Federalism, Tax Base Restrictions, and the Provision of Intergenerational Public Goods

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July 2007

Abstract

We investigate the level of investment in local public goods that will be enjoyed by future generations under decentralized provision of these goods, under both head tax and land tax regimes. We then compare these outcomes to results for the centralized provision of such goods. We find that decentralizing the provision of intergenerational goods always leads to more efficient provision of intergenerational goods, regardless of the tax base available to the centralized and decentralized governments. However, choice of tax base is still important; under a head tax regime, we obtain efficient investment under very general assumptions. Under a land tax regime, we obtain efficient investment only in the limit of perfect competition and noncongestibility of the public good, while investment is inefficiently low if either of these conditions fails.

JEL Codes: D6, D78, H4, H7, R53

*We are grateful to Dennis Epple, Robert Hall, Ken Judd, Ed Lazear, Gerard Padró i Miquel, Ken Shotts, and Romain Wacziarg for helpful discussions. We are particularly indebted to Antonio Rangel, without whom this paper could not have been written. Any comments or suggestions are welcome and may be emailed to hatfield@stanford.edu.

1 Introduction

People who have not yet been born do not vote, and can not sign contracts, and so students of political economy have long despaired that this will lead to intergenerational expropriation and underinvestment in intergenerational public goods (IPGs). Yet this issue is crucially important to the welfare of future generations: environmental protection, investment in infrastructure, and investment in public capital are all political decisions taken today which will have large effects on the welfare of future generations. Assuming that present generations are not perfectly altruistic, what political forms are capable of inducing efficient investment in IPGs and limiting intergenerational expropriation through debt?

We show that to answer this question precisely, one must consider not only the centralization/decentralization of the government in question, but also the form of the tax base the polity has access to. We consider two types of tax bases: land taxes and head taxes, which represent a combination of non-land taxes, such as income taxes, capital gains taxes, usage fees, and the like. In the case of centrally provided intergenerational goods, Rangel (2005) has shown that the first generation will have no incentive to invest in pure IPGs[1], as they enjoy none of the benefits of that investment. For IPGs such that the second generation can invest, there will be more investment by the first generation under a land tax regime than a head tax regime. Further, intergenerational expropriation through debt issued is arbitrarily large under a head tax regime, while debt does not matter under a land tax regime: as was shown by Feldstein (1977), debt will be fully incorporated into the land price.

However, decentralization forces districts to compete for new residents, since the larger the number of residents, the greater the value of the land currently owned by the first generation. This competition leads districts to invest in IPGs in order to attract residents; we find that the decentralization of the provision of public goods leads to more efficient investment, regardless of the tax base chosen. However, interdistrict competition is not enough; in stark contrast to the results for centrally provided IPGs, under decentralization head taxes induce more efficient investment in IPGs; in particular, a decentralized head tax regime will achieve efficient investment in IPGs even under quite weak assumptions, while a land tax regime will not. The results of the previous literature are summarized on the left side of Figure 1; our novel results for locally provided IPGs are summarized on the right.

The head tax regime will be able to achieve efficiency as the first generation is able to set the "price" for the IPG; by changing the debt level, the district can change how much it costs to enjoy the IPG. If some district underinvested in the IPG, it could increase its

[1] A pure IPG is one in which the second generation is completely unable to invest.

	Centrally Provided Purely Intergenerational Public Goods	Locally Provided Purely Intergenerational Public Goods
Head Taxes	1) Debt issued arbitrarily large 2) No intergenerational investment	1) Some debt issued; decreasing in # of districts 2) Optimal intergenerational investment
Land Taxes	1) Debt issued is irrelevant 2) No intergenerational investment	1) Debt issued is irrelevant 2) Less than optimal intergenerational investment

Figure 1: Summary of results.

investment level to the efficient level, given it second generation population, and increase the debt so as to leave second generation agents indifferent. No member of the second generation would change his decision of which district to live in, so land prices would remain fixed; meanwhile, the first generation would gain the surplus created by increasing investment. Under a land tax, however, the issuance of debt will, as in Feldstein (1977), be immediately capitalized into land prices, so districts will be unable to set the "price" for choosing to live in their district. The lack of price-setting ability under a land tax regime has two regrettable effects. First, if there are but a few districts, these districts will exert their market power by lowering their investment in the IPG. In contrast, under a head tax regime, they will increase debt, since a transfer from the second generation, if available, is a more efficient way to exploit their market power, and investment in the IPG will remain at optimal levels. Second, if the public good is partially congestible, then districts under a land tax regime will lower their investment in the IPG so as to lessen the negative effects of the congestibility. In contrast, under a head tax regime, they will use the debt to charge a Pigouvian tax on each incoming resident for the reduction in utility to other residents.

There is a very large literature on the welfare effects of decentralization of public goods, stretching back to Oates (1972), Musgrave (1959), and earlier, and recently summarized in Inman and Rubinfeld (1997) and Oates (1999). There have also been a number of papers on the incentive effects of federalism and the effects of different tax bases in providing public goods; an overview of this literature can be found in Mieszkowski and Zodrow (1989) and McKinnon and Nechyba (1997). Both of these literatures, however, have concentrated on how decentralization and choice of tax base effects the provision of public goods for the current generation. This paper, on the other hand, analyzes how the decentralization and the choice of tax base effects the provision of IPGs. Glaeser (1994) considers this problem, but he assumes that taxation is not controlled by the local governments in question, as well as assuming local governments are Brennan and Buchanan (1977, 1978, 1980) style Leviathans. We, however, assume that district governments can set taxes and debt levels as they choose, and use political mechanisms that choose Condorcet winners when they exist. Kotlikoff and Rosenthal (1993) also touch on this issue in their work, but consider only a two district model where districts are unable to issue debt, fundamentally changing the results.

The paper is organized as follows. Section 2 describes the model. Section 3 quickly characterizes the results in the case of one district, i.e. for a centrally provided IPG. Section 4 characterizes the equilibrium outcomes when there are multiple districts. Section 5 extends the results to durable public goods, i.e. public goods that benefit both the current and future generations. Section 6 concludes.

2 Model

2.1 Economy

We consider an economy with J identical jurisdictions. There are two time periods, and for each period there exists a continuum of agents of size J. Furthermore, there are three goods: a private numeraire good, land, and an intergenerational public good. Land is a durable asset, and we fix the amount of land in each district at 1.

At time 1, the first generation is born and receives the unit endowment of land and $\hat{w}$ units of the numeraire good. There is then an election in which this generation, within each district, decides on how much to spend on the local intergenerational public good, denoted G_j, and a level of debt to issue, denoted D_j.[2] Then, at time 2, the second generation is born, endowed only with w units of the numeraire good.[3] They are free to choose any district to live in, and may only purchase land within that district. The number who choose to live in each district is denoted N_j. After the land market clears, the first generation consumes its wealth, including transfers from the second generation gained from selling their endowment of land, and dies. Generation 2 then pays the debt left from the prior generation, decides how much to spend on the IPG for itself, denoted I_j, and finally enjoys the benefits of individual consumption, land, and the intergenerational good.

There are two different tax regimes that the districts may employ. The first is the

[2] We assume each district can borrow and lend freely at a given interest rate, which for simplicity we fix at 1.

[3] Note that subscripts refer to a district, while a carat denotes generation 1 for variables that appear in both time periods.

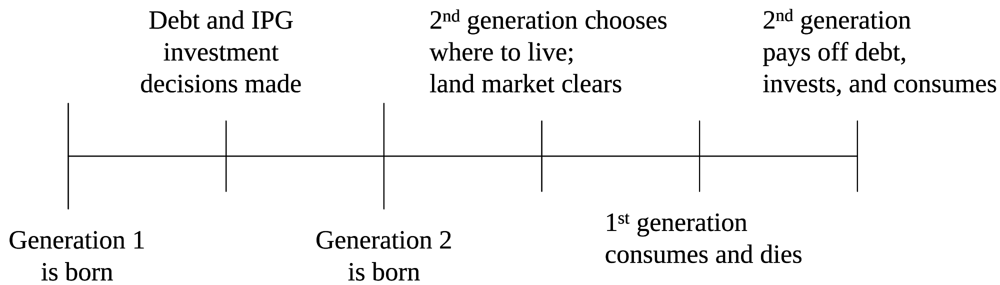

Figure 2: Timeline of events.

head tax regime, in which a resident in district j pays a head tax T_j. Since agents are simply endowed with their wealth, we may think of this equivalently as a tax on income or consumption, but use a head tax for algebraic simplicity. The other tax regime we shall consider is that of a land tax, in which each agent pays a tax equal to τ_j times the amount of land he consumes.

The amount of the intergenerational good enjoyed by a member of the second generation depends both on the amount invested by the first generation and second generations in that district, as well as the number of people within that district. That is, we assume that the IPG may be congestible. Hence, we model the total provision of the IPG to a member of the second generation in district j, g_j as

$$g_j = \frac{G_j + \theta I_j}{N_j^{\beta}}$$

where θ is the parameter that determines how inefficient investment in the good by the second generation is: we assume that $\theta < 1$ so that the efficient solution is for the first generation, and only the first generation, to invest in the IPG.[4] If $\theta = 0$, then the second generation is unable to invest in the IPG and it is a pure intergenerational public good. The other parameter, β, is a measure of the congestibility of the IPG; for $\beta = 0$, the good is fully noncongestible.

Preferences of the second generation are given by

$$u(c) + v(l) + f(g)$$

where c denotes consumption of the numeraire good and l denotes consumption of land. We further assume that $u(\cdot), v(\cdot)$, and $f(\cdot)$ are strictly increasing, strictly concave, twice continuously differentiable and satisfy the Inada conditions.

[4] We assume a specific functional form here for expositional purposes; our results would be qualitatively unchanged if $g_j = h(G_j + \theta I_j, N_j)$.

We assume that there is no intergenerational altruism, so as to ascertain whether decentralization by itself can motivate efficient investment in IPGs. Hence, the preferences of the first generation are given by

$$\hat{u}(c) + \hat{v}(l)$$

where $\hat{u}(\cdot)$ and $\hat{v}(\cdot)$ satisfy the same conditions as $u(\cdot)$ and $v(\cdot)$.

Note that for expositional purposes, we have assumed that the first generation obtains no utility from the IPG; this assumption will be relaxed in section 5.

2.2 Pareto Optimality

We first wish to characterize the set of Pareto optimal outcomes, and in particular we will concentrate on the allocations that provide equal utility to all members of a generation, as our focus is on intergenerational efficiency. It is clear from the concavity of the utility function for land that a Pareto optimal allocation that treats all members of a generation equally must allocate an equal number of agents to each district. Further, since $\theta < 1$, it must be optimal for all investment in the IPG to be done in the first period. Hence, putting a Pareto weight of α on the first generation and $(1-\alpha)$ on the second, we solve[5]

$$\max_{G,D} \left\{ \begin{array}{c} \alpha\left(\hat{u}\left(\hat{w}+D-G\right)+\hat{v}\left(1\right)\right) \\ + \\ \left(1-\alpha\right)\left(u\left(w-D\right)+v\left(1\right)+f\left(G\right)\right) \end{array} \right\}$$

and so, taking first order conditions, we have that

$$\begin{aligned} \alpha\hat{u}'\left(\hat{w}+D-G\right) &= \left(1-\alpha\right)u'\left(w-D\right) \\ \alpha\hat{u}'\left(\hat{w}+D-G\right) &= \left(1-\alpha\right)f'\left(G\right) \end{aligned}$$

and hence any Pareto optimal allocation is characterized by the Samuelson condition

$$\frac{f'(G)}{u'(c)} = 1$$

The intuition behind this result is straightforward: since we can move money between generations using debt, we should equate the marginal benefit of investment in the local IPG to the marginal cost (in terms of lost utility from consumption) for the second generation.

2.3 Equilibrium

We now formally define an equilibrium for our economy. Our definition of equilibrium has four parts. First, the agents must maximize their own welfare as private actors when

[5] Note that since each district has a population of 1, and there is no investment by the second generation, $g_j = \frac{G_j}{N_j^{\beta}} = G_j$.

deciding between land and consumption, and the land market must clear. Second, the second generation's government must maximize the welfare of the representative agent, subject to its budget constraint: the money raised to pay back the debt and invest in the IPG must be raised via local taxation. Third, agents correctly forecast policy and land prices, and given these forecasts, must distribute themselves so that utility is equalized across districts: otherwise, some agents could decide to live in a different district, making themselves better off. Finally, given all of the above, the first generation government must choose policy, that is, debt issuance and intergenerational investment, so as to maximize the welfare of the first generation.

Formally, an equilibrium is a set of intergenerational investments in each district $\{G_j, I_j\}_{j=1}^J$, debt levels in each district $\{D_j\}_{j=1}^J$, head taxes in each district $\{T_j\}_{j=1}^J$ under a head tax regime and land taxes in each district $\{\tau_j\}_{j=1}^J$ under a land tax regime, prices and allocations in each district $\{p_j, l_j\}_{j=1}^J$, locational choices by the second generation $\{N_j\}_{j=1}^J$, and consumptions by each agent $\{\hat{c}_j, c_j\}_{j=1}^J$ such that:

1. Given the locational choices of the second generation, as well as taxes, the second generation agents maximize their utility given the price of land. That is, each second generation agent in district j solves

$$\max_{c_j, l_j} \{u(c_j) + v(l_j)\} \tag{1}$$

   subject to the budget constraint that

$$c_j + T_j + p_j l_j = w$$

   under a head tax regime and

$$c_j + (p_j + \tau_j) l_j = w$$

   under a land tax regime.

2. The land market within each district clears. That is, the market clearing condition

$$N_j l_j = 1 \tag{2}$$

   holds.

3. The members of the second generation that live in district j choose a tax rate to pay off the debt D_j and decide on their investment in the IPG I_j. That is, under a head tax regime the second generation solves

$$\max_{T_j, I_j} \left\{ u(w - p_j l_j - T_j) + v(l_j) + g\left(\frac{G_j + \theta I_j}{N_j^\beta}\right) \right\} \tag{3}$$

subject to

$$D_j + I_j = N_j T_j$$

and under a land tax regime

$$\max_{\tau_j, I_j} \left\{ u\left(w - \left(p_j + \tau_j\right) l_j\right) + v\left(l_j\right) + g\left(\frac{G_j + \theta I_j}{N_j^{\beta}}\right) \right\} \tag{4}$$

subject to

$$I_j + D_j = \tau_j$$

4. Utility is equalized across districts for the second generation. That is,

$$u\left(c_{j'}\right) + v\left(l_{j'}\right) + f\left(g_{j'}\right) = u\left(c_{j''}\right) + v\left(l_{j''}\right) + f\left(g_{j''}\right) \tag{5}$$

for all $j', j'' = 1, ..., J$.

5. The first generation, within each district j, optimally chooses debt D_j and intergenerational investment G_j to maximize their utility. That is, they solve

$$\max_{D_j, G_j} \left\{ \hat{u}\left(\hat{w} + p_j + D_j - G_j\right) \right\}$$

as the initial allocation of land to the first generation is fixed. This reduces to

$$\max_{D_j, G_j} \left\{ p_j + D_j - G_j \right\} \tag{6}$$

taking the above constraints and the actions of agents in other districts as given.

3 Equilibrium Outcomes under Centralization

We first consider the case of centrally provided IPGs, where there is only one district under consideration, and restate some results from the analysis of Rangel (2005) for comparison. Hence, the fourth equilibrium condition becomes vacuous, and the district can act as a monopolist, since every member of the second generation will live there. For simplicity, we drop the district subscripts in this section. We also impose a debt limit $0 < D^{\max} < w$ on the amount of debt the government can issue to ensure an equilibrium exists.

3.1 Head Tax Regime

From the consumer maximization and land market clearing equilibrium conditions (1) and (2), we can calculate the land market equilibrium. In particular,

$$p = \frac{v'(1)}{u'(w - (p + T))} \tag{7}$$

From the second generation's district-level optimization equilibrium condition (3), we can calculate T, the head tax in the second period as $D+I$. Differentiating the above expression, then with respect to D, yields

$$\frac{\partial p}{\partial D} = \frac{pu''(c)}{u'(c) - pu''(c)} \in (-1, 0) \tag{8}$$

Every unit of debt issued lowers the land price through the wealth effect: if the second generation has less to consume, they will be less willing to use that consumption to buy land and so the price of land will fall. However, the price of land will fall by less than the amount of debt issued.

Solving the maximization problem of the second generation (3) we find that

$$\frac{u'(c)}{f'(G+I)} = \theta \tag{9}$$

at an interior solution. Since the second generation can only invest inefficiently in the public good at the rate θ, the marginal utility of consumption must only be a fraction θ of the marginal utility from the IPG.

We now consider the problem of the first generation (6), to maximize $p + D - G$. It is immediate, then, from (8) that the first generation will issue as much debt as possible, as $\frac{\partial p}{\partial D} > -1$. We can also calculate from (7) that the price of land changes with government investment as follows:

$$\frac{\partial p}{\partial G} = \frac{pu''(c)}{u'(c) - pu''(c)} \frac{\partial I}{\partial G} \tag{10}$$

In the case of intergenerational investment, the amount of investment depends on the size of $\frac{\partial I}{\partial G}$. If $\theta = 0$, i.e. the IPG is a pure intergenerational good, then $\frac{\partial I}{\partial G} = 0$ and we will see no investment by the first generation in the IPG. However, even if $\theta > 0$ it is clear that the second generation will not invest if

$$\frac{u'(c)}{f'(G)} \geq \theta \tag{11}$$

is satisfied with no investment by the second generation. Once the first generation invest enough that the second generation has no incentive to invest, the price of land does not depend on additional investment in the IPG. Hence, the first generation will invest at most enough to satisfy this condition, which is less than what is necessary for optimal investment in the IPG, which demands that $u'(c) = f'(G)$.

The results are summarized in the following proposition:

Proposition 1 *The unique equilibrium is characterized by the first generation setting $D = D^{\max}$. If $\theta = 0$, there will be no intergenerational investment, and even for $\theta > 0$, the level of intergenerational investment will be strictly less than optimal.*

3.2 Land Tax Regime

From the consumer maximization and land market clearing equilibrium conditions (1) and (2), we can calculate the land market equilibrium. In particular,

$$p+\tau=\frac{v'(1)}{u'(w-(p+\tau))} \tag{12}$$

From the second generation's district-level optimization equilibrium condition (3), we can calculate T, the head tax in the second period as $D+I$. Differentiating the above expression with respect to D yields

$$\frac{\partial p}{\partial D}=-1 \tag{13}$$

and so we have the standard Feldstein (1977) result that the debt is fully incorporated into the price of land. Hence, the amount of debt issued by the first generation is irrelevant in determining the total transfer to them.

Solving the maximization problem of the second generation (3) we find that

$$\frac{u'(c)}{f'(G+I)}=\theta \tag{14}$$

at an interior solution. As before, since the second generation can only invest inefficiently in the public good at the rate θ, the marginal utility of consumption must only be a fraction θ of the marginal utility from the IPG for the second generation to no longer invest in the IPG.

We now consider the problem of the first generation (6), to maximize $p+D-G$. It is immediate, then, from (8) that the the debt issued by the first generation is irrelevant, as $\frac{\partial p}{\partial D}=-1$. Regardless of the debt issued, the first generation will receive the value of their land, i.e. the solution to $p=\frac{v'(1)}{u'(w-p)}$, assuming no investment by the second generation. We can also calculate from (12) that the price of land changes with government investment as follows:

$$\frac{\partial p}{\partial G}=\begin{cases}\frac{1}{\theta} \text{ if } \frac{u'(c)}{f'(G)}<\theta \\ 0 \quad \text{otherwise}\end{cases}$$

Since the second generation will invest inefficiently in the IPG if $\frac{u'(c)}{f'(G)}<\theta$, any investment that they know they will do will be treated as debt, and so will be fully incorporated into the price of land. Hence, the first generation has an incentive to invest in the IPG exactly up to the point where the second generation will not invest as $\frac{1}{\theta}>1$. The first generation under the land tax regime will invest more in the IPG than under a head tax regime as both 1) $u'(c)$ is smaller under a land tax regime, since the consumption of the second generation is greater as there is no debt to pay off, and 2) investment less than what is necessary to insure no investment by the second generation is fully capitalized into the land price under the land tax regime, while only capitalized at the rate $\frac{-pu''(c)}{u'(c)-pu''(c)} \in (0,1)$ under the head tax regime.

The results are summarized in the following proposition:

Proposition 2 *The unique equilibrium outcome in real variables (i.e. consumptions, investments, and land use) is characterized by a total transfer from the second generation to the first of B where $B = \frac{v'(1)}{u'(w-B)}$. Furthermore, if $\theta = 0$, there will be no investment in the IPG. Otherwise, if $\theta > 0$, the level of investment in the IPG will be strictly greater than that under a head tax regime, but strictly less than optimal.*

4 Equilibrium Outcomes under Decentralization

We now turn to the central focus of the paper, which is to characterize the equilibrium outcomes under different tax bases for the decentralized provision of local IPGs.

4.1 Head Tax Regime

Given a head tax T_j, the problem of the second generation agent, once he has chosen which district to live in, is to decide on how much land to buy, as is given by the first equilibrium condition. As in the case of a centralized head tax regime, we find that

$$p_j = \frac{v'(l_j)}{u'(w - (p_j + T_j)\, l_j)} \tag{15}$$

since the land market must clear; since agents are identical, each will buy an equivalent amount of land. The only difference between this and the result for centralized provision is that the price now depends on the number of agents in the district, and this is no longer fixed at one.

Given that the debt must be paid off, the land market clearing and second generation's district-level optimization equilibrium conditions (2) and (3) imply that

$$T_j = (D_j + I_j)\, l_j$$

Furthermore, using the results above and the utility equalization across districts equilibrium condition (5) we have that

$$\begin{aligned} u(w - (p_j + D_j)\, l_j) + v(l_j) + f(g_j) &= u(w - (p_1 + D_1)\, l_1) + v(l_1) + f(g_1) \qquad (16) \\ p_1 + D_1 &= l_1^{-1} \left(w - u^{-1} \begin{pmatrix} u(w - (p_j + D_j)\, l_j) + \\ v(l_j) - v(l_1) + \\ f(g_j) - f(g_1) \end{pmatrix} \right) \end{aligned}$$

where the second expression characterizes the total transfer of the incoming residents of district 1 to the first generation residents of district 1.

The problem of the first generation residents of district 1 is to solve

$$\max_{p_1, D_1, G_1, l_1} \{p_1 + D_1 - G_1\}$$

subject to (15) and (16). Taking the first-order condition with respect to G_1, and simplifying, we find that

$$N_1^{1-\beta} \frac{f'(g_1)}{u'(c_1)} = 1$$

which, given a population of N_1 in district 1, is the optimal amount of intergenerational investment. Furthermore, within each district the first generation is doing all of the investment in the IPG. Otherwise, if positive investment was done by the second generation, we would have $N_j^{1-\beta} \frac{f'(g_j)}{u'(c_j)} = \theta < 1$.

To see that in any equilibrium each district (given its population) invests efficiently in the IPG, suppose another such equilibrium existed. Then consider district j', one of the districts which, given its population, is not investing efficiently in the IPG. Then that district could change its debt and investment decision such that, holding constant $N_{j'}$, it has efficiently invested in the IPG and the utility level of second generation agents within the district has not changed. Hence, none of the second generation agents has any incentive to change the district he chooses to live in, and so the gains from efficient investing in the IPG must go to the first generation members of the district j'. Hence, we have found a profitable deviation and it can not be an equilibrium for any district to choose an inefficient level of investment in the IPG.

Taking the other first order condition of the first generation's problem, and simplifying, we have

$$D_1 = \beta G_1 + \frac{\left.\frac{\partial p_j}{\partial N_j}\right|_{N_j=1}}{J}$$

at the symmetric equilibrium. The first term in the expression for debt, βG_1, is a Pigouvian tax: it exactly captures the negative externality on the other residents from a given resident choosing to live in that district. The greater the level of congestion β in the public good, the higher the Pigouvian tax. The second term, $J^{-1} \frac{\partial p_j}{\partial l_j}$, is the extra amount the district charges due to imperfect competition among the districts. Districts do not take the reservation utility of agents outside their district as given, but understand that by raising their debt, more agents will enter other districts, lowering the utility of second generation agents in these districts as well. Note that as the number of districts goes to infinity, the amount of intergenerational expropriation through debt approaches βG, a constant.

Note that debt here is positive if either the public good is congestible or the number of districts is finite. By issuing debt, the district can effectively change the price for living in the district. This is important for two reasons: first, in the presence of congestibility, it is necessary to stop overpopulation of the district. Otherwise, investing an efficient amount in the IPG would attract too many outside residents, whose entry would degrade the public good for everyone else. The only way for a district to stop this, while investing efficiently in the IPG, is to charge a fee on second generation agents who enjoy the public good that has been have provided, and that fee is exactly the externality those agents impose on others. Second, in the presence of only a few states, these states will wish to exert their market power. This is not bad *per se*, since every second generation agent must live somewhere, and hence the presence of market power may only induce transfers. That is indeed the case here: market power does allow the first generation to expropriate more from the second generation, but it does not degrade the quality of the IPG provided, for the reasons elucidated above.

The results are summarized in the following proposition:

Proposition 3 *The symmetric equilibrium is characterized by each district having a population of 1, investing efficiently in the IPG, and setting its debt level to* $\beta G + \frac{\left.\frac{\partial p_j}{\partial N_j}\right|_{N_j=1}}{J}$.[6]

4.2 Land Tax Regime

Given a land tax τ_j, the problem of the second generation agent, once he has chosen which district to live in, is to decide on how much land to buy, as is given by the first equilibrium condition. As in case of a centralized land tax regime, we find that

$$p_j + \tau_j = \frac{v'(l_j)}{u'(w - (p_j + \tau_j) l_j)} \tag{17}$$

and since $\tau_j = D_j$, we have that the total transfer to the first generation in district j, $p_j + D_j$, depends only on the number of people within the district j, not the choice of debt level. (As in the single district case, any debt is completely discounted into the land price.) Hence, since debt is irrelevant, we shall assume for purposes of simplification that $D_j = 0$ for all districts. The futility equalization across districts equilibrium condition (5), then, states that

$$u(w - p_1 l_1) + v(l_1) + f(g_1) = u(w - p_j l_j) + v(l_j) + f(g_j) \tag{18}$$

[6] The result that intergenerational investment is effecient holds for a more general model as well. In particular, neither the additive separability nor the specific functional form of the intergenerational production function is necessary.

and the problem for the first generation agents in district 1 becomes to solve

$$\max_{p_1,G_1,l_1} \{p_1 - G_1\}$$

subject to the constraints (17) and (18). Taking the first order condition and simplifying, we obtain

$$\frac{J-1}{J} - \frac{\beta G_1}{\left.\frac{\partial p_j}{\partial N_j}\right|_{N_j=1}} = \frac{u'(c_1)}{f'(g_1)}$$

in a symmetric equilibrium (assuming that the optimal investment by the second generation is 0 at this level of investment by the first generation). Note that the IPG is underprovided if either $J < \infty$ or $\beta > 0$ (as $\frac{\partial p_1}{\partial N_1} > 0$), as then $\frac{u'(c_1)}{f'(g_1)} < 1$. The key issue here is that, unlike in the head tax case, districts are unable to set the price for living in the district. Even in a world of perfect competition, the districts will underprovide the IPG if it is congestible, as this is the second-best solution. When the first generation is unable to charge agents directly for living in the district and imposing a cost (through the congestibility of the public good) on all other citizens of the district, the only way to charge agents for entering is by lowering the quality of the public good. Even without congestion (i.e. $\beta = 0$), the IPG will be underprovided as the districts will exert their market power by underproviding the IPG, since, again, they can not change the price of living in their district directly.

If the level of public investment is such that $\frac{u'(c_j)}{f'(G_j)} \geq \theta$, then the second generation will have no incentive to invest, and the above characterization will define our equilibrium. Otherwise, we will have devolved to the centralized case, and each district will invest exactly as much as a centralized regime would per person.

The results are summarized in the following proposition:

Proposition 4 *The symmetric equilibrium outcome in real variables (i.e. consumptions, investments, and land use) is characterized by each district having a population of 1, and investing less than the efficient amount in the IPG.*

4.3 Regime Comparison

4.3.1 Efficiency Outcomes

We are now able to rank the efficiency of various regimes in providing the IPGs. For purely intergenerational IPGs, a centralized regime will not invest in the IPG at all; in any case it will always underinvest. In contrast, when there are competitive forces at work, both tax regimes induce at least as much intergenerational investment as under a centralized regime, and very likely more. Under the head tax regime, efficient levels of investment in the IPG will always be produced, while under a land tax regime, decentralized provision will always provide more investment in the IPG than either tax regime under centralization.

Further, the implications of Rangel (2005) for the efficiency-enhancing choice of tax regime are reversed if the provision is decentralized. Under a centralized regime, land taxes induce more intergenerational investment and an increase in intergenerational efficiency. Under a decentralized regime, the reverse is true: the head tax regime always generates efficient investment, while the land tax regime only does so under very specific circumstances.

Corollary 5 *A decentralized regime will always induce more efficient investment in the IPG than a centralized regime. Further, a decentralized head tax regime always produces efficient investment in the IPG, while a decentralized land tax regime invests less.*

A useful analogy here can be drawn between our model (with $\theta = 0$) and the standard model of government price setting in the presence of a monopolist who can invest in the quality of his product. With no government intervention, the monopolist will set a price as high as possible and quality as low as possible, assuming demand is inelastic with respect to quality and price. This is like a centralized head tax regime, where every member of the second generation must live in the one district. When such a monopoly exists, government can increase consumer surplus by setting a cap on the price the monopolist can charge; after all, the monopolist is not investing in quality in any case. Since land taxes vitiate debt as a redistributive instrument, switching to a land tax regime from a head tax regime is much like putting a cap on the price the monopolist can charge, and this can indeed increase the welfare of the second generation. In contrast, when there is competition between firms, they will compete on price and set quality to the welfare-maximizing level; if government intercedes by setting a price cap, firms will react by scrimping on quality. A similar mechanism is at work here: a land tax regime imposes a price cap, as debt no longer can be used as a transfer from the second generation to the first, and so the first generation does not invest as much in the IPG.

4.3.2 Distributional Outcomes

It is always better for the second generation to have more competition in the form of a greater number of districts, as this decreases debt in the head tax regime and increases intergenerational investment under the land tax regime. It is also clear from the above results that outcomes will be uniformly more efficient under a decentralized regime, and further that a decentralized head tax regime will efficiently provide the IPG in all cases, while the decentralized land tax regime will, in general, underprovide the IPG. However, this does not mean that the second generation uniformly prefers a decentralized head tax regime.

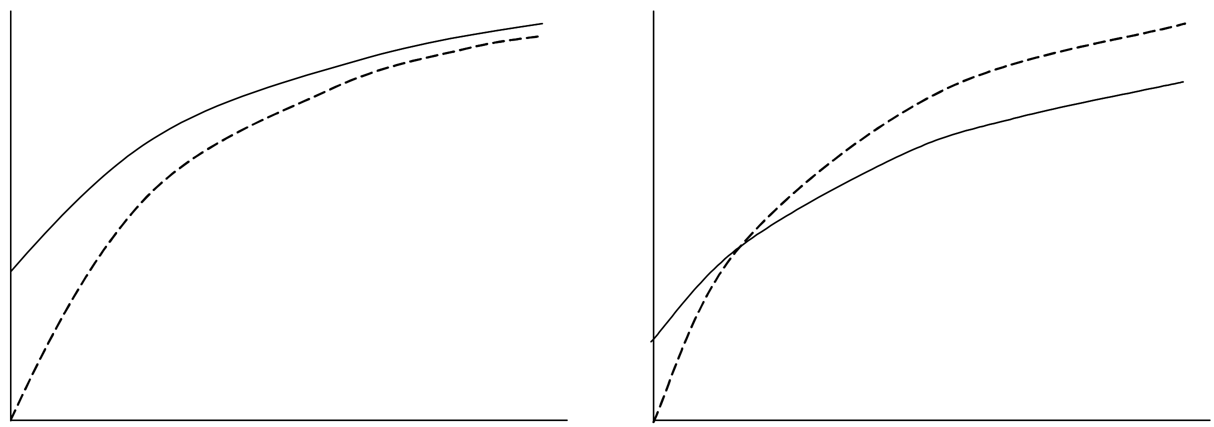

Figure 3: Graph of second generation utility with respect to the number of districts; solid lines represent the head tax regime, dashed lines the land tax regime. The graph on the left shows the case $\beta = 0$, and the graph on the right shows the case $\beta = \frac{1}{2}$.

Consider the case where the utility function of the second generation is given by

$$c + \log(l) + \log(g)$$

which is used in Figure 3. As congestibility increases, the debt load increases linearly, and hence has a large effect on the consumption of the second generation. Under a land tax regime, however, the districts are unable to change the debt, and so only respond by somewhat decreasing the level of the IPG, and so the second generation may be better off under a land tax regime if there are many districts. While total surplus increases when we go from the decentralized land tax regime to the decentralized head tax regime for any level of congestibility, the first generation captures more than all of the gains if β is high.

5 Durable Public Goods

5.1 Model

Many public goods are of a durable character: that is, we expect them to be enjoyed by future generations as well as current ones. Our model can incorporate these types of goods as well, by considering a small change in the utility of 1st period agents; we now let their utility function be

$$\hat{u}(c) + \hat{v}(l) + \hat{f}(G)$$

Note that since the number of first generation agents within each district is fixed, we do not need to consider the congestibility for the first period agents of the public good, nor worry about the amount of land usage enjoyed by each member of the first generation. Furthermore, for this section we shall assume that $\theta = 0$ for simplicity.

The optimal level of IPG investment must now satisfy

$$\max_{G,D} \left\{ \begin{array}{c} \alpha \left(\hat{u} (\hat{w} + D - G) + \hat{v}(1) + \hat{f}(G) \right) \\ + \\ (1-\alpha)\left(u(w-D) + v(1) + f(g)\right) \end{array} \right\}$$

which gives us the Samuelson condition:

$$\frac{\hat{f}'(G)}{\hat{u}'(\hat{c})} + \frac{f'(g)}{u'(c)} = 1$$

given that each district has an equal population.

Now, of course, centralized regimes do have an incentive to invest in the durable public good, but only up to the level that it benefits them: a centralized regime, under both tax regimes, will choose G so that

$$\frac{\hat{f}'(G)}{\hat{u}'(\hat{c})} = 1$$

and hence underinvest in the durable public good.

5.1.1 Head Tax Regime

However, the same forces that ensure efficiency under decentralization and a head tax regime are still at work: the first generation will still have the proper incentives to invest efficiently in the durable public good. The problem for the first generation in district 1 is now

$$\max_{p_1,D_1,G_1,l_1} \left\{ \hat{u}(\hat{w} + p_1 + D_1 - G_1) + \hat{f}(G_1) \right\}$$

subject to the constraint that the price of land is at the market equilibrium (15) and that welfare is the same in each district (16). Taking the first order condition of this problem, we obtain

$$N_1^{1-\beta} \frac{f'(g_1)}{u'(c_1)} + \frac{\hat{f}'(G_1)}{\hat{u}'(\hat{c}_1)} = 1$$

so we see that the durable public good will be provided efficiently. The logic is the same as that for as that for IPGs that benefit only future generations: if the good was not being provided at efficient levels, the first generation could vary the level of debt and investment in order to both maximize surplus and leave the second generation indifferent. By doing so, they capture the surplus and have made themselves better off.

By taking the other first order condition of this problem, we obtain

$$D_1 = \beta G_1 \frac{f'(G_1)}{u'(c_1)} + \frac{\left.\frac{\partial p_j}{\partial N_j}\right|_{N_j=1}}{J}$$

so that, as before, debt is rising with the number of districts and the level of congestion. Note that members of the second generation only pay for the externality they impose on other members of the district, $\beta G_1 \frac{f'(G_1)}{u'(c_1)}$, through the debt instrument.

The results are summarized in the following proposition:

Proposition 6 *Under a head tax regime, the symmetric equilibrium is characterized by each district having a population of 1, investing efficiently in the IPG, and setting its debt level to* $\beta G\frac{f'(G)}{u'(c)}+\frac{\left.\frac{\partial p_j}{\partial N_j}\right|_{N_j=1}}{J}$.

5.2 Land Tax Regime

Under a land tax regime, we again obtain less than efficient investment: since districts are still unable to charge for entry of second generation residents, they will underprovide the durable public good as before, in the presence of either congestion or imperfect competition.

As in the case of IPGs that benefit only future generations, debt is completely incorporated into the land price, and hence does not change real outcomes. Therefore, the problem for the district is to solve

$$\max_{p_1,G_1,l_1}\left\{\hat{u}\left(\hat{w}+p_1-G_1\right)+\hat{f}\left(G_1\right)\right\}$$

subject to the condition that utility is equalized across districts (18).

Taking the first order condition of this problem, we obtain

$$\frac{J-1}{J\left(1-\frac{\hat{f}'(G_1)}{\hat{u}'(\hat{c}_1)}\right)}-\frac{\beta G_1}{\left.\frac{\partial p_j}{\partial N_j}\right|_{l_j=1}}=\frac{u'\left(c_1\right)}{f'\left(g_1\right)}$$

and so, as before, first best will only be achieved when $J\to\infty$ and $\beta=0$.

The results are summarized in the following proposition:

Proposition 7 *Under a land tax regime, the unique equilibrium outcome in real variables is characterized by each district having a population of 1, and investing less than the efficient amount in the IPG.*

6 Concluding Remarks

We have shown that decentralizing the investment in intergenerational public goods will always induce more efficient investment in these goods. The competition for future residents and the resulting increase in the price of land will drive districts to invest strictly more than they would under a centralized regime, regardless of the tax regime. Note that in the case of pure IPGs, this is true even in the presence of externalities from the public good, such as those in Oates (1972) and Besley and Coate (2003); a centralized regime will invest nothing, while a decentralized regime will still invest a positive (if inefficiently low) amount in the IPG. The argument that externalities in the production of public goods point to centralized provision relies crucially on the fact that the central government will be at least as efficient as the district government. However, in our model, the central government does not have incentives that lead it to invest efficiently; district governments, however, do have such incentives, in the form of competition for residents, and hence investment will be closer to optimal, even in the presence of spillovers.

Further, the tax base of the competing districts is of key importance: a head tax allows districts to compete on debt issuance, i.e., the price they charge members of the second generation for living in their district. Since this competition plays itself out using only transfers, it does not affect the efficiency of the investments by the districts. In contrast, land taxes destroy the ability of districts to compete on debt issuance, and so the competition will play itself out along the axis of investment, leading to inefficient investment.

We end with several qualifications of our results. First, this work applies to local IPGs; goods such as scientific research or national environmental protection can not be provided locally, and so the mechanisms described here will not be helpful. Second, we have assumed that local governments face hard budget constraints; we do not consider the issue of intergovernmental bailouts of debt; see Qian and Roland (1998), among others, for discussions of this issue. Finally, we have abstracted from the fact that neither a pure head tax regime or pure land tax regime is seen in practice. The key characteristic, however, of the tax scheme is how a change in tax rates changes the value of land. An instrument such as an income tax would act much like a head tax; we would no longer achieve efficient investment, but only because the first generation will take into account the standard deadweight loss from taxation when raising the money to invest in the IPG. In contrast, a land tax may lead to less deadweight loss from taxation[7], but will not provide as good of incentives for efficient investment in intergenerational public goods.

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7 Appendix

Proof of Propostion 1:

Proof. The problem of the government is to solve

$$\max_{D,G} \{p + D - G\}$$

Since

$$\frac{\partial p}{\partial D}=\frac{pu''(c)}{u'(c)-pu''(c)}\in(-1,0)$$

we have that the optimal amount of debt to issue is as much as possible: i.e. $\hat{D}=D^{\max}$. Further,

$$\frac{\partial p}{\partial G}=\frac{pu''(c)}{u'(c)-pu''(c)}\frac{\partial I}{\partial G}$$

and so, since the second generation will never invest more than is necessary to fulfill $\frac{u'(c)}{f'(G)}\geq\theta$, the first generation will never have any reason to invest more in the IPG than is necessary to fulfill this condition, as $\frac{\partial I}{\partial G}$ for any investment greater than that is 0. Since $\theta>1$, that means the first generation will necessarily underinvest in the IPG. If $\theta=0$, then there is no incentive for the central government to invest in the IPG, as $\frac{\partial I}{\partial G}=0$, and hence $\frac{\partial p}{\partial G}=0$.
■

Proof of Propostion 2:

Proof. The problem of the government is to solve

$$\max_{D,G}\{p+D-G\}$$

Since

$$\frac{\partial p}{\partial D}=-1$$

the government is indifferent over how much debt to issue. Further,

$$\frac{\partial p}{\partial G}=\frac{\partial I}{\partial G}$$

and so , since the second generation will never invest more than is necessary to fulfill $\frac{u'(c)}{f'(G)}\geq\theta$, the first generation will never have any reason to invest more in the IPG than is necessary to fulfill this condition, as $\frac{\partial I}{\partial G}$ for any investment greater than that is 0. Since $\theta>1$, that means the first generation will necessarily underinvest in the IPG. If $\theta=0$, then there is no incentive for the central government to invest in the IPG, as $\frac{\partial I}{\partial G}=0$, and hence $\frac{\partial p}{\partial G}=0$.

To see that the level of investment must be greater than that under a head tax regime, note that c is larger under the land tax regime, as the government can not expropriate wealth using the debt instrument; hence, the necessary level of investment by the first generation is higher to fulfill $\frac{u'(c)}{f'(G)}=\theta$. Second, while under the land tax regime, the government will invest up to this point, under the head tax regime they may not, as land prices go up at the $\frac{pu''(c)}{u'(c)-pu''(c)}\frac{\partial I}{\partial G}$, which is less than $\frac{\partial I}{\partial G}$. ■

Proof of Propostion 3:

Proof. We will assume that there is no investment by the second generation and show that the equilibrium we find involves efficient investment by the first generation: hence the second generation will not invest.

The problem for the first period agents in district 1 is to solve

$$\max_{D_1, G_1, N_1} \{D_1 + p_1 - G_1\}$$

subject to the equilibrium condition #4. Substituting in (16), we have

$$\max_{l_1, g_1} \left\{ l_1^{-1} \left(w - u^{-1} \begin{pmatrix} u\left(w - \left(p_j + D_j\right) l_j\right) + \\ v\left(l_j\right) - v\left(l_1\right) + \\ f\left(g_j\right) - f\left(g_1\right) \end{pmatrix} \right) - G_1 \right\}$$

so taking the FOC with respect to G_1 we have

$$\begin{aligned} l_1^{-1} \left(u^{-1}\right)' \left(u\left(c_1\right)\right) f'\left(g_1\right) l_1^{\beta} &= 1 \\ N_1^{1-\beta} \frac{f'\left(\frac{G_1}{N_1^{\beta}}\right)}{u'\left(c_1\right)} &= 1 \end{aligned}$$

Note that any equilibrium must have each district investing the efficient amount in the IPG, given the number of people it obtains in equilibrium.

Our first order condition with respect to l_1 is

$$\begin{aligned} -l_1^{-2} \left(w - u^{-1} \begin{pmatrix} u\left(w - \left(p_j + D_j\right) l_j\right) + \\ v\left(l_j\right) - v\left(l_1\right) + \\ f\left(g_j\right) - f\left(g_1\right) \end{pmatrix} \right) - & \\ l_1^{-1} \left(u^{-1}\right)' \begin{pmatrix} u\left(w - \left(p_j + D_j\right) l_j\right) + \\ v\left(l_j\right) - v\left(l_1\right) + \\ f\left(g_j\right) - f\left(g_1\right) \end{pmatrix} \begin{pmatrix} -u'\left(c_j\right)\left(p_j + D_j + \frac{\partial p_j}{\partial l_j} l_j\right) \frac{\partial l_j}{\partial l_1} \\ +v'\left(l_j\right) \frac{\partial l_j}{\partial l_1} - v'\left(l_1\right) \\ +\beta l_j^{\beta-1} G_j f'\left(g_j\right) \frac{\partial l_j}{\partial l_1} - \beta l_1^{\beta-1} G_1 f'\left(g_1\right) \end{pmatrix} &= 0 \\ w - u^{-1}\left(c_1\right) + l_1 \left(u^{-1}\right)'\left(c_1\right) \begin{pmatrix} -u'\left(c_j\right)\left(p_j + D_j + \frac{\partial p_j}{\partial l_j} l_j\right) \frac{\partial l_j}{\partial l_1} \\ +v'\left(l_j\right) \frac{\partial l_j}{\partial l_1} - v'\left(l_1\right) \\ +\beta l_j^{\beta-1} G_j f'\left(g_j\right) \frac{\partial l_j}{\partial l_1} - \beta l_1^{\beta-1} G_1 f'\left(g_1\right) \end{pmatrix} &= 0 \end{aligned}$$

In any symmetric equilibrium $l_1 = l_j = 1$, $G_1 = G_j$, $\frac{f'(g_1)}{u'(c_1)} = 1$, $c_1 = c_j$, and $\frac{\partial l_j}{\partial l_1} = -\frac{1}{J-1}$, hence

$$
\begin{aligned}
w-u^{-1}\left(c_1\right)+\left(u^{-1}\right)'\left(c_1\right)\begin{pmatrix} \frac{u'(c_1)\left(p_1+D_1+\left.\frac{\partial p_j}{\partial l_j}\right|_{l_j=1}\right)}{J-1} \\ -\frac{v'(l_1)}{J-1}-v'\left(l_1\right) \\ +\frac{\beta G_1 f'(g_1)}{J-1}-\beta G_1 f'\left(g_1\right)\end{pmatrix} &= 0 \\
p_1+D_1+\left(\frac{(p_1+D_1)}{J-1}+\frac{\left.\frac{\partial p_j}{\partial l_j}\right|_{l_j=1}}{J-1}-\frac{J}{J-1}p_1-\frac{J\beta G_1}{J-1}\right) &= 0 \\
D_1+\frac{\left.\frac{\partial p_j}{\partial l_j}\right|_{l_j=1}}{J-1}-\beta G_1 &= 0 \\
D_1 &= \beta G_1+\frac{\left.\frac{\partial p_j}{\partial N_j}\right|_{N_j=1}}{J-1}
\end{aligned}
$$

■

Proof of Propostion 4:

Proof. It was shown in the text that debt was irrelevant for outcomes in real variables, and we shall assume that all districts choose $D_j=0$ for notational convenience. Consider the case when the second generation does not invest. The problem of the district is thus

$$\max_{G_1}\left\{p_1-G_1\right\}$$

subject to (18),

$$u\left(w-p_1 l_1\right)+v\left(l_1\right)+f\left(g_1\right)=u\left(w-p_j l_j\right)+v\left(l_j\right)+f\left(g_j\right)$$

and so substituting in we have

$$\max_{l_1}\left\{p_1-l_1^{-\beta}f^{-1}\begin{pmatrix} u\left(w-p_j l_j\right)+v\left(l_j\right)+f\left(l_j^{\beta}G_j\right)- \\ \left(u\left(w-p_1 l_1\right)+v\left(l_1\right)\right)\end{pmatrix}\right\}$$

Taking the first order condition with respect to l_1, we have

$$
\begin{aligned}
\frac{\partial p_1}{\partial l_1}+\beta l_1^{-1-\beta}G_1+l_1^{-\beta}\left(f^{-1}\right)'\left(f\left(g_1\right)\right)\begin{pmatrix} \left(u'\left(c_j\right)\left(p_j+l_j\frac{\partial p_j}{\partial l_j}\right)-v'l_j-\beta l_j^{\beta-1}G_j f'\left(g_j\right)\right)\frac{\partial l_j}{\partial l_1}- \\ u'\left(c_1\right)\left(p_1+l_1\frac{\partial p_1}{\partial l_1}\right)-v'\left(l_1\right)\end{pmatrix} &= 0 \\
\frac{\partial p_1}{\partial l_1}+\beta l_1^{-1-\beta}G_1+\frac{l_1^{-\beta}}{f'\left(g_1\right)}\begin{pmatrix} \left(u'\left(c_j\right)l_j\frac{\partial p_j}{\partial l_j}-\beta l_j^{\beta-1}G_j f'\left(g_j\right)\right)\frac{\partial l_j}{\partial l_1}- \\ u'\left(c_1\right)l_1\frac{\partial p_1}{\partial l_1}\end{pmatrix} &= 0
\end{aligned}
$$

In any symmetric equilibrium $l_1 = l_j = 1$, $G_1 = G_j$, $c_1 = c_j$, and $\frac{\partial l_j}{\partial l_1} = -\frac{1}{J-1}$, hence

$$\begin{aligned}
\left.\frac{\partial p_j}{\partial l_j}\right|_{l_j=1} + \beta G_1 - \frac{1}{f'(g_1)}\left(\begin{array}{c}\left(u'(c_1)\left.\frac{\partial p_j}{\partial l_j}\right|_{l_j=1} - \beta G_1 f(g_1)\right)\frac{1}{J-1}+ \\ u'(c_1)\, l_1 \left.\frac{\partial p_j}{\partial l_j}\right|_{l_j=1}\end{array}\right) &= 0 \\
\left.\frac{\partial p_j}{\partial l_j}\right|_{l_j=1}\left(1 - \frac{u'(c_1)}{f'(g_1)}\frac{J}{J-1}\right) + \beta G_1 \frac{J}{J-1} &= 0 \\
\frac{J-1}{J} - \frac{u'(c_1)}{f'(g_1)} + \frac{\beta G_1}{\left.\frac{\partial p_j}{\partial l_j}\right|_{l_j=1}} &= 0 \\
\frac{J-1}{J} - \frac{\beta G_1}{\left.\frac{\partial p_j}{\partial N_j}\right|_{l_j=1}} &= \frac{u'(c_1)}{f'(g_1)}
\end{aligned}$$

Note that if the level of investment implied in the above is such that the second generation will have no additional incentive to invest, then this will indeed characterize the equilibrium.

On the other hand, if the second generation would still invest given this level of investment, then the symmetric equilibrium must be characterized by each district choosing to invest exactly as much as necessary so that the second generation does not invest. If each of them were investing more, then their investment levels would have to satisfy the above first-order condition, and the level of investment implied by the firs order condition, by assumption, is less than that necessary to incentivize the second generation not to invest. ■

Proof of Propostion 5:

Proof. The problem of the first generation is to solve

$$\max_{D_1, G_1}\left\{\hat{u}\left(\hat{w} + p_1 + D_1 - G_1\right) + \hat{f}\left(G_1\right)\right\}$$

subject to the equilibrium conditions that the land price is determined in a market equilibrium and utility is equalized across districts. Substituting this in, we have

$$\max_{l_1, G_1}\left\{\hat{u}\left(\hat{w} + l_1^{-1}\left(w - u^{-1}\left(\begin{array}{c} u\left(w - (p_j + D_j)\, l_j\right) + \\ v(l_j) - v(l_1) + \\ f(g_j) - f(g_1)\end{array}\right)\right) - G_1\right) + \hat{f}\left(G_1\right)\right\}$$

Taking the first order condition with respect to G_1, we have

$$\begin{aligned}
\hat{u}'(\hat{c}_1)\left(l_1^{-1}\left(u^{-1}\right)'\left(u(c_1)\right) f'(g_1)\, l_1^{\beta} - 1\right) + \hat{f}'(G_1) &= 0 \\
N_1^{1-\beta}\frac{f'(g_1)}{u'(c_1)} + \frac{\hat{f}'(G_1)}{\hat{u}'(\hat{c}_1)} &= 1
\end{aligned}$$

which shows that each distrcit will invest efficiently in the IPG, given their population in equilibrium.

Taking the first order condition with respect to l_1, we have

$$\hat{u}'(\hat{c}_1)\left(w-u^{-1}(c_1)+l_1\left(u^{-1}\right)'(c_1)\left(\begin{array}{c}-u'(c_j)\left(p_j+D_j+\frac{\partial p_j}{\partial l_j}l_j\right)\frac{\partial l_j}{\partial l_1}\\ +v'(l_j)\frac{\partial l_j}{\partial l_1}-v'(l_1)\\ +\beta l_j^{\beta-1}G_jf'(g_j)\frac{\partial l_j}{\partial l_1}-\beta l_1^{\beta-1}G_1f'(g_1)\end{array}\right)\right)=0$$

so noting that $\hat{u}'(\hat{c}_1)>0$ and that at a symmetric equilibrium we have $l_1=l_j=1$, $G_1=G_j$, $\frac{f'(g_1)}{u'(c_1)}=1-\frac{\hat{f}'(G_1)}{\hat{u}'(\hat{c}_1)}$, $c_1=c_j$, and $\frac{\partial l_j}{\partial l_1}=-\frac{1}{J-1}$, hence

$$\begin{aligned}p_1+D_1+\left(\frac{(p_1+D_1)}{J-1}+\frac{\left.\frac{\partial p_j}{\partial l_j}\right|_{l_j=1}}{J-1}-\frac{J}{J-1}p_1-\frac{J\beta G_1}{J-1}\frac{f'(g_1)}{u'(c_1)}\right) &= 0\\ D_1 &= \beta G_1\frac{f'(g_1)}{u'(c_1)}+\frac{\left.\frac{\partial p_j}{\partial N_j}\right|_{N_j=1}}{J-1}\end{aligned}$$

■

Proof of Propostion 6:

Proof. It was shown in the text that debt was irrelevant for outcomes in real variables, and we shall assume that all districts choose $D_j=0$ for notational convenience. Consider the case when the second generation does not invest. The problem of the district is thus

$$\max_{G_1}\left\{\hat{u}(\hat{w}+p_1-G_1)+\hat{f}(G_1)\right\}$$

subject to (18),

$$u(w-p_1l_1)+v(l_1)+f(g_1)=u(w-p_jl_j)+v(l_j)+f(g_j)$$

and so substituting in we have

$$\max_{l_1}\left\{\begin{array}{c}\hat{u}\left(\hat{w}+p_1-l_1^{-\beta}f^{-1}\left(\begin{array}{c}u(w-p_jl_j)+v(l_j)+f\left(l_j^{\beta}G_j\right)-\\ (u(w-p_1l_1)+v(l_1))\end{array}\right)\right)+\\ \hat{f}\left(l_1^{-\beta}f^{-1}\left(\begin{array}{c}u(w-p_jl_j)+v(l_j)+f\left(l_j^{\beta}G_j\right)-\\ (u(w-p_1l_1)+v(l_1))\end{array}\right)\right)\end{array}\right\}$$

Taking the first order condition with respect to l_1, we have

$$\begin{aligned}\hat{u}'(\hat{c}_1)\frac{\partial p_1}{\partial l_1}+\left(\hat{u}'(\hat{c}_1)-\hat{f}'(G_1)\right)\left(\begin{array}{c}\beta l_1^{-1-\beta}G_1+\\ l_1^{-\beta}\left(f^{-1}\right)'(f(g_1))\left(\begin{array}{c}\left(\begin{array}{c}u'(c_j)\left(p_j+l_j\frac{\partial p_j}{\partial l_j}\right)-\\ v'l_j-\beta l_j^{\beta-1}G_jf'(g_j)\end{array}\right)\frac{\partial l_j}{\partial l_1}-\\ u'(c_1)\left(p_1+l_1\frac{\partial p_1}{\partial l_1}\right)-v'(l_1)\end{array}\right)\end{array}\right) &= 0\\ \frac{\partial p_1}{\partial l_1}+\left(1-\frac{\hat{f}'(G_1)}{\hat{u}'(\hat{c}_1)}\right)\left(\begin{array}{c}\beta l_1^{-1-\beta}G_1+\\ \frac{l_1^{-\beta}}{f'(g_1)}\left(\begin{array}{c}\left(u'(c_j)l_j\frac{\partial p_j}{\partial l_j}-\beta l_j^{\beta-1}G_jf'(g_j)\right)\frac{\partial l_j}{\partial l_1}-\\ u'(c_1)l_1\frac{\partial p_1}{\partial l_1}\end{array}\right)\end{array}\right) &= 0\end{aligned}$$

In any symmetric equilibrium $l_1 = l_j = 1$, $G_1 = G_j$, $c_1 = c_j$, and $\frac{\partial l_j}{\partial l_1} = -\frac{1}{J-1}$, hence

$$\begin{aligned}
\left.\frac{\partial p_j}{\partial l_j}\right|_{l_j=1} + \left(1 - \frac{\hat{f}'(G_1)}{\hat{u}'(\hat{c}_1)}\right)\left(\beta G_1 - \frac{1}{f'(g_1)}\left(\begin{array}{c} \left(u'(c_1)\left.\frac{\partial p_j}{\partial l_j}\right|_{l_j=1} - \beta G_1 f(g_1)\right)\frac{1}{J-1} + \\ u'(c_1)\, l_1 \left.\frac{\partial p_j}{\partial l_j}\right|_{l_j=1} \end{array}\right)\right) &= 0 \\
\left.\frac{\partial p_j}{\partial l_j}\right|_{l_j=1}\left(1 - \frac{u'(c_1)}{f'(g_1)}\frac{J}{J-1}\right) + \beta G_1 \frac{J}{J-1} &= 0 \\
\frac{J-1}{J\left(1 - \frac{\hat{f}'(G_1)}{\hat{u}'(\hat{c}_1)}\right)} - \frac{u'(c_1)}{f'(g_1)} + \frac{\beta G_1}{\left.\frac{\partial p_j}{\partial l_j}\right|_{l_j=1}} &= 0 \\
\frac{J-1}{J\left(1 - \frac{\hat{f}'(G_1)}{\hat{u}'(\hat{c}_1)}\right)} - \frac{\beta G_1}{\left.\frac{\partial p_j}{\partial N_j}\right|_{l_j=1}} &= \frac{u'(c_1)}{f'(g_1)}
\end{aligned}$$

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