level of governance will be the same as if the owners could self finance. Moreover, there is not necessarily any correlation between the firm's capital structure and its level of governance.*

This result is, in essence, a simple version of Modigliani and Miller (1958). Like Modigliani and Miller, Proposition 5 can be criticized insofar as capital-structure indeterminacy may fail to hold in a richer model. Nevertheless, it indicates that governance need not be a driver of capital structure nor even correlated with it.

A further potential criticism is there is a significant literature that argues that the capital structure is itself part of the governance structure; that is, because  $g$  has entered the model in reduced form, the analysis could be overlooking the possibility that  $g$  is a function of the capital structure. For instance, it has been argued that debt can be used to force managers not to divert funds (see, *e.g.*, Grossman and Hart, 1982, Jensen, 1986, Hart and Moore, 1998). While this literature offers many insights, it remains true that there are a number of other governance instruments, such as incentive schemes, board oversight, and outside auditing, that have nothing to do with the capital structure. Certainly this is true of the models developed in Appendices B and C. Moreover, because of access to these other governance mechanisms, one wonders to what extent firms would utilize capital structure for this purpose. After all, there could be competing motives (*e.g.*, the tax advantage of debt) affecting capital structure; and it could be difficult or costly to adjust capital structure with sufficient precision to deal with governance issues.

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<sup>11</sup>An alternative, but ultimately equivalent, accounting would be to define profit as  $r - C(G(Y))$ . What is relevant for the owners is the cash left in the firm less what they must repay outside investors.

A related concern is that agency issues can lead managers to distort a firm's capital structure (see, *e.g.*, Jensen and Meckling, 1976, Harris and Raviv, 1988, Stulz, 1988; for an empirical examination see Berger et al., 1997). In particular, weak governance could mean more agency problems, which could mean a preference for one form of financing over another; that is, a correlation exists between governance and capital structure. To an extent, this issue can be captured within the framework of the previous section and the interpretation of the agency problem spelled out in Appendix C: Interpret  $S$  as the funds raised via one form of financing and  $Y - S = Y(g)$  the funds raised via another. In light of Lemma 1, stronger governance would be positively correlated with the use of the other form of financing. On the other hand, one might question why the manager is not simply barred contractually from changing the capital structure.

What if the owners must fix and pay for the level of governance before seeking outside capital? Because the owners can subsequently acquire capital from the market at the same rate that their own investments in the market would earn, there is no loss in generality in assuming that, beyond the funding of the governance level, the owners invest none of their own money in the firm. The owners' problem is, therefore,

$$\max_{\{g, Z(\cdot)\}} \int_{-\infty}^{\infty} (r - Z(r)) dF(r|Y(g), \tau) - C(g) \quad (13)$$

subject to

$$C(g) \leq I_0 \text{ and} \quad (14)$$

$$\int_{-\infty}^{\infty} Z(r) dF(r|Y(g), \tau) = Y(g), \quad (15)$$

where, recall,  $I_0$  equals the owners' available funds. Using (15) to substitute out  $\int Z dF$  in (13), the owners' problem becomes

$$\max_g B(Y(g), \tau) - C(g) - Y(g) \quad (16)$$

subject to (14). Expression (16) is equivalent to (10); hence, if  $I_0 > C(G(y^*(\tau)))$ , then the solution is the same as in Proposition 4. If, instead, the constraint binds, then the equilibrium level of governance is less than the unconstrained optimum. The basic conclusion of Proposition 5 continues, however, to hold.

**Corollary 2.** *Suppose a firm's owners are obligated to set and pay for the level of governance before raising capital from the market. Then the level of governance they choose is independent of the firm's capital structure.*

#### 4 MANAGERIAL COMPENSATION

In this section, the focus is on the use of managerial compensation as a governance mechanism. The analysis is facilitated by some changes in the basic model. As shown in Appendix C, a model that hews more closely to the basic model also yields similar insights (see Proposition C.1).

Assume, now, the owners set the manager's compensation and benefits at  $g$ . The manager receives  $g$  unless he is dismissed. There is some level of compensation,  $w$ , that he gets regardless of whether he keeps or loses his job. He is dismissed if he fails an audit. The probability he *passes* the audit is a decreasing function of the ratio  $S/Y$ . Specifically, let  $p : [0, 1] \rightarrow [0, 1]$  be the probability the manager passes the audit. Assume

$$p(0) = 1; p(1) = 0; p'(\cdot) < 0; p'(0) = 0; p'(1) = -\infty; \text{ and } p''(\cdot) < 0.$$

The manager's utility is taken to be

$$w + S + gp(S/Y).$$

An alternative interpretation of the model is that the manager is risk neutral and his pay can be made contingent on a binary signal. The signal indicates success (alternatively, is good or high) with probability  $p(S/Y)$  and indicates failure (alternatively, is bad or low) with probability  $1 - p(S/Y)$ . Given this structure, there is no loss of generality in assuming the manager gets a base wage of  $w$  and a bonus of  $g$  if successful.

Define  $h(\cdot)$  to be the inverse function of  $-p'(\cdot)$ . It is readily shown, by solving the manager's utility maximization problem, that the net funds available,  $N(g)$ , are given by

$$N(g) = Y - S = Y \left( 1 - h\left(\frac{Y}{g}\right) \right).$$

If  $\infty > g > 0$ , the manager would never find it personally optimal to choose  $S = 0$  or  $S = Y$ ; hence  $0 < h(Y/g) < 1$  for  $\infty > g > 0$ . Observe

$$N'(g) = \frac{-Y^2}{p''(h(Y/g))g^2} > 0,$$

where the inequality follows because  $p''(\cdot) < 0$ . In other words, net resources are increasing in the strength of the manager's incentives.

Assume there is a prevailing alternative wage in the economy,  $\underline{w}$ . To hire a manager, a firm must offer expected compensation, which when added to  $S$ , equals  $\underline{w}$ ; that is,

$$w + gp \left( \frac{Y - N(g)}{Y} \right) + \underbrace{(Y - N(g))}_S \geq \underline{w}. \quad (17)$$

Assume, too, the manager is protected by limited liability. Specifically, assume the constraint  $w \geq 0$ ; that is, the manager cannot "buy" his job from the owners.

As previously, let  $B(N, \tau)$  be the owners' benefit function.<sup>12</sup> The owners' problem can be expressed as

$$\max_{w, g} B(N(g), \tau) - w - gp \left( \frac{Y - N(g)}{Y} \right), \quad (18)$$

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<sup>12</sup>It is perhaps best to think of  $B(N, \tau)$  as an expected value to avoid the possibility of the owners' inverting  $B(N, \tau)$  to derive  $S$  exactly and using that to facilitate a contract that



subject to (17) and the limited-liability constraint (LLC) that  $w \geq 0$ .

It seems reasonable to imagine that  $Y > \underline{w}$ ; the resources of most firms exceed the competitive level of compensation. If so, we get the following result:

**Lemma 3.** *Suppose  $Y > \underline{w}$ . Then, in the maximization program given by (18) subject to (LLC) and (17), the former constraint is always binding and the latter constraint is slack.*

Lemma 3 means  $w \equiv 0$ , (17) is slack, and the owners' problem is

$$\max_g B(N(g), \tau) - gp \left( \frac{Y - N(g)}{Y} \right). \quad (19)$$

In what follows, assume that for any  $\tau \in \mathcal{T}$  there exists a  $\hat{g}$ , possibly dependent on  $\tau$ , such that

$$B(N(\hat{g}), \tau) - \hat{g}p \left( \frac{Y - N(\hat{g})}{Y} \right) > 0. \quad (20)$$

Expression (20) is simply the assumption that it is feasible for the owners to make money; that is, that the firm should operate.<sup>13</sup>

**Lemma 4.** *Assume, regardless of type, the owners' marginal benefit at  $N = 0$  exceeds zero (i.e.,  $B_1(0, \tau) > 0$  for all  $\tau$ ). Then, for all  $\tau \in \mathcal{T}$ , the program (19) has an interior solution.*

**Proposition 6.** *Assume, regardless of type, the owners' marginal benefit of net resources ( $N$ ) at zero net resources exceeds zero (i.e.,  $B_1(0, \tau) > 0$  for all  $\tau$ ). Assume resources ( $Y$ ) exceed the market wage ( $\underline{w}$ ) and the manager is protected by limited liability (i.e.,  $w \geq 0$ ). Then*

- (i) *a higher-type firm pays its manager more in expectation than does a lower-type firm; and*

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forces the manager not to divert funds. For instance,  $B(N, \tau)$  could be the expected value

$$\tau \kappa(N) p \left( \frac{Y - N}{Y} \right),$$

where the function  $\kappa(\cdot)$  could be constant or increasing (i.e., the owners' benefit is  $\tau \kappa(N)$  if the manager succeeds and 0 if he fails). Of course, as is well known, there is no requirement in agency models that the principal's (owners') benefit correspond to the signal upon which the agent's (manager's) compensation is contingent.

Alternatively, if  $B(N, \tau)$  is deterministic, one could assume that it cannot be verified, making it difficult to utilize as a contractual contingency.

<sup>13</sup>For instance, if  $p(x) = \sqrt{1 - x^2}$  (the quarter-circle function),  $B(N, \tau) = \tau \sqrt{N}$ , and  $Y > 0$ , then

$$N(g) = Y - \frac{Y^2}{\sqrt{g^2 + Y^2}};$$

hence,

$$B(N(g), \tau) - gp \left( \frac{Y - N(g)}{Y} \right) = \tau \sqrt{Y - \frac{Y^2}{\sqrt{g^2 + Y^2}}} - g \sqrt{\frac{g^2}{g^2 + Y^2}}.$$

It is readily shown that for any  $\tau > 0$ , there exists a  $\hat{g} > 0$  such that this expression is positive.

(ii) if benefits are directly increasing in type (i.e.,  $B_2(N, \tau) > 0$ ), then firm profits and managerial compensation will be positively correlated.

**Proof:** Define

$$\pi(g, \tau) = B(N(g), \tau) - gp \left( \frac{Y - N(g)}{Y} \right).$$

Observe

$$\frac{\partial^2 \pi(g, \tau)}{\partial \tau \partial g} = B_{12}(N(g), \tau) N'(g) > 0.$$

Consider  $\tau' > \tau''$ . Let  $g'$  and  $g''$  be the solutions, respectively, to (19) for  $\tau'$  and  $\tau''$ , respectively. By Lemma 2,  $g' > g''$ . Expected compensation,

$$gp \left( \frac{Y - N(g)}{Y} \right),$$

is increasing in  $g$ , so  $g' > g''$  implies part (i) of the proposition. By the envelope theorem, in equilibrium,

$$\frac{d\pi(g, \tau)}{d\tau} = B_2(N(g), \tau) > 0;$$

hence, higher-type firms earn greater profits (on average) than lower-type firms. Given part (i), part (ii) of the proposition follows. ■

Proposition 6 holds two important implications for empirical analysis. One concerns the cross-sectional relation between pay and performance, the other the cross-sectional relation between pay and firm size. With respect to the first, the proposition predicts that there will be a positive correlation between managerial pay and the financial performance of the firm. Firms that are likely to be more profitable (e.g., higher  $\tau$  firms) will have both a higher level of  $g$  and a higher probability of paying it. Consequently, if one estimated the regression

$$\text{Pay}_i = \delta_0 + \delta_1 \text{Profit}_i + \eta_i, \quad (21)$$

where  $i$  indexes firms, the  $\delta$ s are coefficients to be estimated, and  $\eta_i$  is an error term, then one's estimate of  $\delta_1$  would be positive.<sup>14</sup>

How should such a finding be interpreted? First, it needs to be remembered that the manager of a more profitable firm is paid more than a less profitable firm's in part because he was employed by a firm that anticipated being more profitable. In other words, he is being rewarded, in part, for the inherent profitability of the firm and not solely as a reward to his own efforts.

Second, incentives here are reminiscent of efficiency wages (Shapiro and Stiglitz, 1984). This can be seen most clearly by considering the audit interpretation of the model: A firm that has a greater value for net resources utilized

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<sup>14</sup>For simplicity, any other controls in (21) have been omitted. For purpose of this discussion  $\text{Pay}_i$  and  $\text{Profit}_i$  are in levels. As will become clear, a specification in logs would not change the conclusions of this analysis.

(*i.e.*,  $N$ ), will wish to motivate its manager to divert less. It does so by paying him more, because the more he is paid, the more he cares about passing the audit (keeping his job), which in turn causes him to divert less. Indeed, if, as is often the case in empirical analyses, the data consisted solely of incumbents (*i.e.*, only those in office have their compensation recorded in the data set), then all variation in the pay data is coming from the “efficiency-wage” aspect of pay (*i.e.*, the variation in  $g$  across firms) as opposed to variation in actual performance.

Even under the bonus interpretation of the model, it remains true that (i) *potential* profitability is determining, in part, compensation; and (ii) because firms with different  $\tau$ s will have different values of  $\delta_1$ , heterogeneity across firms could make (21) a questionable specification. To appreciate point (ii), consider the following example: normalize profits so the firm gets  $\tau$  if the manager is successful and 0 if he is not. It follows that, for a  $\tau$  firm,  $\delta_1 = g(\tau)/\tau$ . Unless  $g(\tau)$  is proportional to  $\tau$ ,  $\delta_1$  will vary with  $\tau$  and not be a constant as specification (21) assumes. For instance, if  $p(x) = \sqrt{1-x^2}$ , then it can be shown that  $\delta_1$  varies with  $\tau$ .<sup>15</sup>

This last discussion leads to a final comment about (21). There has been a lengthy debate about whether executive compensation is sufficiently sensitive to firm performance (see, *e.g.*, Jensen and Murphy, 1990, Haubrich, 1994, Hall and Liebman, 1998, Hermalin and Wallace, 2001). A rough summary of the debate is that it is concerned with whether  $\delta_1$  is big enough. But  $\delta_1$  could be the wrong measure on which to focus: The strength of the manager’s incentives are given by  $g$ , not  $\delta_1$ . As the previous example showed, these are not the same; in fact, in that example,  $\delta_1 = g/\tau$  and  $g < \tau$ , implying that  $\delta_1$ , as a measure of incentive strength, is biased downward.<sup>16</sup> To add insult to injury, for that example, it can be shown that  $d\delta_1/d\tau < 0$ . Given that Proposition 6 showed  $dg/d\tau > 0$ , this means that the firms that would appear to have the smallest incentives were their true  $\delta_1$ s observed are, in fact, those that provide their managers with the largest incentives.

Lest one suspect that these results are an artifact of a two-outcome model, Appendix D shows that expression (21) is similarly problematic with a continuum of outcomes and, in particular, that the estimated  $\delta_1$  is biased downward due to firm heterogeneity.

A second cross-sectional relation implied by Proposition 6 is the following. Suppose firm size is positively correlated with firm profits.<sup>17</sup> Given firms that

<sup>15</sup>The formula for  $\delta_1$  in this case is

$$\frac{-12Y^2 \sqrt[3]{2} + (54\tau Y + 2\sqrt{729\tau^2 Y^4 + 864Y^6})^{2/3}}{6\tau(27\tau Y + \sqrt{729\tau^2 Y^4 + 864Y^6})^{1/3}}.$$

<sup>16</sup>See the discussion in Hermalin and Wallace for other reasons why the  $\delta_1$  estimated from (21) using cross-sectional data could be a biased-downward measure of incentive strength.

<sup>17</sup>In empirical analyses, firm size is often measured as market value (see, *e.g.*, Frydman and Saks, 2007), which, as the present discounted value of profits, should be correlated with

will be more profitable (higher  $\tau$  firms) pay more than their counterparts (Proposition 6), the following result obtains.

**Corollary 3.** *If firm size is positively correlated with firm profits, then there will be a positive correlation between firm size and managerial compensation under the assumptions of Proposition 6.*

That there is a positive correlation between firm size and executive compensation is a well-documented phenomenon (see, among others, Baker et al., 1988, Rose and Shepard, 1997, and Frydman and Saks; Gabaix and Landier, in press, provide a brief survey of the literature). Corollary 3 offers a potential explanation for this phenomenon.<sup>18</sup>

The amount of firm resources,  $Y$ , is an alternative measure of size. The following proposition provides comparative statics when  $Y$  varies, but  $\tau$  is fixed (to improve readability,  $\tau$  is, thus, suppressed in the following).

**Proposition 7.** *Assume firms don't vary in type, but have different levels of resources,  $Y$  (but all  $Y > \underline{w}$ , the market wage). Assume the manager is protected by limited liability (i.e.,  $w \geq 0$ ). Assume the owners' marginal benefit of net resources ( $N$ ) at zero net resources is positive (i.e.,  $B'(0) > 0$ ). Assume, too, that the marginal benefit schedule is elastic; that is,*

$$\frac{d \log(B'(N))}{d \log(N)} \leq -1. \quad (22)$$

*Given these assumptions,<sup>19</sup> a firm with more resources will pay its manager more contingent on not dismissing him or, alternatively, a larger bonus than will a firm with less resources (i.e.,  $dg/dY > 0$ ).*

**Proof:** Using the formula for  $N(g)$ , a firm's profits can be written as

$$\hat{\pi}(g, Y) = B\left(Y\left(1 - h\left(\frac{Y}{g}\right)\right)\right) - gp\left(h\left(\frac{Y}{g}\right)\right). \quad (23)$$

Define  $x = h(Y/g)$ , which makes  $g = -Y/p'(x)$ . This substitution in (23) makes the owners' problem

$$\max_{x \in [0,1]} B(Y(1-x)) + \frac{Yp(x)}{p'(x)}. \quad (24)$$

profits.

<sup>18</sup>As such, Corollary 3 complements other explanations that have been offered in the literature. Two such explanations are models based on the distribution of managerial talent (Terviö, 2007, and Gabaix and Landier); and the correlation between firm size, hierarchical depth, and pay at the top of the hierarchy (Calvo and Wellisz, 1979). It is worth noting that both these explanations rely, in part, on heterogeneity in managerial ability whereas Corollary 3 does not.

<sup>19</sup>Note (22) implies  $B(\cdot)$  is concave.

The derivative of (24) at  $x = 1$  is

$$-YB'(0) + Y \left( 1 - \frac{p(1)p''(1)}{(p'(1))^2} \right) = -YB'(0) + Y < 0;$$

hence,  $x = 1$  is not optimal. The derivative of (24) at  $x = 0$  is

$$-YB'(Y) + Y \left( 1 - \frac{p(0)p''(0)}{(p'(0))^2} \right) = -YB'(Y) + Y \times \infty > 0$$

(recall  $p'(0) = 0$ ); hence,  $x = 0$  is not optimal. From Lemma 2, the optimal  $x$  will be increasing in  $Y$  if the cross-partial derivative of the maximand in (24) is positive. That cross-partial derivative is

$$1 - \frac{p(x)p''(x)}{(p'(x))^2} - B'(Y(1-x)) - Y(1-k)B''(Y(1-k)) > 0,$$

where the inequality follows from (22) and because  $p''(x) < 0$ . So the optimal  $x$  is increasing in  $Y$ ; that is,  $dx/dY > 0$ . In terms of  $g$ , this entails

$$\frac{dg}{dY} = \frac{d}{dY} \frac{-Y}{p'(x)} = -\frac{1}{p'(x)} - \frac{Yp''(x)}{(p'(x))^2} \frac{dx}{dY} > 0$$

(recall  $p'(x) < 0$ ). ■

**Corollary 4.** *Under the assumptions of Proposition 7, there is a positive correlation between firm size, measured as available resources (assets), and managerial compensation.*

## 5 ALTERNATIVE FORMULATIONS

### 5.1 GOVERNANCE AS A MULTI-DIMENSIONAL PROBLEM

There are multiple dimensions to governance. There is board structure, compensation, shareholder activism, and so forth. Heretofore, however, governance has been treated as a scalar,  $g$ . In this section, the model is extended to allow governance to be a vector,  $\mathbf{g} \in \mathbb{R}_+^n$ ,  $n > 1$ .

Return to the model of Section 2 and let the manager's utility be

$$u = S + v(Y - S, \mathbf{g}).$$

Assume that  $v(\cdot, \cdot)$  continues to satisfy conditions (1)–(3), with  $\mathbf{g}$  substituted for  $g$ . In lieu of (4), assume

$$v_{1j}(\cdot, \cdot) > 0 \text{ for all } 1 < j < n; \tag{4'}$$

that is, an increase in any dimension of governance lowers the marginal benefit of diverting resources (equivalently, raises the marginal cost of doing so).

**Lemma 1'.** *For all governance levels,  $\mathbf{g} \in \mathbb{R}_+^n$ , there exists an amount  $Y(\mathbf{g})$ , such that, in equilibrium, the manager diverts a positive amount if and only if total resources exceed  $Y(\mathbf{g})$  (i.e., iff  $Y > Y(\mathbf{g})$ ). The equilibrium amount of diversion is  $S = \max\{Y - Y(\mathbf{g}), 0\}$ . Moreover,  $Y(\cdot)$  is strictly increasing and differentiable in each argument (i.e.,  $\partial Y/\partial g_j$  exists and is positive).*

Assume  $v_1(0, \mathbf{0}) = 1$ , so  $Y(\mathbf{0}) = 0$ .

In lieu of (5), assume

$$\forall x \exists \mathbf{g} = (g_1, \dots, g_n) \text{ such that } g_j < \infty \forall j \text{ and } v_1(x, \mathbf{g}) \geq 1. \quad (5')$$

Let the owners' profits be

$$B(Y - S, \tau) - C(\mathbf{g}),$$

where the previous assumptions hold and  $C(\cdot)$  is strictly increasing in each of its arguments. Because increasing  $\mathbf{g}$  along any dimension is costly, the owners will never choose a  $\mathbf{g}$  such that  $Y(\mathbf{g}) > Y$ . Define

$$\mathcal{G}(Y) = \{\mathbf{g} | Y(\mathbf{g}) \leq Y\}.$$

In light of the assumption that  $v_1(0, \mathbf{0}) = 1$ , condition (5'), and the continuity of  $Y(\cdot)$  as established by Lemma 1',  $\mathcal{G}(Y)$  is compact. The owners problem is, thus,

$$\max_{\mathbf{g} \in \mathcal{G}(Y)} B(Y(\mathbf{g}), \tau) - C(\mathbf{g}).$$

Because  $\mathcal{G}(Y)$  is compact and all functions are continuous, at least one solution must exist. Let  $\mathbf{g}(\tau)$  be the solution adopted by a type- $\tau$  firm.

The main comparative static result is the following:

**Proposition 8.** *Higher-type firms spend at least as much on governance as do lower-type firms (i.e., if  $\tau > \tau'$ , then  $C(\mathbf{g}(\tau)) \geq C(\mathbf{g}(\tau'))$ ). Moreover, if a lower-type firm has not blocked all resource diversion (i.e.,  $Y(\mathbf{g}(\tau')) < Y$ ), then the higher-type firm spends strictly more (i.e.,  $C(\mathbf{g}(\tau)) > C(\mathbf{g}(\tau'))$ ).*

Recall the interpretation set forth in Section 2 that  $g$  be thought of as the expenditure on governance; that is,  $g = C(\mathbf{g})$ . In light of Proposition 8, one is free to view the owners as solving a two-step process: first, for each  $y$  solve the problem

$$\min_{\mathbf{g}} C(\mathbf{g}) \text{ subject to } Y(\mathbf{g}) = y.$$

Let  $\mathbf{g}(y)$  denote the solution. Then associate to each  $y$  a  $g \equiv C(\mathbf{g}(y))$ ; this yields a one-to-one strictly monotonic mapping. This mapping can be inverted to yield a function mapping  $g$  into  $y$ . By construction, that function is equivalent to the  $Y(g)$  function used in Section 2. Observe, in this case, the cost of  $g$  is just  $g$ .<sup>20</sup>

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<sup>20</sup>To rule out corner solutions at no governance (i.e.,  $g = 0$ ) it was assumed in Section 2 that  $C'(0) = 0$ . Under the interpretation here,  $C'(g) \equiv 1$ . Hence, an alternative solution is needed to rule out solutions; one such assumption would be  $B_1(0, \tau) > 1$  for all  $\tau$ .

A related question is whether higher-type firms employ stronger governance on all dimensions than lower-type firms; that is, does  $\tau > \tau'$  imply  $\mathbf{g}(\tau) \geq \mathbf{g}(\tau')$ , where the order over vectors is the usually piecewise ordering? It is readily shown this implication cannot be true generally. For instance, suppose  $n = 2$ ,  $v(Y - S, \mathbf{g}) = v(Y - S, \max\{g_1, g_2\})$ , and

$$C(\mathbf{g}) = g_1 + \frac{1}{2}g_2 + \frac{3}{2} \left( g_2 - \min \left\{ g_2, \frac{2}{3} \right\} \right),$$

then the optimal  $\mathbf{g}$  to achieve effective governance level  $g = \max\{g_1, g_2\}$  is  $(0, g)$  for  $g \leq 1$  and  $(g, 0)$  for  $g > 1$ . Hence, if  $g(\tau') < 1 < g(\tau)$ , then  $\mathbf{g}(\tau')$  and  $\mathbf{g}(\tau)$  cannot be compared (*i.e.*, neither  $\mathbf{g}(\tau) \geq \mathbf{g}(\tau')$  nor  $\mathbf{g}(\tau') \geq \mathbf{g}(\tau)$  are true).

In the preceding example, the two dimensions of governance are perfect substitutes. If the dimensions of governance are complements, then the desired implication,  $\tau > \tau' \Rightarrow \mathbf{g}(\tau) \geq \mathbf{g}(\tau')$ , follows from Topkis's Monotonicity Theorem (Milgrom and Roberts, 1990, p. 1262):

**Proposition 9.** *Suppose that the manager's marginal benefit from behaving in a manner desired by the owners,  $v_1(y, \mathbf{g})$ , exhibits complementarities in governance; specifically, assume it is supermodular in  $\mathbf{g}$ . Suppose it also exhibits increasing differences; that is, if  $y > y'$  and  $\mathbf{g} \geq \mathbf{g}'$ , then*

$$v_1(y, \mathbf{g}) - v_1(y', \mathbf{g}) > v_1(y, \mathbf{g}') - v_1(y', \mathbf{g}').$$

Finally suppose that the marginal cost of governance in one dimension is non-increasing in any other dimension (*i.e.*,  $\partial^2 C(\mathbf{g}) / (\partial g_i \partial g_j) \leq 0$ ,  $i \neq j$ ).<sup>21</sup> Then the governance of a higher-type firm on any given dimension is no less than that of a lower-type firm on that dimension; that is,  $\tau > \tau'$  implies  $\mathbf{g}(\tau) \geq \mathbf{g}(\tau')$ .

Observe that if  $v(y, \mathbf{g})$  has the form

$$v(y, \mathbf{g}) = \gamma(\mathbf{g})v(y) \tag{25}$$

plus, possibly, additional terms with zero cross-partial derivatives in  $g_i$  and  $y$  for all  $i$ , if  $\gamma(\cdot)$  is increasing in its arguments, with positive cross-partial derivatives, and if  $v(\cdot)$  is increasing, then the conditions on  $v_1(y, \mathbf{g})$  set forth in Proposition 9 all hold. Observe further that (25) has the following interpretation: given a choice of  $\mathbf{g}$ , the firm has an "effective" level of governance  $\gamma(\mathbf{g})$ .<sup>22</sup> The owners' problem can, thus, be seen as choosing, for each effective level of governance  $\gamma$ ,

<sup>21</sup>Observe this condition would hold if  $C(\mathbf{g})$  were additive across the dimensions; that is, if

$$C(\mathbf{g}) = \sum_{i=1}^n c_i(g_i),$$

where  $c_i(\cdot)$  is strictly increasing for all  $i$ .

<sup>22</sup>The effective level of governance is increasing in each dimension of governance and the marginal effective level in any one dimension is increasing in any other dimension (*e.g.*,  $\gamma(\cdot)$  exhibits complementarities).

the cost-minimizing vector  $\mathbf{g}(\gamma)$ . The cost of such an effective level is  $C(\mathbf{g}(\gamma)) \equiv \hat{C}(\gamma)$ . Viewing  $\gamma$  as the equivalent of  $g$  in Sections 2 and 3, the analysis in those earlier sections can be seen as short-hand for a more elaborate model in which the owners set governance on many dimensions in a cost-minimizing way to achieve an effective level of governance (the  $g$  in those sections).

Whether different dimensions of governance are complements or substitutes is an empirical question. This question does not seem to have attracted much attention. One partial exception is Hermalin and Wallace (2001), which studies, *inter alia*, whether firms base incentive compensation on the same measures or different measures. They find evidence that if a firm heavily weights one measure, it will tend not to weight another; whereas if a firm heavily weights the other, it will tend not to weight the one. With respect to compensation, these findings support a view that dimensions of governance are substitutes. On the other hand, they neglect many other dimensions, so the overall issue of complements versus substitutes must be seen as open.

## 5.2 HETEROGENEITY IN COST OF GOVERNANCE

An alternative to assuming heterogeneity in the benefit function,  $B(\cdot, \cdot)$ , is to assume heterogeneity in the cost-of-governance function. To explore this, consider the model of Section 2, except write the owners' payoff as  $B(Y - S) - C(g, \theta)$ , where  $\theta$  denotes firm type in this alternative specification. As a definition of type, assume

$$C_{12}(\cdot, \cdot) < 0; \tag{6'}$$

that is, higher-type firms have lower marginal costs of governance. A straightforward modification of the proof of Proposition 1 shows that

**Proposition 1'.** *In a heterogeneous costs model, higher-type firms adopt at least as great a level of governance as lower-type firms (i.e., if  $\theta > \theta'$ , then  $g(\theta) \geq g(\theta')$ ). Moreover, if a lower-type firm has not adopted the maximum level of governance (i.e.,  $g(\theta') < \bar{g}$ ),<sup>23</sup> then a higher-type firm will have a strictly greater level of governance (i.e.,  $g(\theta) > g(\theta')$ ).*

If  $C_2(0, \theta) \leq 0$  for all  $\theta$ , then it also follows that

**Proposition 2'.** *Under the assumptions of the heterogenous costs model and assuming a common level of resources,  $Y$ , a firm that will be more profitable in equilibrium than another has at least as high a level of governance as the other firm.*

and

**Proposition 4'.** *Assuming (6') and endogenous investment, there will be a strictly positive correlation between the amount the owners invest in a firm and its level of governance. Furthermore, if governance cost is decreasing in firm type (i.e.,  $C_2(g, \theta) < 0$ ), then there will be a strictly positive, but non-causal, correlation between firm profit and level of governance.*

<sup>23</sup>Recall  $\bar{g}$  is the solution to  $Y(g) = Y$ .



In short, the model operates the same whether it is assumed that heterogeneity stems from different profit potentials or it is assumed that heterogeneity stems from different marginal costs of governance across firms.

There are numerous reasons firms could have different marginal costs of achieving a given level of effective governance (*i.e.*, a level that deters a given amount of agency behavior). Monitoring in some settings could be more difficult than in others. For example, it could be more difficult to determine what is going on with firms in fast-changing or highly innovative industries than with firms in staid and predictable industries. Or for instance, it could be more difficult to monitor a conglomerate operating in many industries than a firm operating in a single industry. Variation in laws and regulations across time or place could lead to differences in governance costs across time or space.

Although there is no reason to think that heterogeneity is solely on the benefit or cost side, one might ask which is more relevant. This is, obviously, an empirical question. One piece of evidence that points to the benefit side is the trend towards increased use of outside directors on the board in the United States and other countries over the past quarter decade or more (see, for instance, Borokhovich et al., 1996, Dahya et al., 2002, and Huson et al., 2001). It is difficult to see this trend as reflecting a drop in the marginal cost of outside directors. If anything, the evidence suggests the marginal cost of outside directors has increased; certainly, outside director compensation has increased (see, *e.g.*, Huson et al.). Another piece of evidence is the increased payout to executives, particularly from incentive pay (Hall and Liebman report a nearly seven-fold increase in the amount of options granted managers from 1980 to 1994). It is unclear what, if anything, has changed over the past thirty years to make the marginal cost of incentive pay decrease.

## 6 TRENDS IN GOVERNANCE

There have been numerous trends in corporate governance over the past twenty to thirty years (see, *e.g.*, Becht et al. and Holmstrom and Kaplan, 2001, for surveys). As noted above, the proportion of outside directors on boards has steadily increased in the United States and other countries (Borokhovich et al.; Dahya et al.; and Huson et al.). There has been growing use of stock-based incentives for directors over the period 1989 to 1997 (Huson et al.). Kaplan and Minton (2006) find evidence that CEO turnover rates in the period 1998–2005 are significantly greater than in the period 1992–1998; and the rate in that period is greater than found in studies for the pre-1992 period. They interpret this as evidence of better monitoring by boards of directors. Consistent with this interpretation is the finding of Huson et al. that firings, as a percentage of all CEO successions, were trending upward in the period 1971 to 1994. These trends can all be interpreted as evidence that governance has been getting stronger over the past twenty to thirty years.

At the same time, there is evidence that firms' profit potential and resources have been increasing during this period. As Gabaix and Landier note, there has been a six-fold increase in the market value of the top 500 US firms be-

tween 1980 and 2003. From 1973 to 2003, there has been a three-fold increase in patents granted in the US; and, since the late 1980s, evidence of increased productivity in R&D (Hall, 2004). Technological progress has been remarkable in this period, especially with respect to information technology and telecommunications. Since the mid-1990s, there has been a growth in productivity that has not resulted in a significant increase in wages (DeLong, 2003).

The analysis in Sections 2–4 offers a way of tying these two trends together. As the potential profitability and resources of firms increased, the value of improved governance also rose. Consequently, governance got stronger.

This is not to say the process was necessarily smooth. As noted by many authors, one might expect management to resist improved governance. This resistance could have led to more aggressive forms of change, such as the takeovers, leveraged buyouts, and proxy fights that characterized the 1980s. It could also have motivated shareholders to seek change through legislation or changes in the listing requirements of exchanges. But over time, as suggested by Holmstrom and Kaplan, a new equilibrium with stronger governance has emerged.

Tying the change in governance to changes in the resources and potential of firms also serves to explain why changes in governance occurred when they did. After all, commentators have been complaining about the state of governance for a long time (consider, *e.g.*, Berle and Means, 1932), so presumably something had to occur to motivate action. Until the point that the payoff from improved governance made imposing it worthwhile, investors were not willing to walk the talk.

The model set forth above offers a broader explanation for change than that set forth by Terviö or Gabaix and Landier, which are concerned with executive compensation only; moreover, their models suggest a relatively smooth process in which growth in firm size increases executive compensation.<sup>24</sup> Like the model here, Hermalin (2005) offers an explanation for improvements, broadly, in governance, but his explanation is based on the rise of institutional investors.<sup>25</sup> His explanation can be seen as complementary to the one set forth here insofar as greater institutional holding could reduce the free-riding problem among equity holders with respect to taking action; hence, greater institutional holding could serve to reduce the owners' marginal cost of governance, which, following Proposition 1', would yield higher levels of governance. Alternatively, an increase in holdings by institutional investors can be seen as a rise in  $\tau$ —as a larger proportion of profits accrue to these investors, their incentive to push for stronger governance increases.

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<sup>24</sup>Another explanation primarily focused on executive compensation is the model of Murphy and Zábojník (2003), which argues that changes in the CEO labor market, specifically a greater emphasis on general versus firm-specific knowledge, explains the rise in CEO compensation.

<sup>25</sup>Huson et al. report that the percentage of US equity held by institutional investors has increased from 20% in 1971 to 45% in 1994. Gompers and Metrick (2001) report a similar doubling from 1980 to 1996.

## 7 CONCLUSIONS

This paper has sought to make the case that firms are not profitable because they have good corporate governance, rather they have good corporate governance because they are profitable. This is not to say, of course, that corporate governance is irrelevant, but instead to say that the observed variation in governance across firms is not the cause of observed variation in their profits.

This insight holds important implications for empirical work in corporate governance and, to an extent, in the study of organizations generally. The endogenous characteristics of an organization are, presumably, chosen to facilitate the organization's objectives. If organizations are behaving optimally, then variation in how well they do cannot be explained by variation in their characteristics. However, as shown here, the variation in their characteristics could well be a tied to variation in the potentials they have to do well. Consequently, any cross-sectional regression of performance on characteristics is finding evidence of correlation, not causation.

To be sure, real-life optimizing is often a trial-and-error process. Hence, at any moment in time, organizations could be making errors and, thus, some variation in chosen characteristics is explaining some of the variation in performance. But if the variation in characteristics persists over time, then the weight one can place on cross-sectional regressions' representing evidence of causation is *de minimus*.

How, then, might empirical work proceed? One course suggested by this paper is to consider exogenous firm attributes that plausibly predict profitability and see whether they predict patterns in governance. A second course is, for some aspects of governance such as compensation, to utilize panel data and employ random-coefficient or similar models to estimate firm-specific coefficients. For instance, in the pay-for-performance regression (21), estimate the coefficients  $\delta_0$  and  $\delta_1$  on a firm-by-firm basis.<sup>26</sup> A third course is to examine the consequences of regulated changes that are binding on some firms (*e.g.*, those resulting from the Sarbanes-Oxley Act). If firms were optimizing prior to the regulated change (*i.e.*, their behavior was as shown in Figure 2), then those firms for which the regulations are binding should suffer poorer performance subsequent to the regulations than firms for which the regulations were not binding (*i.e.*, than firms that were meeting the regulations prior to their enactment). For example, if the regulation required a minimum level of the relevant governance parameter between  $L$  and  $H$  in Figure 2, then firm A should see a drop in performance, while firm B's performance should remain unaffected.<sup>27</sup>

Beyond empirical work, future research may wish to model the dynamics of governance change. Although some sense of how the model might be extended to a dynamic setting was given in Section 6, this is far from a complete analysis.

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<sup>26</sup>Hermalin and Wallace essentially employ this approach in their study of pay for performance; they find evidence for a much stronger pay-for-performance relationship using this approach than suggested by a cross-sectional analysis of the same data.

<sup>27</sup>It is possible that harm to firm A, as for instance caused by stiffer regulation, could benefit firm B if the two compete in product or factor markets.

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Moreover, in a dynamic setting, many models presume managers can entrench themselves or otherwise gain influence over how they are governed (surveys by Becht et al. and Hermalin and Weisbach discuss such models). In terms of the model presented above, this suggests that the cost of governance (*i.e.*,  $C(g)$ ) could rise over time if the same management team remains in office. Particularly if the *marginal* cost increases, then governance strength will decline and, consequently, so will profits. In terms of Figure 2, it could be as if firm B becomes firm A over time; it's still optimizing, but its circumstances are worse.

To an extent, entrenchment and managerial influence are, themselves, a product of the initial governance system, which suggests that, when a dynamic perspective is taken, initial governance might be overly strong when viewed at that point in time, but is optimal *vis-à-vis* the dynamic game. Other factors that influence the play of a dynamic game would be adjustment costs related to governance. In short, numerous issues will arise when this analysis is extended to a dynamic framework. Nonetheless, the basic messages of the paper concerning causality and the importance of firm heterogeneity are unlikely to be overturned by such an extension.

## APPENDIX A: PROOFS

**Proof of Lemma 1:** Fix a  $g$ . From (3) and the continuity and monotonicity of  $v_1(\cdot, g)$ , there exists a  $Y(g) < \infty$  such that

$$v_1(Y(g), g) = 1. \quad (26)$$

The manager's problem is

$$\max_{S \geq 0} S + v(Y - S, g). \quad (27)$$

$$\text{The derivative} = 1 - v_1(Y - S, g) \begin{cases} < 0, & \text{if } Y - S < Y(g) \\ > 0, & \text{if } Y - S > Y(g) \end{cases},$$

where the inequalities follow from (26) because (27) is strictly concave. It follows the manager does best to set  $S = 0$  if  $Y < Y(g)$  and  $S > 0$  if  $Y > Y(g)$ . In the latter case, it is readily seen the manager's optimal  $S = Y - Y(g)$ . The moreover part follows because raising  $g$  increases the left-hand side of (26) by (4), hence, by concavity,  $Y(g)$  must increase to restore equality. That  $Y(\cdot)$  is differentiable follows from the implicit function theorem. ■

**Proof of Lemma 2:** By the definition of an optimum (revealed preference):

$$f(\hat{x}, z) \geq f(\hat{x}', z) \text{ and} \quad (28)$$

$$f(\hat{x}', z') \geq f(\hat{x}, z'). \quad (29)$$

Expressions (28) and (29) imply

$$\begin{aligned} 0 &\leq (f(\hat{x}, z) - f(\hat{x}', z)) - (f(\hat{x}, z') - f(\hat{x}', z')) \\ &= \int_{\hat{x}'}^{\hat{x}} (f_1(x, z) - f_1(x, z')) dx = \int_{\hat{x}'}^{\hat{x}} \left( \int_{z'}^z f_{12}(x, \zeta) d\zeta \right) dx, \end{aligned}$$

where the integrals follow from the fundamental theorem of calculus. The inner integral in the rightmost term is positive because  $f_{12}(\cdot, \cdot) > 0$  and the direction of integration is left to right. It follows that the direction of integration in the outer integral must be weakly left to right; that is,  $\hat{x}' \leq \hat{x}$ . To establish the moreover part, because  $f_1(\cdot, \zeta)$  is a differentiable function for all  $\zeta$ , if  $\hat{x}'$  is an interior maximum, then it must satisfy the first-order condition

$$0 = f_1(\hat{x}', z').$$

Because  $f_{12}(\cdot, \cdot) > 0$  implies  $f_1(\hat{x}', z) > f_1(\hat{x}', z')$ , it follows that  $\hat{x}'$  does not satisfy the necessary first-order condition for maximizing  $f_1(x, z)$ . Therefore  $\hat{x}' \neq \hat{x}$ ; so, by the first half of the lemma,  $\hat{x}' < \hat{x}$ . ■

**Proof of Lemma 3:** Given the optimization program, it is not feasible for both constraints to be slack given that reducing  $w$  always increases (18). Given  $Y > \underline{w}$  and (LLC), (17) cannot bind if

$$gp \left( \frac{Y - N(g)}{Y} \right) - N(g) \geq 0. \quad (30)$$

Observe that  $\lim_{g \rightarrow 0} h(Y/g) = 1$  given  $p'(1) = -\infty$  and  $p(1) = 0$ . Hence, the left-hand side of (30) is 0 for  $g = 0$ . Its derivative is

$$p\left(\frac{Y - N(g)}{Y}\right) - \underbrace{\left(\frac{g}{Y}p'\left(\frac{Y - N(g)}{Y}\right) + 1\right)}_{=0 \text{ (manager's FOC)}} N'(g) > 0.$$

Consequently, (30) holds for all  $g$ . ■

**Proof of Lemma 4:** At  $g = 0$ , the derivative of (19) with respect to  $g$  is proportional to

$$B_1(0, \tau) - p\left(\frac{Y - 0}{Y}\right) > 0$$

because  $N(0) = 0$  and  $B_1(0, \tau) > 1 > 0 = p(1)$ . So  $g = 0$  is never optimal.

$$\lim_{g \rightarrow \infty} B(N(g), \tau) - gp\left(\frac{Y - N(g)}{Y}\right) = B(Y, \tau) - \infty \times p(0) = -\infty.$$

So the  $\hat{g}$  defined in (20) is superior to letting  $g \rightarrow \infty$ . ■

**Proof of Lemma 1':** The proof up to the “moreover” part mimics that of Lemma 1 and is omitted for the sake of brevity. The moreover part follows because raising any element of  $\mathbf{g}$  increases the left-hand side of (26) by (4'), hence, by concavity,  $Y(\mathbf{g})$  must increase to restore equality. That  $Y(\cdot)$  is differentiable follows from the implicit function theorem. ■

**Proof of Proposition 8:** Let  $\tau > \tau'$ . To reduce notational clutter, let  $\mathbf{g} = \mathbf{g}(\tau)$  and  $\mathbf{g}' = \mathbf{g}(\tau')$ . To prove the first part of the proposition it is sufficient to show that  $Y(\mathbf{g}) \geq Y(\mathbf{g}')$ ; because, if  $Y(\mathbf{g}) \geq Y(\mathbf{g}')$  but  $C(\mathbf{g}) < C(\mathbf{g}')$ , then the  $\tau'$ -type firm cannot be optimizing—it could weakly increase its benefit and strictly lower its costs by switching to  $\mathbf{g}$ . By revealed preference:

$$B(Y(\mathbf{g}), \tau) - C(\mathbf{g}) \geq B(Y(\mathbf{g}'), \tau) - C(\mathbf{g}') \quad \text{and} \quad (31)$$

$$B(Y(\mathbf{g}'), \tau') - C(\mathbf{g}') \geq B(Y(\mathbf{g}), \tau') - C(\mathbf{g}). \quad (32)$$

Expressions (31) and (32) can be combined to yield

$$B(Y(\mathbf{g}), \tau) - B(Y(\mathbf{g}'), \tau) \geq B(Y(\mathbf{g}), \tau') - B(Y(\mathbf{g}'), \tau').$$

Twice applying the fundamental theorem of calculus, this last expression can be rewritten as

$$\int_{\tau'}^{\tau} \int_{Y(\mathbf{g}')}^{Y(\mathbf{g})} B_{12}(y, t) dy dt \geq 0. \quad (33)$$

Because  $B_{12}(\cdot, \cdot) > 0$  and  $\tau > \tau'$ , (33) can be non-negative only if the direction of integration for the inner integral is left to right; that is, only if  $Y(\mathbf{g}) \geq Y(\mathbf{g}')$ . As noted, this implies  $C(\mathbf{g}) \geq C(\mathbf{g}')$ , as was to be shown.

Turning to the moreover part, the goal is to show  $Y(\mathbf{g}) > Y(\mathbf{g}')$ . Suppose, instead, that  $Y(\mathbf{g}) = Y(\mathbf{g}')$ . One of the types would, therefore, have to be playing non-optimally if  $C(\mathbf{g}) \neq C(\mathbf{g}')$ ; hence, this supposition implies  $C(\mathbf{g}) = C(\mathbf{g}')$ . The  $\tau'$ -type firm is at an interior solution, so there must be at least one  $g'_j$  such that

$$B_1(Y(\mathbf{g}'), \tau') Y_j(\mathbf{g}') - C_j(\mathbf{g}') = 0.$$

Because  $B_{12}(\cdot, \cdot) > 0$ , this implies

$$B_1(Y(\mathbf{g}'), \tau) Y_j(\mathbf{g}') - C_j(\mathbf{g}') > 0.$$

It follows there exists a governance vector  $\tilde{\mathbf{g}}$  that has slightly more on the  $j$ th dimension such that

$$B(Y(\tilde{\mathbf{g}}), \tau) - C(\tilde{\mathbf{g}}) > B(Y(\mathbf{g}'), \tau) - C(\mathbf{g}') = B(Y(\mathbf{g}), \tau) - C(\mathbf{g}),$$

where the equality follows because  $Y(\mathbf{g}) = Y(\mathbf{g}')$  and  $C(\mathbf{g}) = C(\mathbf{g}')$ . But then,  $\mathbf{g}$  was not optimal for the  $\tau$ -type firm, a contradiction. By contradiction,  $Y(\mathbf{g}) \neq Y(\mathbf{g}')$ , which, given the first part of the proposition, entails  $Y(\mathbf{g}) > Y(\mathbf{g}')$ . It must then be that  $C(\mathbf{g}) > C(\mathbf{g}')$  because otherwise the  $\tau'$ -type firm is not behaving optimally. ■

**Proof of Proposition 9:** In light of Proposition 8, define

$$\tilde{C}(y) = \min_{\mathbf{g} \in \mathcal{G}(Y)} C(\mathbf{g}) \text{ subject to } y = Y(\mathbf{g}). \quad (34)$$

The cost,  $C(\mathbf{g})$ , is increasing in each dimension and so is  $Y(\mathbf{g})$  (the latter follows from Lemma 1'). Consequently,  $\tilde{C}(\cdot)$  is an increasing function. The owners' problem can be reexpressed as

$$\max_{y \leq Y} B(y, \tau) - \tilde{C}(y). \quad (35)$$

Let  $y(\tau)$  be the solution to (35) selected by a type- $\tau$  firm. Utilizing Lemma 2, it is readily shown that  $\tau > \tau'$  implies  $y(\tau) \geq y(\tau')$ . The proposition follows if it can be shown that  $y(\tau) \geq y(\tau')$  implies  $\mathbf{g}(\tau) \geq \mathbf{g}(\tau')$ .

To that end, observe that minimization program in (34) is equivalent to the program

$$\max_{\mathbf{g} \in \mathcal{G}(Y)} -C(\mathbf{g}) \text{ subject to} \quad (36)$$

$$v_1(y, \mathbf{g}) = 1 \quad (37)$$

Let  $\lambda$  be the Lagrange multiplier on (37). The program given by (36) is, thus, equivalent to

$$\max_{\mathbf{g}, \lambda} -C(\mathbf{g}) + \lambda(v_1(y, \mathbf{g}) - 1). \quad (38)$$

This expression is supermodular in  $(\mathbf{g}, \lambda)$  in light of Topkis's characterization theorem (Milgrom and Roberts, 1990, p. 1261) because  $v_{1i}(y, \mathbf{g}) > 0$  by (4') and

$v_{1ij}(y, \mathbf{g}) \geq 0$  and  $-C_{ij}(\mathbf{g}) \geq 0$  by the assumptions of the proposition.<sup>28</sup> By the increasing-differences assumption of the proposition, (38) exhibits increasing differences in  $y$  and  $g_i$  for any  $i$ . Let  $\mathbf{g}(y)$  denote the solution to (38). It follows, therefore, from Topkis's monotonicity theorem (Milgrom and Roberts, p. 1262) that  $y > y'$  implies the  $\mathbf{g}(y) \geq \mathbf{g}(y')$ . ■

## APPENDIX B: A GOVERNANCE MODEL

This model is based on the career-concerns model of Holmstrom (1999). The manager has an ability  $\alpha$ . *Ex ante*, the value of  $\alpha$  is unknown by anyone, but it is commonly known that  $\alpha \sim \mathcal{N}(0, 1/\rho)$ , where  $\mathcal{N}(\mu, \sigma^2)$  denotes a normal distribution with mean  $\mu$  and variance  $\sigma^2$ . An amount  $Y$  of funds or assets is available for utilization, of which  $S \geq 0$  may be diverted by the manager for his private benefit. The amount the manager diverts and, hence, the amount actually utilized represent his private information. A private signal,  $\omega$ , is then observed by the owners. Assume

$$\omega = (Y - S)e^\xi,$$

where  $e$  is the base of the natural logarithm and  $\xi$  is a random variable distributed  $\mathcal{N}(\alpha, 1/g)$ , where  $g$ —essentially the precision of the signal—is a measure of the level of governance. It can be interpreted as investments by the owners to facilitate accurate information, a board of directors who are paying attention and can interpret information, etc.<sup>29</sup> Based on this signal, the owners can decide to retain the manager, in which case their payoff would be

$$(Y - S)e^\alpha.$$

Observe, if the owners knew  $S$ , then they would know  $\omega$ . Instead, they infer  $S$ —denote the inferred value by  $\hat{S}$ —but in equilibrium their inference must be correct. Consequently, their *posterior* distribution of  $\alpha$  is

$$\alpha \sim \mathcal{N}\left(\frac{g\xi}{g + \rho}, \frac{1}{g + \rho}\right) \quad (39)$$

(see DeGroot, 1970, p. 167, for the derivation of (39)).

The alternative to retaining the manager is to dismiss him, in which case the owners employ a new manager. Their payoff from doing so is

$$(Y - S)e^\beta,$$

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<sup>28</sup> $v_{1ij}$  denotes the third partial derivative of  $v$  with respect to  $y$ ,  $g_i$ , and  $g_j$ .

<sup>29</sup>An alternative model would be one in which  $g$  is the probability of the owners' obtaining a signal; that is,  $g$  is a measure of their vigilance. As Hermalin shows, however, the two approaches lead to similar conclusions.



where  $\beta$  is the ability of the *new* manager. Assume  $\beta \sim \mathcal{N}(0, 1/\rho)$  ( $\alpha$  and  $\beta$  are independent random variables). The owners will prefer to retain the original manager if  $\mathbb{E}\{e^\alpha|\xi\} \geq \mathbb{E}\{e^\beta\}$ ; that is, if

$$\exp\left(\frac{g\xi}{g+\rho} + \frac{1}{2(g+\rho)}\right) \geq \exp\left(\frac{1}{2\rho}\right). \quad (40)$$

Algebra reveals that (40) holds for all  $\xi$  if  $g = 0$  and holds for

$$\xi \geq \frac{1}{2\rho} \quad (41)$$

if  $g > 0$ .

Recall the owners do not observe  $\xi$  directly, but  $\omega$ . Hence, their *estimate* of  $\xi$  is

$$\log(\omega) - \log(Y - \hat{S}) = \xi + \log(Y - S) - \log(Y - \hat{S}).$$

Observe, as will be the case in equilibrium, if the owners' inferred  $S$  equals the true  $S$ , then the right-hand side is  $\xi$ . Using (41), the original manager *loses* his job if

$$\xi < \frac{1}{2\rho} - \log(Y - S) + \log(Y - \hat{S}).$$

It can be shown (see, *e.g.*, Hermalin) that the *ex ante* distribution of  $\xi$  is

$$\mathcal{N}\left(0, \frac{g+\rho}{g\rho}\right).$$

Hence, letting  $\Phi(\cdot)$  denote the distribution function of the standard normal (*i.e.*,  $\mathcal{N}(0, 1)$ ), the *ex ante* probability the original manager will be dismissed is

$$\Phi\left(\sqrt{\frac{g\rho}{g+\rho}}\left(\frac{1}{2\rho} - \log(Y - S) + \log(Y - \hat{S})\right)\right).$$

Suppose the original manager's utility is  $S$  if he keeps his job and  $S - \ell$ ,  $\ell > 0$  if he loses it. It follows, therefore, that

$$v(\underbrace{Y - S}_N, g) = -\ell\Phi\left(\underbrace{\sqrt{\frac{g\rho}{g+\rho}}}_{H(g,\rho)}\underbrace{\left(\frac{1}{2\rho} - \log(\underbrace{Y - S}_N) + \log(\underbrace{Y - \hat{S}}_{\hat{N}})\right)}_{\Lambda(\rho, N, \hat{N})}\right).$$

Observe

$$\frac{\partial v(N, g)}{\partial N} = \ell\phi(H(g, \rho)\Lambda(\rho, N, \hat{N}))H(g, \rho)\frac{1}{N}, \quad (42)$$

where  $\phi(\cdot)$  is the density function for the standard normal. Recall that  $\phi'(z) = -z\phi(z)$ , so

$$\begin{aligned} v_{11}(N, g) &= \ell\phi H^2\Lambda\frac{1}{N^2} - \ell\phi H\frac{1}{N^2} \propto H\Lambda - 1 \\ v_{12}(N, g) &= -\ell\phi H^2\Lambda^2\frac{1}{N}H_1(g, \rho) + \ell\phi\frac{1}{N}H_1(g, \rho) \propto H_1(g, \rho)(1 - H^2\Lambda^2), \end{aligned}$$

where the arguments of most functions have been dropped to facilitate readability. In a pure-strategy equilibrium  $N = \hat{N}$  and  $\Lambda(\rho, N, N) = 1/(2\rho)$ . Parameter values exist such that a unique pure-strategy equilibrium exists.<sup>30</sup> It is readily seen that  $H_1(g, \rho) > 0$ . Algebra reveals that, in equilibrium,  $H\Lambda < 1$  for all  $g \geq 0$  if  $\rho \geq 1/4$ . Hence, it follows that parameters exist such that conditions (2) and (4) hold. If one restricts

$$Y < \ell\phi\left(\frac{1}{\sqrt{\rho}}\right)\sqrt{\rho},$$

then it can be seen from (42) that condition (5) is satisfied.

### APPENDIX C: A STANDARD AGENCY MODEL

Consider an agency model in which the manager allocates some portion,  $S$ , of total available resources  $Y$  to his private benefit. For instance,  $Y$  could be his total available time and  $S$  the amount of it he devotes to private pursuits. Alternatively,  $Y$  are firm assets and  $S$  the amount diverted (*e.g.*, misuse of corporate jet, having underlings run private errands, etc.). Assume, as is customary in agency models, the manager's utility is additively separable in income,  $w$ , and consumption of  $S$ . Specifically, assume it is  $S + V(w)$ , where  $V(\cdot)$  is increasing and strictly concave.<sup>31</sup> Suppose that there are two possible outcomes, success and failure, upon which the manager's compensation can be based. Let  $w_s$  and  $w_f$  be compensation for success and failure, respectively. Let the probability of success be  $P(Y - S)$ , where  $P'(\cdot) > 0$  and  $P''(\cdot) < 0$ . Assume  $P(0) = 0$ . Define  $g = V(w_s) - V(w_f) - 1/P'(0)$ .<sup>32</sup> It follows that

$$v(Y - S, g) = P(Y - S) \left( g + \frac{1}{P'(0)} \right) + V(w_f).$$

Observe  $v_{11} = P''(Y - S)g < 0$  and  $v_{12} = P'(Y - S) > 0$ , as required.

Here,  $C(g)$ , the cost of governance, must be determined as part of the analysis. To keep the analysis straightforward, assume  $Y$  is sufficiently big that the constraint  $S \leq Y$  never binds in equilibrium. Given  $g$ , the manager will choose  $S$  to solve

$$-P'(Y - S) \left( g + \frac{1}{P'(0)} \right) + 1 = 0.$$

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<sup>30</sup>For example, if  $\rho = 1$  and  $\ell = 3$ , then a unique pure-strategy equilibrium exists for all  $g$  (*e.g.*, if  $g = 1$ , then the equilibrium value of  $Y - S \approx .795$ ). The Mathematica program for calculating equilibria available from author upon request.

<sup>31</sup>Perhaps a more natural specification would be  $b(S) + V(w)$ , where  $b(\cdot)$  is an increasing and concave function. Such a generalization has no qualitative consequences for the model and is, thus, ignored for convenience.

<sup>32</sup>The subtraction of the  $1/P'(0)$  term ensures that incentives vary in a meaningful way as  $g$  increases from zero.

Observe if  $g = 0$ , the manager will choose  $S = Y$ . In that case, because  $P(0) = 0$ , it is irrelevant what  $w_s$  is because the manager will be paid  $w_f$  with certainty.

The manager chooses  $S$  such that

$$P'(Y - S) = P'(\hat{N}(g)) = \frac{P'(0)}{1 + gP'(0)};$$

note the implicit definition of  $\hat{N}(\cdot)$ . Because  $P(\cdot)$  is concave,  $\hat{N}(\cdot)$  is an increasing function.

To close the model, assume that the manager has, as an alternative to working for the firm in question, an opportunity that would yield him utility  $\underline{U}$ . Normalize this reservation utility to zero (*i.e.*,  $\underline{U} = 0$ ). Conditional on  $V(w_s) - V(w_f) = g + 1/P'(0)$ , the firm will set  $w_s$  and  $w_f$  as low as possible, which means the manager's participation constraint,

$$P(\hat{N}(g)) \underbrace{(V(w_s) - V(w_f))}_{g+1/P'(0)} + V(w_f) \geq 0,$$

is binding. It follows that

$$w_f(g) = V^{-1} \left( -P(\hat{N}(g)) \left( g + \frac{1}{P'(0)} \right) \right) \text{ and, thus}$$

$$w_s(g) = V^{-1} \left( \left( 1 - P(\hat{N}(g)) \right) \left( g + \frac{1}{P'(0)} \right) \right).$$

The expected cost of providing  $g$  in incentives is, therefore,

$$C(g) = P(\hat{N}(g))(w_s(g) - w_f(g)) + w_f(g).$$

**Lemma C.1.** *The function  $C(\cdot)$  is increasing.*

**Proof:** Given that  $\hat{N}(\cdot)$  is increasing, it is sufficient to show that the standard agency problem of implementing  $N$  at minimum cost yields a cost function  $c(N)$  that is increasing in  $N$ . The standard problem is

$$\min_{\{v_s, v_f\}} P(N)V^{-1}(v_s) + (1 - P(N))V^{-1}(v_f) \quad (\text{P})$$

subject to

$$P'(N)(v_s - v_f) - 1 = 0 \text{ and} \quad (\text{IC})$$

$$P(N)v_s + (1 - P(N))v_f \geq 0. \quad (\text{IR})$$

The solution is readily shown to be

$$v_f = -\frac{P(N)}{P'(N)} \text{ and } v_s = \frac{1 - P(N)}{P'(N)}.$$

Hence,

$$c(N) = P(N)V^{-1}\left(\frac{1-P(N)}{P'(N)}\right) + (1-P(N))V^{-1}\left(-\frac{P(N)}{P'(N)}\right).$$

Observe that  $c(N)$  is the expected value of a convex function over a two-point distribution that has a mean of zero for all  $N$ ; that is,

$$P(N)v_s + (1-P(N))v_f = \frac{P(N)(1-P(N))}{P'(N)} - \frac{P(N)(1-P(N))}{P'(N)} = 0.$$

Because the left point of the distribution is falling in  $N$  (i.e.,  $dv_f/dN < 0$ ) and the right point is increasing in  $N$  (i.e.,  $dv_s/dN > 0$ ), an increase in  $N$  represents a mean-preserving spread. From Jensen's inequality, it follows that  $c(N)$  must be increasing in  $N$ . ■

By the choice of functional forms, one can insure that  $C'(0) = 0$ .<sup>33</sup> For example, if  $P(N) = \sqrt{1 - (N-1)^2}$  (a quarter-circle function with center  $(1, 0)$ ) and  $V(w) = \log(w)$ , then it can be shown  $\lim_{g \rightarrow 0} C'(g) = 0$ .

Applying this model to the issues in Section 4, suppose that owners get  $\tau$  if the manager is successful and 0 if he fails. The benefit function is, thus,  $B(N, \tau) = \tau P(N)$ . As in the proof of Lemma C.1, let  $c(N)$  denote the minimum cost to the owners of inducing the manager to divert only  $S = Y - N$ . Note  $c(N)$  is the manager's expected compensation, which by Lemma C.1 is an increasing function of  $N$ . Because  $\hat{N}(g)$  is increasing in  $g$  it is invertible, so a higher  $N$  also means the manager has more high-powered incentives. Given that  $B_{12}(\cdot, \cdot) > 0$  and  $B_2(\cdot, \cdot) > 0$  when  $B(N, \tau) = \tau P(N)$ , it follows that  $N$ , and thus expected compensation and the power of the manager's incentives will (i) be non-decreasing in  $\tau$  by Lemma 2; and (ii) that therefore there will be a non-negative correlation between firm profits and managers' expected compensation and between profits and strength of incentives. By Lemma 2 "non-decreasing" and "non-negative" can be replaced by "increasing" and "positive" if the owners' problem always admits an interior solution (for instance, if  $c(\cdot)$  is convex and  $c'(0) < \tau P'(0)$  for almost every  $\tau \in \mathcal{T}$ ). To summarize:

**Proposition C.1.** *Under the agency model in this appendix and assuming the owners get profits  $\tau$  if the manager is successful and 0 if he fails, an increase in  $\tau$  causes*

- (i) net resources,  $N$ , not to decrease;
- (ii) the manager's expected compensation,  $c(N)$ , not to decrease; and
- (iii) the power of the manager's incentives,  $g$ , not to decrease.

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<sup>33</sup>Of course, since this assumption was made primarily for convenience, it is not essential that it hold.

Coefficient	Estimate
$\delta_0$	.884 (72.7)
$\delta_1$	.366 (209.)
$R^2$	.813

**Table C.1:** Estimation of equation (21) using data generated as described in this appendix. Dependent variable is realized pay. Independent variable is profit (gross of compensation), its coefficient is  $\delta_1$ . The coefficient  $\delta_0$  is the intercept. Numbers in parentheses are t-statistics.

If the owners' problem has an interior solution for all  $\tau \in \mathcal{T}$ , then “not to decrease” can be replaced by “to increase.” Furthermore, the following correlations will hold:

- (iv) Firm profits and managerial compensation will be non-negatively correlated; and
- (v) Firm profits and the power of managerial incentives will be non-negatively correlated.

If the owners' problem has an interior solution for all  $\tau \in \mathcal{T}$ , then “non-negatively” can be replaced by “positively.”

How does this model fair with respect to the critique of the cross-sectional regression specification given by (21)? To analyze this, it was assumed that  $P(N) = \frac{N}{N+1}$  and  $V(w) = \log(w)$ . Data were created for 10,000 firms as follows. For each firm, its  $\tau$  was a random draw from the Pareto distribution  $\tau \sim 1 - (6/\tau)^3 : [6, \infty)$ . Gabaix and Landier provide evidence for why a Pareto distribution reflects reality. Then, for each firm, the optimal contract was calculated, as was the  $N$  its manager would choose in equilibrium. Whether that firm was successful or not was determined by whether a uniformly distributed random variable on the unit interval was less than  $P(N)$ —successful—or was above  $P(N)$ —failure. Once the data were constructed, equation (21) was estimated. The results are shown in Table C.1.<sup>34</sup>

The estimated  $\delta_1$ , while estimated with great accuracy, is a poor measure of actual incentives in this sample. For any given firm, the true coefficients solve

$$w_f = \delta_0 + \delta_1 \times 0 \quad \text{and} \quad w_s = \delta_0 + \delta_1 \tau.$$

Hence a firm's true  $\delta_1$  is given by

$$\delta_1 = \frac{w_s - w_f}{\tau}.$$

<sup>34</sup>The Mathematica program used to generate the data is available from the author upon request.

In the sample, the range of  $\tau$  proved to be from 6.0010 to 132.32. True  $\delta_1$  is falling in  $\tau$  from .670 to .147. This is considerable variation. Moreover, the estimated  $\delta_1$  is the true  $\delta_1$  of a firm with a  $\tau \approx 18.6$ . This implies that 96.6% of the firms have true  $\delta_1$ s greater than the estimated  $\delta_1$ !<sup>35</sup>

As in Section 4, there is an insult to this injury: even if the true  $\delta_1$ s were known, they would be a misleading measure. While true  $\delta_1$ s fall with  $\tau$ , the true measure of incentive power,  $g$ , is rising with  $\tau$ . In other words, the firms that appear to have the smallest incentives in terms of  $\delta_1$  are really those that provide their managers the largest incentives.

## APPENDIX D: ISSUES IN THE ESTIMATION OF THE PAY-FOR-PERFORMANCE RELATION

Consider the following agency model. The manager's utility is

$$\frac{-1}{\exp(\underbrace{Y - N}_S + \delta_0 + \delta_1 \pi)},$$

where his compensation contract is  $\delta_0 + \delta_1 \pi$ ,  $\pi$  being realized profits gross of compensation. Assume that<sup>36</sup>

$$\pi \sim \mathcal{N}(\tau \log(N), \sigma^2),$$

In what follows, assume  $Y$  is sufficiently large that the constraint  $N \leq Y$  never binds.

Using the formula for the moment-generating function of a normal random variable, the manager's expected utility is

$$\frac{-1}{\exp(Y - N + \delta_0 + \delta_1 \tau \log(N) - \frac{1}{2} \delta_1^2 \sigma^2)}.$$

A monotonic transformation is

$$Y - N + \delta_0 + \delta_1 \tau \log(N) - \frac{1}{2} \delta_1^2 \sigma^2. \quad (43)$$

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<sup>35</sup>Even if one dropped outliers from the data, a considerable majority of firms would have true  $\delta_1$ s greater than the estimated  $\delta_1$ . For example, if the  $\tau$ s were distributed uniformly on  $[6, 16]$ , then the estimated  $\delta_1$  corresponds to a  $\tau \approx 12.8$ ; that is, 68% of the firms have true  $\delta_1$ s greater than the estimated  $\delta_1$ .

<sup>36</sup>Note the assumed compensation contract is not second-best optimal under these assumptions (see Mirrlees, 1974). A reformulation of this simple model along the lines of Holmstrom and Milgrom (1987)—in particular, assuming the manager decides how much to divert on a continuous basis with the resulting net funds controlling the drift of a Brownian motion—would, however, yield an optimal compensation contract of this form. One can thus view the model here as a simple approximation of that more complex model. In any case, given the issue is the econometric consequences of heterogeneity, the optimality of the contract is not essential to the analysis.

Given  $\delta_0$  and  $\delta_1$ , the manager maximizes (43) with respect to  $N$ . This yields

$$N = \delta_1 \tau.$$

Hence, if the owners want to induce a particular  $N$ , they must set  $\delta_1$  to satisfy

$$\delta_1 = \frac{N}{\tau}. \quad (44)$$

Assume, as an alternative to working for the firm, the manager could get a job that paid a flat wage of  $\underline{w}$ . Hence, the owners must set  $\delta_0$  to satisfy

$$Y - N + \delta_0 + \underbrace{\frac{N}{\tau}}_{\delta_1} \tau \log(N) - \frac{1}{2} \underbrace{\frac{N^2}{\tau^2}}_{\delta_1^2} \sigma^2 \geq \underline{w}.$$

Given the owners' profit is decreasing in  $\delta_0$ , the constraint will bind and, thus,

$$\delta_0 = \underline{w} - Y + N - N \log(N) + \frac{1}{2} N^2 \frac{\sigma^2}{\tau^2}. \quad (45)$$

The owners seek to choose  $N$  to maximize their expected profits. Using (44) and (45), their expected profits can be written as

$$\tau \log(N) - \underline{w} + Y - N - \frac{1}{2} N^2 \frac{\sigma^2}{\tau^2}.$$

The first-order condition is

$$\frac{\tau}{N} - 1 - N \frac{\sigma^2}{\tau^2} = 0.$$

Solving for the root that satisfies the second-order condition yields

$$N = \frac{\tau^{3/2} \sqrt{4\sigma^2 + \tau} - \tau^2}{2\sigma^2}. \quad (46)$$

Plugging (46) into (44) and (45) yields expressions for  $\delta_1$  and  $\delta_0$ , respectively, in terms of the model's primitives ( $\tau$ ,  $\sigma^2$ ,  $Y$ , and  $\underline{w}$ ).

Data were created for 10,000 firms as follows. For each firm, its  $\tau$  was a random draw from the uniform distribution  $\tau \sim \mathcal{U} : [10, 20]$ . To insure that  $N \leq Y$  was never binding,  $Y = 400$ . The reservation wage,  $\underline{w}$ , was set to zero. Then, for each firm, the optimal contract was calculated, as was the  $N$  its manager would choose in equilibrium. Profits gross of compensation,  $\pi$ , were then generated for that firm as a draw from  $\mathcal{N}(\tau \log(N), 1)$  and the resulting compensation for the manager calculated. Once the data were constructed, equation (21) was estimated with  $\pi$  as the independent variable. The results are shown in Table D.2.<sup>37</sup>

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<sup>37</sup>The Mathematica program used to generate the data is available from the author upon request.

Coefficient	Estimate
$\hat{\delta}_0$	-396.46 (-15321.)
$\hat{\delta}_1$	.27568 (440.69)
$R^2$	.95104

**Table D.2:** Estimation of equation (21) using data generated as described in this appendix. Dependent variable is realized pay. Independent variable is profit (gross of compensation), its coefficient is  $\delta_1$ . The coefficient  $\delta_0$  is the intercept. Numbers in parentheses are t-statistics.

The true  $\delta_1$ s for firms with  $\tau \in [10, 20]$  ranges from [.916, .954]; hence, the estimate  $\hat{\delta}_1$  is less than the true  $\delta_1$  for 100% of the firms. The reason why (21) does so poorly is that while the true  $\delta_1$ s are fairly constant across  $\tau$ , the intercepts—the true  $\delta_0$ s—vary considerably. Because low  $\tau$  firms have greater  $\delta_0$ s and tend to have lower  $\pi$ s, while high  $\tau$  firms have lower  $\delta_0$ s and tend to have higher  $\pi$ s, the estimated slope between pay and profits is flattened. Consequently, the estimated  $\hat{\delta}_1$  is biased downward.



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