The Impact of the Internet on Advertising Markets for News Media

by

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We provide a model of the market for advertising on news media outlets when consumers have opportunities to switch between outlets. We hypothesize that the move to online news content has facilitated greater consumer switching, as well as heterogeneity in consumer switching patterns. The news outlets are modeled as competing two-sided platforms bringing together heterogeneous, partially multi-homing consumers with advertisers with heterogeneous valuations for reaching consumers, and the multi-homing behavior of the advertisers is determined endogenously. The presence of switching consumers means that, in the absence of certain consumer tracking technologies, scarce advertising capacity is taken up by advertisers purchasing wasted impressions on outlets, as a given advertiser may reach the same consumer too many times. This has subtle effects on the equilibrium price for ad impressions and the profits of outlets, and it may lead to heterogeneity in the multi-homing behavior of advertisers. We characterize the impact of greater consumer switching on outlet profits and the impact of technologies that track consumers both within and across outlets on those profits. Somewhat surprisingly, superior tracking technologies may not always increase outlet profits. In addition, we analyze the impact of blogs, aggregators and paywalls on outlet profits from advertising, which ultimately determine market structure and outlet quality investment.

1   Introduction

The issue of whether the Internet will permanently destroy the news media is currently a big news topic. This is, in part, a response to falling advertising revenues and issues associated with the loss of related services (such as classified ads) that shared the costs associated with news delivery. However, it is also a reflection of the increased competition for attention that news outlets face from new media (including web-only news, blogs and news aggregators). This underlies concerns that a loss of complementary revenue as well as fragmentation in the media might undermine incentives to invest in quality journalism.

While new technologies and competition can often explain why individuals firms may see decreased revenue and profitability, what is important about the adverse impact of the Internet on the news media is that it is widespread – specifically, with the decline in total advertising revenue.[1] This represents an economic puzzle because, in many respects, the fundamental drivers of supply and demand appear to be as strong if not stronger than before. To see this, consider the alternative assertion that advertising revenues are being destroyed by the Internet because of the flood of available advertising space. From the *New York Times*,

> *… online ads sell at rates that are a fraction of those for print, for simple reasons of competition. "In a print world you had pretty much a limited amount of inventory — pages in a magazine," says Domenic Venuto, managing director of the online marketing firm Razorfish. "In the online world, inventory has become infinite." (Rice, 2010)*

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[1] According to the Newspaper Association of America (www.naa.org), since 2000 total advertising revenue earned by its member US newspapers declined by 57% in real terms to be around $27 billion in 2009. Much of this decline was in revenue from classifieds but total display advertising revenue fell around 40%. In contrast, circulation over the same period declined by 18%. Ad revenue as a share of GDP also declined by 60%. According to ComScore, total US display advertising revenue online was around $10 billion in 2010 which includes all sites and not just newspapers.

While there may be space for every advertiser on the Internet, it is still the case that effective advertising inventory requires those ads to be viewed by an actual consumer. The attention of those consumers is still limited and that scarcity that will limit the available advertising capacity. Moreover, it is that attention that advertisers still wish to compete for and hence, the price of ads is unlikely to become zero.

Moving beyond this, while it is true that delivery of content and advertising is less costly this should not reduce overall profitability in the industry. Moreover, while the Internet had afforded advertisers new options to each consumers (e.g., through search-related ads), though significant, it is suggested that this has opened up opportunities to new advertisers rather than changed the strategy of existing ones.[2] Countering this are improved metrics regarding ad impressions and performance as well as new opportunities to target consumers in advertising choices (Evans, 2009).[3]

The puzzle of the adverse impact of the Internet is related to long-standing puzzles in media economics. The standard approach (e.g., Anderson and Coate, 2005) has media outlets competing for consumers and then selling advertisers access to those consumers. On the advertising side of the market, even if there is fragmentation amongst outlets, total advertising revenue in the industry would reflect monopoly levels. Indeed, if anything, competition amongst media outlets predicts higher ad prices as those outlets scale back levels of annoying advertising. Nonetheless, there is evidence that competition is associated with falling ad prices including mergers that increase them (Anderson, Foros and Kind, 2010). In addition, traditional media economics predicts that ad revenue per consumer should equalize across outlets (that is, attention

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[2] Evans (2008, 2009)

[3] Some hypothesize that online or digital ads are far less effective than ads that are on paper. However, to date, the evidence is not consistent with that hypothesis (see Dreze and Hussherr, 2003; Lewis and Reiley, 2009; Goldfarb and Tucker, 2010).

is worth the same regardless of where it is allocated) whereas there is evidence that larger outlets command a premium suggesting an unmodeled source of market power.[4] Finally, rather than welcome policy moves to require public broadcasters to raise revenue from ads rather than be subsidized, existing media outlets have typically opposed the lifting of advertising restrictions.[5] All of these factors suggest that competition in advertising markets is not working in the manner that traditional media economics predicts.

This paper's aim is to present a formal analysis of the prospects for advertising-funded content on the Internet. One important starting point is that an economic analysis of advertising markets should be grounded in fundamentals – namely, of supply and demand – so that changes, such as those brought about by the Internet, can be examined in a consistent manner. Our model set-up (in Section 2) stresses that fixed nature of consumer attention as well as the constant demand from advertisers for that attention. Thus, rather than seeing advertising as a revenue stream accompanying consumers, we model revenue as arising from equilibrium outcomes in the market for advertising.

We demonstrate that there are two model elements – imperfect consumer tracking and increased consumer switching – that can together lead to outcomes that match the stylized facts as we have stated them – namely, that increases in outlet competition can reduce ad prices and total ad revenue and consequently that the Internet can account for these recent changes in the newspaper industry. First, consider the problem of consumer switching.

> *Newspaper readers are "better" than Web visitors. Online readers are a notoriously fickle bunch, and apparently are getting more so by the day. Web visitors barely stick around, yet they are counted in broad traffic statistics as if*

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[4] Recently, this has been referred to as the "ITV Premium Puzzle" (Competition Commission, 2003). However, the relationship has been noticed previously by Fisher et.al. (1980) and Chwe (1988).
[5] Ambrus and Reisinger (2006) document the opposition of German broadcasters to allowing public television broadcasters to show advertisements after 8pm.

they were the same as the reader who lingers over his Sunday paper. (Farhi, 2009)

This reflects the proposition that the web enables consumers to more readily switch between outlets. In the offline world, consumers of print and other media would face some constraints in accessing news and other content from multiple sources. This is not to say that consumers literally allocated all of their attention to one outlet, but just that their ability to switch between outlets and bundle a variety of content was limited in comparison to their options today. Thus, while consumers may have spent 25 minutes reading the morning print newspaper, they may spend on average 90 seconds on a news website (Varian, 2010). This is not a reduction in the amount of consumption but instead a reduction in 'loyalty' to any one outlet. Web browsers make it easy for consumers to move between outlets while free access removes other constraints. But, going beyond this, intermediaries such as search engines, aggregators and social networks facilitate switching. Indeed, we examined empirically the news consumption patterns of several million internet users, and found that among users who consumed at least 10 news articles per week, the concentration of a user's consumption among different news outlets, as measured by a news consumption Herfindahl index, was strongly and negatively associated with the users' frequency of using Google news and Bing news.[6]

Second, consider the problem of imperfect tracking. We postulate that outlets have a superior ability to track the behavior of consumers within their outlets rather than between them.[7] If consumers were to visit only one outlet, this would not be an issue for advertisers. If they wanted to reach many consumers, they could purchase impressions on a wide number of outlets (i.e., multi-home) and achieve those goals. However, when consumers switch between outlets,

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[6] See also Chiou and Tucker (2010) for additional evidence that news aggregators facilitate consumer switching between outlets.
[7] This is consistent with current practice (Edelman, 2010).

advertisers have a harder task. An advertiser who multi-homes will find that some of their impressions are viewed by the same consumer more than once as they switch between outlets. Those impressions are potentially wasted.[8] Maximizing ad reach now carries the additional cost of paying for wasted impressions. In contrast, an advertiser who single-homes, will miss some proportion of consumers even if they can minimize payments for wasted impressions.

Consumer switching and imperfect tracking together interact to generate an outcome whereby an increase in consumer switching leads to a reduction in impression prices as advertisers are not willing to pay as much due to the potential waste. This result is derived in an environment whereby outlets are symmetric (both in terms of readership and the number of ads they can place in front of consumers) and without requiring any change in their number – although an increased number of outlets will, in the context of this model, reduce total advertising revenues further. Consequently, not only can these elements provide a consistent story of the impact of the Internet on the news media but also rationalize the lack of a similar competitive outcome in traditional economics models of media competition.

To be sure, it is worth emphasizing here that it is *the combination* of consumer switching and imperfect tracking that generates this outcome. While most models in the media economics literature assume that consumers single-home – that is, choose to allocate attention to only one outlet – there are some that have considered what happens when consumers multi-home. Gabszewicz and Wauthy (2004) and Anderson and Coate (2005) considered this but demonstrated that advertisers would all single-home in this case resulting in no change in overall

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[8] Some advertisers target an optimal number of impressions per consumer that is greater than one. Imperfect tracking makes it difficult to target that optimal number of impressions, however, for concreteness in our model we set the optimal number equal to one.

advertising revenues.[9] Recently, Ambrus and Reisinger (2006) considered a model of horizontally differentiated outlets whereby only some share of consumers multi-homed; specifically, consumers who are on the margin of choosing one outlet or the other. They then posited that those consumers were less valuable to outlets than consumers who single-homed. Consequently, outlets adjust their advertising levels (creating more annoying ads) to reduce consumer multi-homing. The overall impact on prices is ambiguous but competition does reduce outlet profits in their model.[10]

As noted earlier, we move away from the notion that consumers come to outlets with an associated revenue stream and instead model revenue as arising from the effective impressions advertisers are able to procure. This involves constructing a model whereby consumers may switch outlets within the time period advertisers want to place impressions in front of them. This requires us to consider the mixed single and multi-homing consumer outcomes and to solve for the resulting equilibrium in the advertising market. The modeling challenge arises because the price that clears the market also impacts on the 'quality' of likely matches between consumers and advertisers. Nonetheless, we demonstrate that a sorting equilibrium exists where high value advertisers multi-home and, in some cases, increase the frequency of impressions so as not to miss consumers, while lower value advertisers single-home. As the share of switching consumers rises, advertisers prefer to single rather than multi-homing. This frees up ad capacity on each outlet for lower valued advertisers who set the price in the market. Consequently, prices and total ad revenue decline.

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[9] Ashlagi, Edelman & Lee (2010) examine competing ad auctions for search engines where consumers single-home but advertisers face costs that make multi-homing costly.

[10] Anderson, Foyos and Kind (2010) develop a related model but focus on the impact of changes on consumer ad disutility of ad prices and outlet profits.

Interestingly, we demonstrate that this result is not straightforward and depends critically on the total available ad capacity. When that capacity is very high, while single-homing advertisers are always the marginal advertiser in the market, high value advertisers have an incentive to purchase multiple impressions and absorb inframarginal capacity. The balance between the marginal and inframarginal means that, in some cases, for fixed (and large) ad capacity, a higher ability of consumer to switch between outlets may be associated with higher, not lower, outlet profits. Indeed, profits may exceed levels that can be achieved when either switching or imperfect tracking is not a problem. Nonetheless, we demonstrate that it always the case that a reduction in competition (say through a merger) results in higher industry ad revenues.

The paper proceeds as follows. In Section 2, we set-up our baseline model of consumer attention and advertiser value focusing on the drivers of advertising demand and supply. Section 3 then considers benchmark cases including efficient allocations of advertisers to outlets, the single-homing consumer case (i.e., no consumer switching) and the no tracking case. We demonstrate that the latter cases cannot explain the stylized media facts and the impact of the internet.

Section 4 then outlines the advertiser's dilemma (in choosing to multi- or single-home) and turns to hypothesize the existence of a technology – perfect tracking – that would solve that dilemma. That is, we assume that a platform exists that can allow advertisers who place ads on outlets if the consumers they are impressing are really 'new' thereby eliminating any waste in the impressions they pay for. Such technologies have been heralded as ones that would allow the Internet to improve matching efficiency in advertising markets. In particular, it has always been the case that consumers' attention was divided amongst different outlets (e.g., newspaper, radio,

television) creating the problem of wasted impressions. Technology can resolve this online. Indeed, it has already begun with respect to geo-tracking that ensures that local advertisers placing ads on national websites are only delivering ads to local customers (Athey and Gans, 2010). However, in the future, it is conceivable that technologies could track a consumer across outlets very widely (from the net, to mobile phones, etc). Ad platforms may develop that serve many outlets and offer the following proposition: "place an ad with our platform and we will deliver it to the consumer exactly three times during this period on some outlet." We demonstrate that such technologies can generate outcomes whereby the degree of consumer switching does not lead to inefficiency and reduced advertising revenues. Indeed, it generates the straightforward business model for media outlets to focus on attracting more consumer attention as this will bring with it a revenue stream per consumer.

Section 5 then presents our model of imperfect tracking. This derives the results on competition as forecast above but also allows us to consider whether outlets will have an incentive to adopt tracking technologies or not. Indeed, we demonstrate that when the share of consumer switchers and ad capacities are relatively high, outlets may not be better off by adopting technologies that improve market efficiency. Nonetheless, we also show that mergers will increase outlet advertising revenue. In addition, we demonstrate that when some outlets cannot sell ads (as they might if they are regulated public broadcasters or smaller blogs) then this will lead to higher ad prices compared with situations where such restrictions are lifted. This is because those outlets capture consumer attention – reducing the supply of capacity that can be sold to advertisers in the market – and because movements to and from such outlets do not created wasted impressions – increasing advertisers' willingness to pay for impressions. Thus,

our model provides a rationalization of private media outlet objections against public broadcasters being allowed to sell ads.

Our baseline model assumed that outlets were symmetric. In Section 6 we relax this and demonstrate under what conditions outlets might have a positional advantage in advertising markets – thereby allowing them to earn higher advertising revenue per consumer than their rivals. This happens when one outlet has a lower ad capacity than the other although it may not increase their total profits. Significantly, an outlet with a higher readership can, in the face of consumer switching, command a higher impression price than its rival. This is because the marginal advertiser who is a single-homer in that case will prefer to purchase impressions on the outlet with the higher readership share and is willing to pay a premium to do so. Consequently, higher valued single-homing advertisers sort onto the high readership outlet first giving them a positional advantage. We demonstrate that the extent of this positional advantage can drive competition for those consumers and, indeed, may cause outlets to invest more in quality than they would under benchmark cases or perfect tracking.

We also demonstrate that an outlet can gain a positional advantage by having limited content but of high value to consumers – something we term magnet content. If outlets can ensure that a high share of consumers will at some point allocate attention to them, those outlets can command a premium in advertising markets and a positional advantage over outlets who provide a greater quantity of content. This suggests that outlets may focus their efforts on producing offerings that regularly attract the attention of many consumers rather than the focused attention of fewer consumers. Relatedly, we demonstrate that paywalls unilaterally imposed by an outlet can have the effect of reducing their positional advantage or giving their rivals a

positional advantage in advertising markets. As a result, we identify additional competitive costs to outlets from introducing paywalls.

A final section concludes and identifies paths to future research in this area.

2   Model Set-Up

We begin by setting out the fundamentals of consumer and advertiser demand and behavior that drive our model. These are the core elements that do not change as consumers face lower costs of switching between outlets (that is, as we move from a traditional advertising world to one delivered electronically).

2.1   Consumer Attention and Advertiser Value

Consumers have scarce attention that they devote to consuming various bits of media. In addition, consumers are potential purchasers of products and can be matched with firms through advertising. We assume that consumers purchase products at a slower rate than they consume media. For example, a consumer might purchase one soda in a day but have numerous opportunities to consume media over that same period of time. A soda-maker is concerned about putting an impression in front of a consumer sometime during the day and so is indifferent as to which period of the day that occurs. Consequently, what matters for advertising is the total attention a consumer devotes to viewing media over the course of a day. However, which media outlets consumers view during that day impacts upon the opportunities to place ads in front of them.

Formally, suppose there are T periods in which advertisers who put an impression in front of a consumer in that period receive a value (strictly, value of a lead), $v \in [0,1]$; distributed

identically across consumers with cumulative distribution function, $F(v)$.[11] The T periods may represent hours of the day or days of the week or something similar. It is assumed that a consumer can only read one unit of content per period. In this respect, T is a measure of the amount of consumer attention.

We let a_i be the quantity of advertising that outlet i presents to consumers per unit of time. We assume that all advertising is equally effective regardless of the quantity, and our base model ignores consumer disutility of ads for simplicity. We assume that ad capacity is exogenous (although we explore the robustness of our results to relaxing this assumption below). In each period that a consumer visits outlet i, a consumer is impressed by a_i ads and if that happens in all periods, the consumer is impressed by Ta_i ads. Ta_i is the total (maximum possible) amount of *advertising inventory* introduced to the market by outlet i, as well as the maximum quantity of advertisers who could possibly reach an *individual* consumer that stays with outlet i for all periods (if each advertiser purchased at most one ad per consumer). If this quantity is available, and advertisers are ranked by value in terms of rationing of access to consumer attention, then the marginal advertiser, v_i, is defined by $1 - F(v_i) = Ta_i$. We restrict attention, therefore, to cases where $\max_i a_i < 1/T$ so there is an interior solution.

2.2 Outlet Demand and Advertising Inventory

How do consumers allocate attention to different media outlets? We assume that whenever a consumer has an opportunity to choose, outlet i will be chosen with probability x_i. Thus, x_i is a measure of an outlet's intrinsic quality.[12] Note that, if a consumer chooses an outlet,

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[11] An alternative specification might have advertisers desiring to reach a specific number of consumers (Athey and Gans, 2010) or a specific consumer type (Athey and Gans, 2010; Bergemann and Bonatti, 2010).

[12] Outlet quality may be endogenous and below we will consider what happens when outlets invest in quality prior to competing for consumers. One modeling possibility is that this probability is drawn from a logit choice model that can be applied whenever a consumer has an opportunity to choose (or switch) outlets.

$i i \in \{1,...,I\}$, and then had no opportunity to switch thereafter, outlet i's maximum possible advertising inventory would be $x_i a_i T$.

We assume, however, that an opportunity for a consumer to switch outlets arrives (independently) each period with probability, ρ.[13] For convenience, throughout this paper we assume that $T = 2$ so, in effect, there is, at most, a single opportunity to switch.[14] Thus, the total expected amount of attention going to i is:

$$x_i + x_i\left((1-\rho)+\rho x_i\right)+(1-x_i)\rho x_i = 2x_i \tag{1}$$

However, for an outlet, what is important is distinguishing between consumers who are loyal to it (either by choice or lack of opportunity) and those who are expected to switch between outlets. Thus, suppose that D_i^l is the share of consumers loyal to i (i.e., single-homers) and D_{ij}^s is the share consumers who switch between outlets i and j (i.e., multi-homers) in any given period, then:

$$D_i^l = x_i - x_i(1-x_i)\rho \tag{2}$$

$$D_{ij}^s = 2\rho x_i x_j \tag{3}$$

Note that when there are no switching opportunities (i.e., $\rho = 0$), $D_i^l = x_i$ and $D_{ij}^s = 0$ for all $\{i,j\}$.

We observe that for much of our analysis, because we take outlet size (x_i) to be exogenous, the key variables are the proportion of switchers and the proportion of loyal consumers for each outlet; the specific model of consumer switching does not affect the results.

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[13] Here we treat this probability as independent of history (i.e., outlets a consumer may have visited earlier) or the future (i.e., outlets that they may visit later). Below we explore the implications of relaxing this assumption.
[14] If there are more than two periods, then there are many opportunities to switch, and there will be many different types of switchers, e.g. those who spend different fractions of their time with different outlets. This complicates the modeling substantially without changing the basic economic tradeoffs.

3   Benchmarks

To begin, it is useful to examine two benchmarks for the advertising market: the first-best and pure single-homing consumers. This will allow us to examine the impact of possible tracking technologies as well as the impact of the Internet when such technologies are not available.

3.1   First Best Allocation

Given outlet quality and consumer behavior (in switching), note that consumers loyal to an outlet i will generate $2a_i$ in advertising inventory while a consumer switching between outlets i and j will generate $a_i + a_j$ in advertising inventory. As there are I outlets, this means that there are $I + \frac{1}{2}I(I-1)$ different consumer types (i.e., I loyals and the remainder switchers between two different outlets).

As each advertiser's value is constant in the number of consumers reached, we can consider allocating advertisers to each consumer separately; as the only distinction among consumers is their "switching type," we consider allocating advertisers to each such type. To achieve first best the highest value advertisers should be allocated first to advertising inventory generated by each switching type. Let v_i denote the marginal advertiser allocated to consumers loyal to outlet i and let $v_{s,ij}$ denote the marginal advertiser allocated to consumers who switch between outlets i and j (in most of the analysis that follows we will consider only two outlets so that, with only two periods, we can drop the ij subscript on switchers). Then a first best allocation involves allocating all advertisers with $v \geq v_i$ to outlet i's loyal consumers and those with $v \geq v_{s,ij}$ to those who switch between i and j. This is done for all i and all $\{i, j\}$ with $i \neq j$. Thus, the marginal advertisers will be determined by: $2a_i = 1 - F(v_i)$ and $a_i + a_j = 1 - F(v_{s,ij})$.

3.2   Pure Single-Homing Consumers

Another benchmark comes from traditional media economics that assumes that consumers pay attention to only a single outlet (e.g., Anderson and Coate, 2005); that is, in the language of the two-sided markets literature, all consumers single-home. Here, this corresponds to a situation where there are no opportunities for consumer switching (i.e., $\rho = 0$); say, corresponding to a world where consumers read one newspaper per day. We term this environment NS. In this case, outlet i attracts a x_i share of consumers and then sells its advertising inventory, $2x_i a_i$, to advertisers. In effect, this environment gives outlets a monopoly over access to a share of consumers and will set advertising pricing terms to reflect that.[15]

To see this, recall our assumption that advertisers place the same marginal value on reaching any consumer. Thus, advertisers will multi-home and pay for impressions on all outlets. Consequently, an advertiser with value, v, will advertise on any outlet whose impression price, p_i, is less than v.

There is an issue, however, in that when an outlet has many consumers, it needs to track when an ad is placed in front of a given consumer. This is an issue we will return to below. The common assumption is that *outlets can track consumers within their own outlets and so to access all an outlet's consumers an advertiser need only pay for one impression per consumer*. Thus, if it has advertising inventory of a_i per period, the market clearing price for outlet i is the p_i that satisfies $1 - F(p_i) = 2a_i$. If $P(z) \equiv F^{-1}(1-z)$ then $p_i = P(2a_i)$. Outlet i's profits will be:

$$\pi_i = x_i P(2a_i) 2a_i.$$

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[15] Note that this is the usual assumption in many models of media competition. For example, Anderson and Coate (2005) assume that broadcasters compete for viewers and then are able to earn an advertising revenue, $R(a)$ per consumer contingent upon the number of ads shown to them.

Note that, contingent upon the assumption that $\rho = 0$, this is a first-best allocation when advertising capacity is exogenous. Of course, if advertising capacity were endogenously chosen by each outlet, then the capacity chosen will be less than what would be chosen by a social planner—this is just a standard monopoly output problem, since each outlet acts as a monopolist over its loyal consumers. In much of our analysis we will consider the special case of advertiser values with a uniform distribution, $F(v) = v$. Then, as is standard, the monopoly problem for outlet i would choose a_i to maximize π_i resulting in $\hat{a}_i^{NS} = \frac{1}{4}$ and a profit level of

$$\pi_i^{NS} = x_i(1 - 2a_i)2a_i = x_i \frac{1}{4}.$$

3.3   No tracking:

Another benchmark is to consider what happens when outlets are unable to internally (or externally) track impressions and to control matching between advertisers and consumers. This is the assumption made by Butters (1977) and Bergemann and Bonatti (2010). In the early days of the Internet, websites had no ability to track consumers even within outlets and even today with privacy settings such tracking may not be possible. What that means is that if an outlet had x consumers in a given time period, if an advertiser placed n impressions on that outlet in that time period, the expected number of unique consumer impressions the advertiser would receive would be $x\left(1 - (1 - \frac{1}{x})^n\right) \approx x(1 - e^{-n/x})$.

To examine advertiser behavior in this environment, let each outlet's own shares of loyal and switching consumers' attention be:

$$y_i^l \equiv \frac{2D_i^l}{2D_i^l + D^s} \text{ and } y_i^s \equiv \frac{D^s}{2D_i^l + D^s} \tag{4}$$

If n_i is the number of impressions purchased on outlet i, an advertiser, v, solves the following:

$$\max_{\{n_1 \geq 0, n_2 \geq 0\}} \pi := D_1^l(1-e^{-n_1 y_1^l/D_1^l})v + D_2^l(1-e^{-n_2 y_2^l/D_2^l})v + D^s(1-e^{-(n_1 y_1^s + n_2 y_2^s)/D^s})v - n_1 p_1 - n_2 p_2 \quad (5)$$

Let $n^*(v, p_1, p_2) := (n_1^*(v, p_1, p_2), n_2^*(v, p_1, p_2))$ denote the solution this problem. Notice that if $\rho > 0$, then impressions on different outlets are (imperfect) substitutes from the advertisers' perspective as they allow them to reach the shared customers. The higher is ρ, the higher is the degree of substitutability. Using this, we can demonstrate the following:

Proposition 1a. *For all values* $v, p_1, p_2 \geq 0$, *a solution to the advertisers' problem exists and is unique. Suppose* $n^* \neq (0,0)$. *Then* $\frac{n_1^*}{2D_1^l + D^s} \geq \frac{n_2^*}{2D_2^l + D^s}$ *if and only if* $p_2 \geq p_1$. *Moreover* $\frac{\partial n_1^*}{\partial \rho} \geq 0$ *and* $\frac{\partial n_2^*}{\partial \rho} \leq 0$ *if and only if* $p_2 \geq p_1$.

The proofs of all propositions are in the appendix. The first part of the proposition says that if an advertiser is active (i.e. buys any impression), then the number of impressions purchased per unit of attention captured must be higher on the cheaper outlet. The second part says that the relative price pins down the comparative statics. To build intuition, consider the case of equal prices $p_1 = p_2 = p$ which implies that $n_1^*/n_2^* = (2D_1^l + D^s)/(2D_2^l + D^s)$. Using this fact we can easily demonstrate that advertisers choose a number of impressions on each outlet directly proportional to the outlet's attractiveness, x_i. Specifically if $p_1 = p_2 = p$ then:

$$n_1^* = x_1 \ln[v/p], \qquad n_2^* = x_2 \ln[v/p],$$

if $v \geq p$ and zero otherwise. Note that the optimal advertising strategy as well as advertiser surplus equal to $(1-e^{-\ln[v/p]})v - \ln[v/p]p$ are independent of the share of switchers. On the other hand if $p_1 \neq p_2$, then switching matters as advertisers substitute away impressions on the more expensive outlet. For instance, if $p_2 > p_1$, then $n_1^*/(2D_1^l + D^s) > n_2^*/(2D_2^l + D^s)$. Higher types, who have a higher opportunity cost of missing users, multi-home, whereas lower types with

$v \geq p_1$ single-home. A higher ρ reduces the relative attractiveness of the more expensive outlet 2 in favor of outlet 1 since now a larger fraction of users are shared.

The above analysis implies that switching and asymmetries in readership shares affect the advertisers' incentives only due to "economic" and not "technological" considerations. Thus, absent supply-side asymmetries to account for gaps in market prices, equilibrium total advertising revenues will be constant in D^s and x_i will only determine how of this amount is allocated to each i.

Proposition 1b. *Suppose that $a_1 = a_2 = a$. Then outlets' profits are independent of ρ and total industry revenue per consumer is equal to $2ap^*$ where p^* is the unique solution to*

$$2a = \int_p^1 \ln(v/p)dF(v).$$

Thus, an increasing share of switchers will not account for declining ad revenue in the no tracking case despite asymmetries in readership shares. Asymmetry is something Bergemann and Bonatti (2010) assume and they demonstrate that adding more outlets with increasing asymmetric readership shares can *increase* total advertising revenue by virtue of facilitating improved targeting. They assume, however, that all consumers read one outlet while some also switch to other outlets. Thus, in their case, there is a hierarchy in readership preferences.

4   Perfect Tracking

4.1   The Advertiser's Dilemma

When there are no switching consumers, an advertiser who places an ad on one outlet impresses those consumers only and one that places ads on multiple outlets, impresses all of the consumers of those outlets. Importantly, none of those impressions is wasted and no consumer

on an outlet is missed. Consequently, save for any shortfalls in ad capacity, all advertisers multi-home.[16]

When consumers switch between outlets, advertisers, in general, face a dilemma. If advertisers multi-home, they access all of the loyal consumers on each outlet but they may only reach a fraction of the switchers. While some switchers may see distinct ads when they traverse between outlets, others may see the same ad from a multi-homing advertiser twice. The advertiser then faces a trade-off. If advertisers multi-home, they impress all of the loyals but pay for some wasted impressions on switchers. Moreover, they are not necessarily guaranteed impressing all switchers by this strategy. Some advertisers may then prefer to single-home; sacrificing loyals on another outlet but not wasting any impressions. Other advertisers may decide to increase the number of impressions across all outlets. This increases their number of wasted impressions in return for impressing a greater proportion of switchers.

Given the two period structure of attention, one might think that this dilemma could be resolved by *coordinating on a time period*. For instance, an advertiser could pay for impressions only in the first period across all outlets and none in the second. However, this would require that consumers were overlapping completely in time in terms of the reading habits.[17] Instead, there is nothing in the two period structure that requires such synchronization. Consequently, we assume that coordination of impressions in a given period of time will not resolve the advertiser's dilemma.

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[16] If ad capacities differ between outlets, then, by definition, there must exist some advertisers who do not multi-home.

[17] In the context of coordinating attention, the Superbowl commands such a large share of attention at a given period of time that advertisers can be assured of impressing that share of consumers. Consequently, the coordination opportunity afforded by this may be a reason why ad space commands such high payments per viewer during that event. We explore a similar effect below.

Advertisers face a dilemma not so much because consumers may see repeat impressions (although in a broader model this may increase annoyance and be harmful) but that they pay for them when they do not add value. This is a consequence of our assumption that advertisers pay per impression (as is common in display advertising both online and offline). Instead, advertisers could pay per click (as is common in sponsored search advertising). In this case, they would not be paying for waste as it could be presumed that consumers may only click once. However, it may be that the value of a lead from an impression is not the rate of click-through but the display itself. In this case, under common pay per click algorithms, fewer impressions will be given compared to what the advertiser would find optimal. Thus, pay per click pricing is not likely to resolve the issue.

The advertiser's dilemma arises when outlets cannot easily track consumers as they move across outlets. Indeed, as noted earlier, in some situations outlets may face difficulties in tracking consumers within an outlet. We will explore the precise nature of this below. First, however, it is instructive to consider cases where there are switching consumers but where the missed/wasted impressions problem does not arise. This will allow us to consider the impact of a technological 'benchmark' on the efficiency of advertising markets and competition within them.

4.2   Perfect ad-tracking

The advertiser's dilemma arises because, when an advertiser purchases impressions, they cannot tell whether those impressions will be placed in front of unique consumers or not. Here we imagine a technology – the elements of which currently exist (at least online) but the implementation is far from achieving ideal working – whereby consumers can be tracked both within and across outlets with information kept as to the ads they have seen. In this situation, a consumer could be impressed by an ad at most once and advertisers could, with certainty, pay for

an impression to a consumer and receive it.[18] Thus, there are neither wasted impressions nor missed impressions. We term this *perfect ad-tracking*, as the advertising platform (or broker or exchange) is able to track consumers across web-sites and control the ads they see in a given period of time. Here, we assume that this service is provided competitively and we assume that advertisers pay only for an impression. This too is a heroic assumption that we relax in a later section.

As noted earlier, there are $I + \frac{1}{2}I(I-1)$ types of consumer; I who single-home on a given outlet and the remainder who switch between two outlets. Perfect ad-tracking means that each consumer can be tracked and so the platform can charge advertisers for access to that consumer. That is, the platform can *price discriminate* based on consumer-type. We assume in this section that the outlets choose a single level of ad capacity for all consumers, and sell those impressions using the ad platform.[19,20]

For instance, a consumer single-homing on outlet i, will generate $2a_i$ in advertising inventory. Advertisers will choose to advertise to a consumer so long as their value exceeds the impression price. Consequently, the price per impression to a single-homer on outlet i, p_i, will be

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[18] Of course, some advertisers may have an optimal number of impressions per consumer other than one. The technology could ensure that optimum so, without loss in generality, we restrict that optimum to one here.

[19] To see how this would work, consider the allocation and pricing problem faced by the ad platform. Consumers who end up loyal to the highest-capacity outlet see ads from the largest interval of advertisers, while consumers who end up loyal to the lowest-capacity outlet see ads from the smallest interval of advertisers, but allocative efficiency requires that all consumers see ads from the highest-value advertisers. The challenge is that before the resolution of switching behavior, the total set of advertisers a consumer should see and the market-clearing price cannot be determined. In a stable environment, the ad platform can offer a set of prices for each type of consumer, such that supply equals demand for each type. In the first period, the platform allocates the highest-value advertisers first to each consumer (as revealed by their willingness to place an order for the most expensive consumer types). Then in the second period, the ad platform knows the total supply of ad space for each consumer and allocates the remainder of the advertisers who place an order for those types of consumers.

[20] An alternative (but probably less realistic) assumption would be that the ad platform shares information with the outlet about the consumer type, so that the outlet can set different capacities for different types.[20] This additional flexibility would lead to a scenario with essentially distinct markets, so that firms compete for switchers and but have a monopoly over access to loyal users. It is a bit more complicated to think how this would work in practice, since consumer types would only be fully determined in the second period, after the consumer had already experienced a first-period ad capacity. We omit the formal analysis of this case.

determined by $1 - F(p_i) = 2a_i$. In contrast, a multi-homing consumer, switching between outlets i and j, generates $a_i + a_j$ units of advertising inventory and so the price per impression on them is determined by $1 - F(p_{ij}) = a_i + a_j$. Note that this allocation corresponds to the first-best allocation. Note also that this implies that, if $a_i = a_j$, then $p_i = p_j = p_{ij}$. In contrast, if $a_i > a_j$, then $p_i < p_{ij} < p_j$.

It is useful to emphasize here that competition between outlets over advertisers for multi-homing consumers is limited by the number of attention periods. In this respect, when the advertising platform distinguishes between switchers moving between outlets, the number of advertising impressions it can sell is limited to the capacities of those two outlets. If there were more attention periods, then for some switchers who traverse more outlets advertising capacity will be supplied by a greater number of outlets. As we observe below, when advertising capacity is endogenous, this will drive more competition between outlets in the provision of such capacity.

In a given period, outlet i receives all of its loyal consumers, D_i^l, and half of the switchers between it and a given outlet j, D_{ij}^s. Given this specification, outlet i's profits are:

$$\pi_i = \sum_{j \neq i} P(a_i + a_j) a_i D_{ij}^s + P(2a_i) 2a_i D_i^l \tag{6}$$

Examining this profit function, it is easy to see that an increase in ρ causes a greater share of consumers to become switchers. An outlet i will benefit from this change if they earn more, on average, from switchers than from loyal consumers; i.e., if and only if:

$$\sum_{j \neq i} \left(P(a_i + a_j) - P(2a_i) \right) x_j > 0 \tag{7}$$

In particular, an outlet with a high capacity relative to other outlets that have a relatively large readership share will become more profitable as a greater number of consumers become multi-homers. The opposite is the case for an outlet with relatively low advertising capacity. This insight leads to the following result.

Proposition 2. *For an outlet i with $a_i \geq \max_j a_j$ ($a_i \leq \min_j a_j$), π_i is non-decreasing (non-increasing) in ρ. If all outlets have equal advertising capacities, then π_i does not change with ρ for all i.*

The result is due to an externality that the 'high capacity' outlets exerts on the low capacity one through p_{ij}. As ρ increases, the low capacity outlets lose their most valuable readers in favor of the high capacity ones and vice versa. Since the only impact of switching is to change the total capacity of ads a consumer sees, if all outlets have the same capacity, clearly switching has no impact when capacity is exogenous.

It is then straightforward to compare the profits achieved under perfect ad-tracking with those that arise in the benchmark case with single-homing consumers.

Proposition 3. *If $\rho = 0$ and/or $a_i = a$ for all i, then expected outlet profits under perfect ad tracking is the same as the benchmark case with single-homing consumers. Otherwise, there exists, under perfect ad-tracking, at least one outlet whose profits will be higher and one whose profits will be lower than the benchmark case.*

Note that when $\rho = 0$, there are no switchers and profits in (6) equal $\pi_i = x_i P(2a_i)2a_i$, the profits in the benchmark case. Similarly, if $a_i = a$, then (6) becomes $\pi_i = P(2a)a\left(\sum_{j \neq i} D_{ij}^s + 2D_i^l\right) = x_i P(2a)2a$. Intuitively, when there is no switching and no ad tracking, an outlet earns revenue for each reader it attracts and can divide that revenue between the two attention periods. When there is perfect ad-tracking, the same can be achieved as the outlet is paid per reader attracted per period. As the expected attention the outlet received is the

same in both cases (that is, $2x_i$ units), their profits are the same. The final result is a direct corollary of Proposition 2.

Importantly, relative profits are not driven by differences of relative market shares. That is, with symmetric ad capacities, profits are proportional to x_i. Thus, acquisition of an additional unit of readership share by an outlet transfers the profits associated with that reader directly from another outlet, and the marginal acquisition value of a reader is $P(2a)2a$ regardless of how many readers an outlet already has. Assuming an exogenous level of ad capacity, this is precisely the same acquisition incentive that outlets in the case where consumers single home.

In the appendix, we examine the case where ad capacity is endogenous. It is demonstrated there that Cournot-like competitive outcomes result and become more intense as ρ becomes higher. In this situation, compared with the case of single-homing consumers, total outlet profits may be lower than when capacity is exogenous.

5   Imperfect Tracking

As argued in the introduction, outlets do not currently operate either at an extreme of not being able to track consumers nor are they able to track consumers across outlets. Instead, tracking is imperfect – being available to varying degrees internally to an outlet and unavailable externally. Here we examine the equilibrium in the advertising market that arises when tracking is imperfect. Our purpose is twofold. First, as discussed in the introduction, current models in media economics do not explain that fact that outlets believe that competition reduces total advertising revenues in the news media industry and that they believe mergers may improve those revenues. Relatedly, one of the primary factors associated with the Internet has been an increase in the ability of consumers to switch between outlets. Here we demonstrate that in a

model of imperfect tracking, this fact can explain declining total advertising revenue in the news industry.

Second, as noted earlier, one of the primary benefits of the Internet for the news media industry is to utilize tracking technology to ensure that there is a tighter match between advertisers and readers. By providing a model of advertising markets under imperfect tracking we can compare those outcomes to ones generated with the benchmark and perfect tracking cases. This will allow us to understand both the costs to the industry from increased consumer switching as well as the incentives to adopt tracking technologies.

5.1 Tracking technologies

There are distinct technologies that might be employed by outlets to tracking consumer internally. Here we describe those technologies and identify their common characteristics.

Content-Based Impressions: We begin our examination of imperfect tracking with content-based impressions. This is the basis most commonly associated with offline content. That is, print newspapers agree to place an advertiser's ad on a particular page associated with a given piece of content. Thus, all consumers reading that content view the ad. In the online world, the idea here is that an advertiser can bid for an impression tied to an item of content but cannot specify the content itself. That ability limits potential wasted impressions as it can be assumed that a consumer will view a piece of content only once. Nonetheless, advertisers might want to increase their number of impressions (that is, the number of different content pieces) on an outlet as these will increase the number of switchers the advertiser is likely to impress. This repetition, however, comes at the cost of wasted impressions on loyal consumers.

Table 1: Expected Advertiser Surplus under Imperfect Tracking			
Advertiser Choice	Content-Based Tracking	Frequency-Based Tracking	Internal Tracking

Single home on i, 1 impression	$\left(D_i^l + \frac{1}{2}D^s\right)(v-p)$	$(D_i^l + \frac{1}{2}D^s)(v-p)$	$(D_i^l + \frac{1}{2}D^s)(v-p)$
Single home on i, 2 impressions	$(D_i^l + \frac{3}{4}D^s)v - (2D_i^l + D^s)$	$(D_i^l + D^s)v - (2D_i^l + D^s)p$	$(D_i^l + D^s)(v-p)$
Multi-home, 1 impression each	$(D_1^l + D_2^l + \frac{3}{4}D^s)v - p$	$(D_1^l + D_2^l + \frac{3}{4}D^s)v - p$	$(D_1^l + D_2^l + \frac{3}{4}D^s)v - p$
Multi-home, 2 on i and 1 on j	$(D_1^l + D_2^l + \frac{7}{8}D^s)v$ $-(2D_i^l + D_j^l + \frac{3}{2}D^s)p$	$v - (2D_i^l + D_j^l + \frac{3}{2}D^s)p$	$v - (D_i^l + D_j^l + \frac{3}{2}D^s)p$
Multi-home, 2 impressions on each	$(D_1^l + D_2^l + \frac{15}{16}D^s)v - 2p$	$v - (2D_1^l + 2D_2^l + 2D^s)p$	$v - (D_1^l + D_2^l + 2D^s)p$

In order to reach a greater share of switchers, advertisers need to purchase more impressions on a greater range of content. Specifically, if an advertiser pays for n impressions, it will hit $(1-\frac{1}{2^n})D^s$ switchers. Table 1 states the expected advertiser surplus for five possible choices. Interestingly, like no tracking, under content-based tracking, the expected number of switchers is unchanged regardless of which outlet those impressions appear on; beyond, of course, the first on each outlet. Consequently, if you want to increase the probability of impressing a switcher, it is better to place an additional impression on the outlet with the lower number of loyal consumers as that determines the volume of impressions that are wasted but you still have to pay for.

This fact makes it difficult to characterize and establish the equilibrium in the market. Specifically, the just excluded advertiser in the market will be an advertiser who will want to single home on one outlet. This is the same for both outlets and so long as ad capacities are not too different across outlets, this will drive a common impression price, p, across outlets. The problem is that, given this common price, multi-homers with more than one impression on each outlet will want to purchase additional impressions on the smaller outlet (with the lowest D_i^l). Thus, in equilibrium, those additional purchases should be concentrated there. However, it is far from clear that there would exist a single market clearing price under that condition.

Frequency-Based Tracking: While placing impressions in association with content can ensure that some impressions are not wasted, there is clearly a game being played in targeting switching consumers. It can be imagined that the number of pieces of content exceeds the rate at which consumers might switch between sites – here a maximum of one time. Instead, we can imagine that outlets can offer to place impressions based on frequency. That is, an advertiser bids for a frequency of impressions on a given outlet – either one or two per attention period. If an advertiser bids for two, that means that all visitors to an outlet will be expected to see that impression but that loyal consumers will see it twice. Thus, it shares with content-based impressions that there are wasted impressions on loyal consumers but, at least for an individual outlet, does not create wasted impressions on switchers. Of course, as switchers move between sites, some waste may still occur. Table 1 lists the expected advertiser surplus. Importantly, the expected surplus an advertiser receives from multi-homing is less than the sum of single homing on each outlet with the same number of overall impressions.

Internal tracking: Perfect tracking is a platform technology that works because consumers are tracked across outlets. However, what if outlets themselves could more effectively track consumers? For instance, suppose that outlets could determine whether a consumer was impressed by an advertiser already and ensure that they were not impressed again. In effect, it could ensure that there were no wasted impressions on loyal consumers. For switchers, this would not be an issue for the outlet itself but would remain an issue across outlets. This type of tracking is currently offered by many outlets, and it is known in the industry as "frequency capping."[21]

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[21] Though ad platforms might also offer frequency capping across outlets, here we focus on capping impressions that individual consumers see within an outlet. This practice is outlined and analyzed by Ghosh et.al. (2009).

The idea would be that advertisers could apply for a greater frequency of impressions (2 rather than 1) but with the caveat that impressions to any one consumer are capped at 1. In this case, if both outlets did this, the expected surplus to advertisers depending on their choice would be as listed in Table 1. Notice that, in this case, there are no wasted impressions on loyal consumers. Indeed, with two impressions on a single outlet, advertisers have no wasted impressions. However, in order to target all consumers, advertisers must purchase impressions on both outlets and, in this case, they purchase some wasted impressions. In contrast to frequency-based impressions, however, they can target additional switchers without incurring any wasted impressions on loyal consumers. Nonetheless, the wasted impressions on switchers is sufficient to generate diminishing returns to multi-homing for advertisers.

The fact that the marginal advertiser in the market will single-home due to diminishing returns to multi-homing and the fact that multi-homers, if they exist, will be high value advertisers tells us something about the resulting market equilibrium. First, as the next result demonstrates, imperfect tracking allows a greater quantity of advertisers to access outlets.

Proposition 4. *Suppose there is (i) exogenous and symmetric advertising capacity; (ii) $\rho > 0$; and (iii) there is imperfect tracking. If, in equilibrium, advertiser, $v = 1$, purchases no more than one impression per consumer, the equilibrium number of advertisers active in the market is greater than when $\rho = 0$.*

The proof is a straightforward accounting exercise. Due to diminishing returns to multi-homing that arises when $\rho > 0$, the marginal advertiser in any equilibrium is a single-homer. As no advertiser single-homes when $\rho = 0$, and by the assumption that the highest value advertiser is multi-homing with only one impression per consumer (as they would do when there are no switchers), the total number of advertisers purchasing ad space necessarily increases when

$\rho > 0$. Note, however, that as the number of impressions are fixed (as supply is fixed), this implies that impression prices will be lower.

When the highest value advertiser purchases more than one impression per consumer, the result in Proposition 4 may not hold. As will be demonstrated below, this has implications for impression pricing. Put simply, as it is inframarginal advertisers who are most likely to buy multiple impressions per consumer on one or more outlets, marginal advertisers determine the market clearing price. Therefore, as the number of switchers becomes large, inframarginal demand becomes large enough that the marginal advertiser may have higher value than when there are no switchers. Consequently, impression prices may rise.

5.2   Market Equilibrium with Frequency-Based Tracking

Because the market equilibria under each type of imperfect tracking is driven by similar factors, we will concentrate here on the equilibrium under frequency-based tracking as this corresponds to what many online outlets are able to provide at the present time.

There are several important things to observe about expected advertiser surplus as listed in Table 1. First, that having 2 impressions on each outlet is purely wasteful and can be ruled out. Consequently, the highest number of impressions any advertiser will purchase is 3. Second, if outlets are symmetric (i.e., $D_1^l = D_2^l = D^l$ and $a_1 = a_2 = a$), multi-homing with one impression on each outlet is preferable to single-homing with two-impressions as $D^l > \frac{1}{4}D^s$ which is true as $D^s \leq \frac{1}{2}$. Finally, multi-homing (2 and 1) advertisers will, all other things equal, purchase their third impression on the outlet with the fewest number of loyal consumers.

Under symmetry, therefore, there is a clear ranking of options based on advertiser value. Low value advertisers prefer single-homing, then there is multi-homing (1 impression on each)

and then there is multi-homing (2 and 1) that may be chosen by the highest value advertisers. Assuming a single price for impressions across outlets (which arises under symmetry), the marginal multi-homing (2 and 1) advertiser is $v_{12'} = \frac{D_i^l + \frac{1}{2} D^s}{\frac{1}{4} D^s} p$ while the marginal multi-homing (1 and 1) advertiser is $v_{12} = \frac{D_j^l + \frac{1}{2} D^s}{D_j^l + \frac{1}{4} D^s} p$. The marginal advertiser in the market is a single-homer, $v_i = p$. Note that $v_{12'} > v_{12} \Rightarrow \frac{D_i^l + \frac{1}{2} D^s}{\frac{1}{4} D^s} > \frac{D_j^l + \frac{1}{2} D^s}{D_j^l + \frac{1}{4} D^s} \Rightarrow D_j^l > 0$ (under symmetry). There are therefore, three regimes of equilibrium possible: (i) if $v_{12} \geq 1$, both outlets will only have single-homing advertisers on them; (ii) if $v_{12'} \geq 1 > v_{12}$, there will be multi-homing advertisers on each outlet; and (iii) if $1 > v_{12'}$, there will be multi-homing advertisers (with additional impressions) on each outlet.

To solve for the market equilibrium, each outlet's demand has to equal its supply. For an outlet, its total supply of advertising inventory is given by:

$$2a_i D_i^l + a_i D^s \tag{8}$$

It will often be convenient in what follows to express variables in a per customer basis. In this case, total advertising inventory on outlet i is $2a_i$.

On the demand side, for each consumer it expects to attract, an outlet receives a share of single-homers $(F(v_{12}) - F(v_i))$, an impression from each multi-homer $(1 - F(v_{12})$ or $F(v_{12'}) - F(v_{12})$ as the case may be) and a further half (under symmetry) of multi-homers (if any) who have 2 impressions on one outlet $(1 - F(v_{12'}))$. Thus, outlet demand is:

$$\begin{array}{ll} (D_i^l + \frac{1}{2} D^s)\left(\sigma_i(F(v_{12}) - F(v_i)) + (1 - F(v_{12})) + \frac{1}{2}(1 - F(v_{12'}))\right) & \text{if } \frac{1}{2} D^s > p \\ (D_i^l + \frac{1}{2} D^s)\left(\sigma_i(F(v_{12}) - F(v_i)) + (1 - F(v_{12}))\right) & \frac{1}{2} D^s \leq p \end{array} \tag{9}$$

where

$$\sigma_i = \begin{cases} \max\left[\dfrac{2a_i-(F(v_{12'})-F(v_{12})+\frac{3}{2}(1-F(v_{12'})))}{2a_i-(F(v_{12'})-F(v_{12})+\frac{3}{2}(1-F(v_{12'})))+2a_j-(F(v_{12'})-F(v_{12})+\frac{3}{2}(1-F(v_{12'})))},0\right] & \text{if } \frac{1}{2}D^s > p \\[2em] \max\left[\dfrac{2a_i-(1-F(v_{12}))}{2a_i-(1-F(v_{12}))+2a_j-(1-F(v_{12}))},0\right] & \frac{1}{2}D^s \le p \end{cases} \tag{10}$$

That is, σ_i is outlet i's spare capacity after sales to multi-homing advertisers and we assume that single-homers are allocated in equilibrium to each outlet according to their spare capacity (if any). Under symmetry, if these are positive for both outlets then $\sigma_i = \frac{1}{2}$.

5.3   Symmetric outlets

To focus on the core drivers of competition in the advertising market, we consider here the symmetric case where $D_1^l = D_2^l = D^l$ and $a_1 = a_2 = a$. In this case, each outlet's available capacity is auctioned on a first price basis. The marginal single-homing advertiser on outlet i, v_i, will have a willingness to pay for an impression determined by their expected surplus of $(D^l + \frac{1}{2}D^s)(v_i - p)$ while the marginal multi-homing advertiser in the market, v_{12}, will have a willingness to pay for an impression on outlet i, determined by:

$$(2D^l + \tfrac{3}{4}D^s)v_{12} - 2(D^l + \tfrac{1}{2}D^s)p - (D^l + \tfrac{1}{2}D^s)(v_{12} - p) \tag{11}$$

while the marginal multi-homing (2 and 1) advertiser is determined by

$$(2D^l + D^s)v_{12'} - 3(D^l + \tfrac{1}{2}D^s)p - \left((2D^l + \tfrac{3}{4}D^s)v_{12'} - 2(D^l + \tfrac{1}{2}D^s)p\right) \tag{12}$$

These equations holding with equality determine the threshold values – $v_i = p$, $v_{12} = \frac{D^l + \frac{1}{2}D^s}{D^l + \frac{1}{4}D^s}p$,

and $v_{12'} = \frac{D^l + \frac{1}{2}D^s}{\frac{1}{4}D^s}p$ – that sort advertisers in the market.

We now turn to consider possible equilibrium allocations of advertisers to outlets. First, is it possible that $\sigma_1 = \sigma_2 = 0$ and there are only multi-homing advertisers in the market? For this to be an equilibrium, the willingness to pay of a multi-homing advertiser for an impression on an

outlet must exceed the willingness to pay of a single-homing advertiser for an impression on an outlet. That is, the following two inequalities (derived from (11)) must hold:

$$(D_1^l + \tfrac{1}{4}D^s)v_{12} - (D_1^l + \tfrac{1}{2}D^s)p_1 \geq (D_1^l + \tfrac{1}{2}D^s)(v_1 - p_1) \tag{13}$$

$$(D_2^l + \tfrac{1}{4}D^s)v_{12} - (D_2^l + \tfrac{1}{2}D^s)p_2 \geq (D_2^l + \tfrac{1}{2}D^s)(v_2 - p_2) \tag{14}$$

Note that the marginal advertiser on each outlet would have to be a multi-homer and so $v_i = v_{12}$.

Note also that because the 'just excluded advertiser' (with value $v_{12} - \varepsilon$) would be willing to pay that for a single impression on an outlet, $p_i > v_{12} - \varepsilon$ for each outlet. It is clear that as ε goes to zero, the willingness to pay of the just excluded advertiser to single-home exceeds the willingness to pay of the marginal multi-homing advertiser for its marginal impression. That is, the LHS of (13) and (14) becomes negative while the RHS is zero if $D^s > 0$. If $D^s = 0$ Consequently, at least one outlet must, in equilibrium, sell to single-homing advertisers. That advertiser sets the marginal price in the market. If $D^s = 0$, (13) and (14) hold with equality and so a pure multi-homing equilibrium can arise.

Second, is an equilibrium where each outlet has both multi-homing and single-homing advertisers possible? That is, an equilibrium involving $\sigma_i > 0$ for all i. For this to arise, demand from (9) must equal supply from (8) with symmetry implying that $\sigma_i = \tfrac{1}{2}$. Without a distributional assumption, this does not yield a closed-form solution for price. However, assuming that $F(v) = v$ (i.e., a uniform distribution), we can solve for the market clearing impression prices:

$$p = \begin{cases} \frac{D^s(2-D^s)}{4+D^s(2-D^s)}(3-4a) & \text{if } \tfrac{1}{2}D^s > p \\ \frac{2(2-D^s)}{4-D^s}(1-2a) & \tfrac{1}{2}D^s \leq p \end{cases} \tag{15}$$

Note that under symmetry, $D^s = 2\rho x^2 < \frac{1}{2}$. Thus, the number of switchers cannot exceed that level.

Importantly, when there are no advertisers purchasing multiple impressions on a single outlet, price declines with D^s. However, as D^s rises, there comes a point at which price is low enough that advertisers do purchase multiple impressions. The ones that do so are the inframarginal advertisers and so as D^s rises beyond this point, price, and hence, outlet profits, $p(D^l 2a + D^s a) = pa$, rise.[22]

Finally, is it possible that there are only single-homing advertisers in equilibrium? This would arise if for the highest value advertiser ($v=1$), its willingness to pay for an additional impression on an additional outlet were negative; that is, $v_{12} = \frac{D^l + \frac{1}{2}D^s}{D^l + \frac{1}{4}D^s} p > 1$. Using (15), it is easy to determine that this will arise if $2a < \frac{D^s}{4}$. In this case, all advertisers on each outlet would be single-homing so that $2a = \sigma_i(1-p) \Rightarrow p = (1-4a)$. To confirm that this can be an equilibrium note that when $2a < \frac{D^s}{4}$, $v_{12} = \frac{D^l + \frac{1}{2}D^s}{D^l + \frac{1}{4}D^s}(1-4a) > \frac{D^l + \frac{1}{2}D^s}{D^l + \frac{1}{4}D^s}(1-\frac{D^s}{2}) > 1$ which always holds.

The following proposition summarizes how equilibrium profits depend on D^s.

Proposition 5. *Assume that F(.) is uniform and there are two symmetric outlets. Suppose also that $a_1 = a_2 = a$. Then an outlet's equilibrium profits are as follows:*

    (i)   *For $D^s \leq \min\left\{8a, 4(1-a) - 2\sqrt{2(1-2a) + 4a^2}\right\}$, $\pi_i = \frac{1}{2}\frac{2(2-D^s)}{4-D^s}(1-2a)2a$;*

    (ii)   *For $D^s > 4(1-a) - 2\sqrt{2(1-2a) + 4a^2}$ and $D^s < 8a$, $\pi_i = \frac{1}{2}\frac{D^s(2-D^s)}{4+D^s(2-D^s)}(3-4a)2a$*

    (iii) *For $D^s \geq 8a$, $\pi_i = \frac{1}{2}(1-4a)2a$.*

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[22] It is useful to check whether multiple equilibria are possible. To rule this out as a concern note that market clearing prices in both cases above are equal if: $\frac{D^s(2-D^s)}{4+D^s(2-D^s)}(3-4a) = \frac{2(2-D^s)}{4-D^s}(1-2a) \Rightarrow D^s = 2\left(2(1-a) - \sqrt{2(1-2a) + 4a^2}\right)$. At this level of D^s, $p = 2(1-a) - \sqrt{2(1-2a) + 4a^2}$; i.e., $D^s / 2$. So, for given ad capacities, there is no issue of multiple equilibria.

Perfect

This characterization of equilibrium profits provides some insight into the impact of the Internet

on the news media. To the extent that the Internet has facilitated switching, these results suggest

that profits will decline but will eventually rise as switching becomes easier (see Figure One).

When the share of switchers is low, competition for the marginal advertiser pushes down total

outlet ad revenue. However, as the switcher share becomes large, the comparative static changes

sign and profits rise with the number of switchers. This is because high value advertisers begin to

purchase multiple impressions on individual outlets. This takes up scarce capacity and excludes

lower valued advertisers who were setting the impression price. The end result is that more

switchers drive higher impression prices and profits.

Figure One: Outlet Profits as a function of D^s ($a = 0.4$)
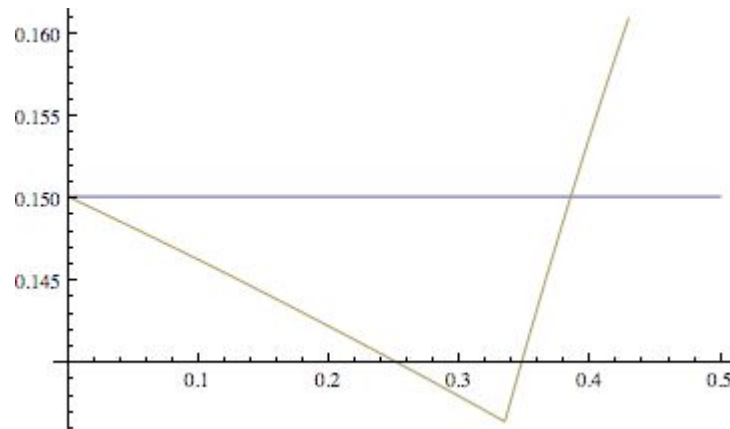
It is important to note, however, that the result that profits will rise with D^s relies on ad

capacity being high enough. If ad capacity is scarce, impression prices never fall to a level that

makes it worthwhile for infra-marginal advertisers to purchase multiple impressions on

individual outlets.

5.4   Incentives to adopt perfect tracking

Having characterized the equilibrium outcomes under imperfect tracking, we can now

examine incentives to adopt perfect tracking under the assumption that advertising capacity can

be adjusted prior to and following such adoption. The following proposition compares profits here with profits under perfect tracking.

Proposition 6. *Assume that F(.) is uniform and there are two symmetric outlets. Suppose also that $a_1 \approx a_2$. For low levels of D^s, outlet profits under perfect tracking exceed profits without tracking. For high levels of D^s, profits under perfect tracking may be lower than profits without tracking.*

This result is depicted in Figure One. Our earlier analysis identified that outlets with symmetric capacities, perfect tracking yields the benchmark profit outcome. Nonetheless, here we have demonstrated that when ad capacities are sufficiently high, profits for both outlets may be higher under no tracking than under perfect tracking. The reason is that higher value advertisers are induced to purchase more impressions. This crowds-out lower value advertisers who are setting price at the margin and consequently, impression prices are higher. This suggests that perfect tracking technology might not be adopted despite their ability to generate efficient outcomes in advertising markets.[23]

It is useful to note that outlets do not have a unilateral incentive to adopt perfect tracking as it has no value unless the other outlet is on board. This fact also makes it challenging for a provider of perfect tracking services to appropriate the rents from that activity as we would expect each outlet to have some hold-out power.

## 5.5 The Impact of Prohibiting Tracking

In 2010, the Federal Trade Commission was exploring a policy that would give consumers the right to 'opt out' of tracking of any kind by websites. If widely adopted, this

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[23] Of course, this also highlights the importance of how ad capacities are chosen; something we analyze in the appendix. That analysis demonstrates that it is, in fact, an inability to commit to not selling advertisements when ad capacity is relatively high that permits the outcome that perfect tracking may lead to lower profits than imperfect tracking.

would eliminate tracking options for media outlets. The analysis here allows us to examine the

No
Tracking

impact of that on advertising markets.

Figure Two demonstrates that, for the most part, removing tracking options lowers outlet profits.[24] Moreover, as tracking eliminated wasted impressions, its removal also reduces allocative efficiency in advertising markets. That said, from Proposition 6, we know that, in some cases, when tracking technologies are not available, outlet profits rise. Nonetheless, a prohibition only prevents tracking from being used and thus, weakly reduces profits.

Figure Two: Outlet Profits as a Function of a ($D^s = \frac{1}{2}$)
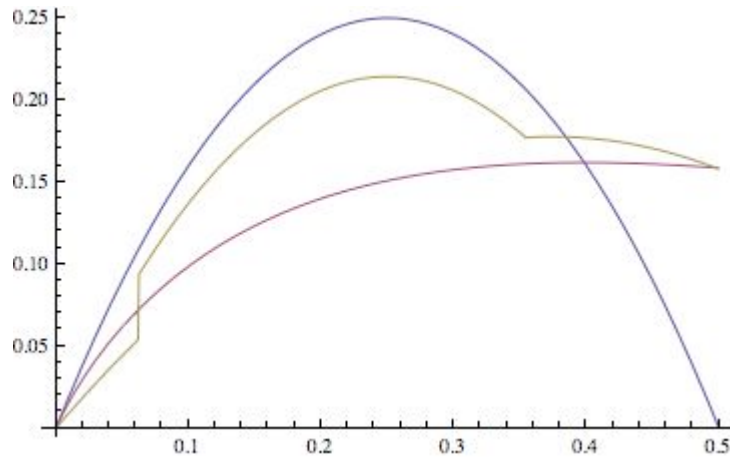
5.6   The Impact of Mergers

The evaluation of mergers between media outlets has always posed some difficult issues for policy-makers. On the one hand, if it is accepted that outlets have a monopoly over access to their consumers, then such mergers are unlikely to reduce to competitive outcomes in advertising markets. On the other hand, it is argued that a merger may indeed reduce competitive outcomes in advertising markets, increasing ad revenue, and stimulating outlet's incentives to attract consumers. While a full delineation of these views is not possible here, the analysis thusfar can

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[24] With a uniform, $F(.)$, under no tracking, equilibrium impression prices are given by $p = -\text{ProductLog}[-e^{-1-2a}]$.

speak to the question of whether a merger between outlets would reduce competitive outcomes (i.e., increase total revenue) on the advertising side of the media industry.

To begin, suppose that a merger between two outlets allows them to improve inter-outlet tracking. In this case, this will reduce the number of wasted and missed impressions in the advertising market. While impression prices would rise, so would allocative efficiency. As noted earlier, a move to perfect tracking will generate, for a fixed ad capacity, the first best outcome. Interestingly, by Proposition 6, it is not clear that outlets would choose to merge in order to facilitate this. While allocative efficiency may rise, total advertising profits could fall in cases where D^s and a are sufficiently high.

Alternatively, it may be that the technology is not readily available to improve inter-outlet tracking (even with common ownership). In this case, if the outlet charges a single price to advertisers on each outlet, the total ad revenue generated will be the same as the case where both outlets are separately owned. Here, it is only where commitments to reduce ad capacity were possible, that the merger may allow the exercise of market power in the advertising market.

However, what if the merger allowed the commonly owned outlet to price discriminate in a novel way; specifically, to identify and charge differential prices to single and multi-homing advertisers? That is, suppose that, on each outlet, the monopoly owner can commit to an ad capacity allocated to multi-homers, a_m, and an ad capacity allocated to single homers, a_s. Suppose also that no advertiser wants to purchase multiple impressions on one outlet and that outlet readership quality is symmetric. Price discrimination is achieved by charging all advertisers the same price for their first impression on one of the outlets and a different price for their second impression. The price the outlet can charge multi-homers, p_m for their second impression and single-homers, p_s, for their single impression are determined by:

$$a_m = 1 - v_{12} \text{ and } a_s = v_{12} - v_i \tag{16}$$

where $v_i = p_s$ and v_{12} is determined by:

$(2D^l + \frac{3}{4}D^s)v_{12} - (D^l + \frac{1}{2}D^s)(p_m + p_s) = (D^l + \frac{1}{2}D^s)(v_{12} - p_s)$ or $v_{12} = \frac{2}{4D^l + D^s}p_m$ given the

symmetric readership assumption. Solving for prices and substituting into the profit function,

$(p_s + p_m)a_m + p_s a_s$, gives:

$$\left(\frac{4D^l + D^s}{2}(1 - a_m) + (1 - a_s - a_m) \right)a_m + (1 - a_s - a_m)a_s \tag{17}$$

Maximizing with respect to (a_m, a_s) and subject to $a_s + 2a_m = 2a$ yields:

$$a_m = \frac{8a - D^s}{2(4 - D^s)} \text{ and } a_s = \frac{D^s}{4 - D^s}(1 - 2a) \tag{18}$$

so long as $8a > D^s$.[25] Profits are: $\frac{32a(2 - D^s)(1 - a) + D^{s2}}{8(4 - D^s)}$ which are greater than profits in the absence of

price discrimination. Interestingly, price discrimination does not imply that $p^m > p^s$. Here,

$p^m = \frac{(6 - D^s)(2 - D^s)}{4(4 - D^s)}$ and $p^s = \frac{8 + 4a(2 - D^s) - 3D^s}{4(4 - D^s)}$ so that $p^m > p^s$ only if $a > \frac{4 + 2D^s - D^{s2}}{8(2 - D^s)}$. When a is relatively

small, i.e., $a \in \frac{1}{8}[D^s, \frac{4 + 2D^s - D^{s2}}{2 - D^s}]$, $p^m < p^s$ and the outlet charges less for the second impression.

Nonetheless, the price of both impressions exceeds the price that would be charged in the

absence of price discrimination. Intuitively, the outlet encourages infra-marginal demand by

have a lower price for the second unit. This increases the value of the marginal advertiser and so

impression prices are higher.

5.7   The Impact of Blogs and Public Broadcasting

One of the factors that traditional newspapers have argued are contributing to their

decline is the rise of blogs and also competition from government-subsidized media. Both of

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[25] If this condition does not hold, the outlet would not choose to price discriminate.

those types of outlets have in common that they either do not accept advertising or accept very little of it. Somewhat in contradiction to this position, newspapers and television broadcasters have objected to plans to allow public broadcasters to sell advertisements rather than rely on subsidies. This latter objection remains a puzzle from the perspective of traditional media economics as there requiring competing public broadcasters to sell ads will cause more annoyance for their consumers and benefit other outlets. Here we explore the impact of competition from non-advertising media outlets.

We do this by assuming that the probability that consumers visit such outlets if given the choice is x_b. We also assume that the two mainstream (advertising) outlets have symmetric readership shares with $x_1 = x_2 = \frac{1}{2}(1-x_b)$. This implies that:

$$D^l = \tfrac{1}{2}(1-x_b)\left(1-\tfrac{1}{2}(1+x_b)\rho\right) \tag{19}$$

$$D_{12}^s = \rho\tfrac{1}{2}(1-x_b)^2 \tag{20}$$

$$D_{ib}^s = \rho x_b(1-x_b) \tag{21}$$

Given this, the advertiser expected surplus from given advertising strategies are:

Advertiser Choice	Frequency-Based Tracking
Single home on i, 1 impression	$(D_i^l + \tfrac{1}{2}D_{12}^s + \tfrac{1}{2}D_{ib}^s)(v-p)$
Single home on i, 2 impressions	$(D_i^l + D_{12}^s + D_{ib}^s)v - (2D_i^l + D_{12}^s + D_{ib}^s)p$
Multi-home, 1 impression each	$(D_1^l + D_2^l + \tfrac{3}{4}D_{12}^s + \tfrac{1}{2}(D_{1b}^s + D_{2b}^s))v$ $-p(D_1^l + D_2^l + D_{12}^s + \tfrac{1}{2}(D_{1b}^s + D_{2b}^s))$
Multi-home, 2 on i and 1 on j	$(D_i^l + D_j^l + D_{12}^s + D_{ib}^s + \tfrac{1}{2}D_{jb}^s)v$ $-(2D_i^l + D_j^l + \tfrac{3}{2}D_{12}^s + D_{ib}^s + \tfrac{1}{2}D_{jb}^s)p$
Multi-home, 2 impressions on each	$(D_i^l + D_j^l + D_{12}^s + D_{ib}^s + D_{jb}^s)v$ $-(2D_i^l + 2D_j^l + 2D_{12}^s + D_{ib}^s + D_{jb}^s)p$

The main difference between this case and the previous two outlet model is that some advertisers may choose to multi-home with two impressions on each outlet so as to impress a greater share of those switching between blogs and mainstream outlets. Indeed, under symmetry, the threshold advertiser rates become (under symmetric ad capacities):

$$v_i = p \tag{22}$$

$$v_{12} = 2 \frac{2D^l + D_{12}^s + D_{ib}^s}{4D^l + D_{12}^s + 2D_{ib}^s} p \tag{23}$$

$$v_{12'} = 2 \frac{2D^l + D_{12}^s + D_{ib}^s}{D_{12}^s + 2D_{ib}^s} p \tag{24}$$

$$v_{12''} = \frac{2D^l + D_{12}^s + D_{ib}^s}{D_{ib}^s} p \tag{25}$$

where $v_{12''}$ is the threshold between multi-homing with 2 on one outlet and multi-homing with 2 impressions on each outlet. It is clear that, under symmetry, $v_{12''} > v_{12'} > v_{12} > v_i$ when $\rho > 0$. This implies that there are three demand 'cases' but that supply in the market is $D_1^l 2a_1 + D_2^l 2a_2 + D_{12}^s (a_1 + a_2) + D_{1b}^s a_1 + D_{2b}^s a_2$. So long as ad capacities are symmetric, the market clearing price is given by:

$$p = \begin{cases} \frac{2D_{ib}^s(2D_{ib}^s + D_{12}^s)(2D_{ib}^s + 4D^l + D_{12}^s)}{(2(D_{ib}^s + D^l) + D_{12}^s)(8D_{ib}^s(D_{ib}^s + 2D^l) + 4(2D_{ib}^s + D^l)D_{12}^s + D_{12}^{s2})} 2(1-a) = \frac{2x_b(1+3x_b)\rho(4-(1-x_b)\rho)}{(1+x_b\rho)(4-\rho+x_b(28-(2-3x_b)\rho))} 2(1-a) \\ \frac{(2D_{ib}^s + D_{12}^s)(2D_{ib}^s + 4D^l + D_{12}^s)}{12D_{ib}^{s2} + 16D^{l2} + 20D^l D_{12}^s + 5D_{12}^{s2} + 16D_{ib}^s(2D^l + D_{12}^s)} (3-4a) = \frac{(4-(1-x_b)\rho)(\rho+3x_b\rho)}{16+4(1+7x_b)\rho-(1-x_b)(1+3x_b)\rho^2} (3-4a) \\ 2\frac{4D^l + D_{12}^s + 2D_{ib}^s}{8D^l + 3D_{12}^s + 4D_{ib}^s} (1-2a) = 2\frac{4-(1-x_b)\rho}{8-(1-x_b)\rho} (1-2a) \end{cases} \qquad \begin{matrix} p \le x_b\rho \\ \text{if } p \in [x_b\rho, \frac{1}{4}(1+3x_b)\rho] \\ p \ge \frac{1}{4}(1+3x_b)\rho \end{matrix} \tag{26}$$

It can be seen here that as the number of blog readers increases and/or the probability of switching rises, that inframarginal advertisers will demand more impressions.

Given this, we can prove the following:

Proposition 7. *For $\rho > 0$ and exogenous $a_1 \approx a_2$, equilibrium impression prices are increasing in x_b.*

The proof of the proposition requires a simple examination of (26) and is omitted. Intuitively, an increase in x_b has two effects. First, it decreases the effective supply of advertising capacity in the market. Because blog readers do not see advertisements, as attention is diverted to blogs, less attention is available for ads to be placed in front of. Second, unlike switchers between mainstream outlets, switchers between blogs and mainstream outlets do not contribute to the wasted impressions problem. Consequently, a greater share of blog readers increases the share of blog-mainstream switchers as well and so improves the efficiency of matching. This increases the demand for advertisements. These two effects – a decrease in supply and an increase in demand – combine to raise equilibrium impression prices. It is instructive to note that, even under perfect tracking, the supply-side effect remains and so impression prices would be expected to rise with blog readership share in that case too.

Nonetheless, in terms of the impact on overall outlet profits, the price effect of an increased blog share may not outweigh the quantity effect (in terms of lost readers) and so the impact on those profits is ambiguous. What is possible is that comparing markets with large and small blog shares, an increase in overall switching (ρ) will have a smaller impact on profits in the large blog market compared with the small one.[26] However, if it is the case that we are comparing a situation where one output sells advertising to one where it does not (absent any quantity changes in readership), then it is clear that advertising-selling outlets prefer the situation where its rival is prohibited from selling ads thus resolving the puzzle in traditional media economics.

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[26] This analysis has yet to be done and we hope to include it in a future version of the paper.

6 Positional Advantages

Thusfar, in analyzing imperfect tracking, we have focused on a situation where outlets are symmetric along various dimensions and, consequently, have a similar competitive position in advertising markets. Here we now consider the impact of different types of asymmetries between outlets in order to understand whether positional advantages are possible and what drives them in advertising markets. This is important because, as noted in the introduction, it has been observed that display advertisements both across media types but, most interestingly, within media types have considerable variation in their revenue earned per impression. By examining asymmetries we can hypothesize what factors might drive such heterogeneity.

6.1 Asymmetric ad capacities

While the above analysis allowed for some differences between outlets in ad capacities, the main results on imperfect tracking assumed symmetry. Here we consider what happens when ad capacities can be asymmetric. Specifically, in this situation, it becomes of interest as to whether asymmetry can permit a single market clearing price for advertising and, if not, what do prices look like? Importantly, does an outlet have an incentive to reduce ad capacity in order to exercise market power in advertising markets?

The following proposition summarizes the equilibrium outcomes.

Proposition 8. *Suppose that outlets are symmetric in readership and* $F(v)=v$ *but that* $a_1 < a_2$. *If* $a_1 \in [0, \frac{4a_2 - D^s}{2(2-D^s)}]$ *and* $a_2 \in [\frac{1}{4}(2a_1(2-D^s)+D^s,1]$, *then, in equilibrium,* $p_1 > p_2$. *Otherwise,* $p_1 = p_2$.

The proof (in the appendix) demonstrates that profits are:

$$\pi_1 = (1 - \tfrac{1}{2}D^s)(1 - 2a_1)2a_1 \tag{27}$$

$$\pi_2 = \begin{cases} (1-2a_2)2a_2 & \\ \frac{2D^s}{2+D^s}(1-a_2)2a_2 & \end{cases} \text{if } \begin{matrix} a_2 \leq \frac{2-D^s}{4} \\ a_2 > \frac{2-D^s}{4} \end{matrix} \tag{28}$$

Here it is clear that having a smaller ad capacity is not necessarily an advantage for outlets even if it does result in a higher impression price.

What does this imply for the incentive of an outlet to use capacity to exercise market power? When ad capacities are symmetric, the analysis of endogenous ad capacity in the appendix demonstrates that outlets have incentives akin to those of quantity duopolists in choosing their ad capacities. However, while locally this may be the case, each can unilaterally generate an asymmetric equilibrium of the form described in Proposition 8. When its rival's capacity is low, an outlet has an incentive to expand capacity so that there are no single-homers on the rival outlet. In contrast, when a rival outlet has very high capacity, an outlet may choose a low capacity so as to only sell to multi-homing advertisers. The appendix demonstrates that, over a non-trivial range of D^s, no pure strategy equilibrium exists. However, if outlets choose capacities sequentially, the resulting equilibrium is asymmetric with one outlet choosing a low and the other a high ad capacity converging to symmetry as D^s becomes small. Nonetheless, if each ad capacity is constrained to be no greater than ¼, then that is the resulting equilibrium and no asymmetric outcome occurs.

6.2   Asymmetric outlets

Asymmetric capacity choices can lead to differential prices but do not confer absolute positional advantages on outlets. Here we now consider what happens when outlets have different content quality with one outlet being able to generate a higher readership share than the other; in particular, when $x_1 > x_2 \Rightarrow D_1^l > D_2^l$. In this case, we demonstrate that outlet 1

commands a positional advantage in the advertising market that leads to it being able to earn higher impression prices than outlet alongside having a higher readership share.

To see this, observe that, if there is sufficient capacity on both outlets, single homing advertisers will sort on to outlet 1 first. This is because, for a given v, if impression prices were the same on each outlet (equal to p) then $(D_1^l + \frac{1}{2} D^s)(v - p) > (D_2^l + \frac{1}{2} D^s)(v - p)$. However, as impression prices will differ in equilibrium (specifically, it must be the case that $p_1 > p_2$ if there are single homers on outlet 2), the marginal single-homer on outlet 1 will be given by $v_1 = \frac{2(D_2^l p_2 - D_1^l p_1) + D^s(p_2 - p_1)}{2(D_2^l - D_1^l)}$ while $v_2 = p_2$. Note that $v_1 > v_2 \Rightarrow (2D_1^l + D^s)(p_2 - p_1) < 0$.

It is important to emphasize that it is the existence of switching consumers (i.e., $D^s > 0$) that generates this sorting. If there are no switchers, then the marginal advertiser on each outlet is competing with a multi-homing advertiser for their marginal impression. In this case, as there are no diminishing returns to additional impressions, a higher value multi-homing advertiser will outbid a smaller value single-homing advertiser for that slot. It is only when there are switchers that single-homing advertisers – competing against one another – determine the impression price on an outlet.

Some set of advertisers will multi-home with one impression on each outlet. The marginal multi-homing advertiser will be determined by:

$$
\begin{aligned}
&(D_1^l + D_2^l + \tfrac{3}{4} D^s)v_{12} - (D_1^l + \tfrac{1}{2} D^s)p_1 - (D_2^l + \tfrac{1}{2} D^s)p_2 \\
&= \max\left[(D_1^l + \tfrac{1}{2} D^s)(v_{12} - p_1), (D_2^l + \tfrac{1}{2} D^s)(v_{12} - p_2) \right]
\end{aligned}
\tag{29}
$$

Note that if $p_1 \leq p_2$ or there are single-homers on outlet 1, then $(D_1^l + \frac{1}{2} D^s)(v_{12} - p_1) \geq (D_2^l + \frac{1}{2} D^s)(v_{12} - p_2)$ implying that $v_{12} = \frac{D_2^l + \frac{1}{2} D^s}{D_2^l + \frac{1}{4} D^s} p_2$. Of course, it is also possible that some advertisers will multi-home with 2 impressions on one outlet. Note that, in

this case, the outlet receiving the additional impression will be outlet 2 as it has the smallest number of loyal consumers. Hence, $v_{12'} = \frac{2(2D_2^l+D^s)}{4(1-D_2^l-D_1^l)+3D^s} p_2$.

Given this, market clearing implies that the following equations (for each outlet) be simultaneously satisfied:

$$\underbrace{1 - F(v_1) = 2a}_{\text{Demand for 1}} \tag{30}$$

$$\underbrace{2(1 - F(\min\{v_{12'}, 1\})) + F(\min\{v_{12'}, 1\}) - F(v_{12}) + F(v_1) - F(v_2) = 2a}_{\text{Demand for 2}} \tag{31}$$

The following proposition characterizes the equilibrium outcome when ad capacities are symmetric. The derived profits are found by solving (30) and (31) for outlet prices and substituting them into outlet profits while checking to see what allocations of advertising choices these imply (in the same manner as those derived in Proposition 5).

Proposition 9. *Assume that $F(.)$ is uniform, $a_1 = a_2 = a$ and $x_1 > x_2$. Then each outlet's equilibrium profits are as follows:*

(i)   *For* $\frac{D^s}{2D_2^l + D^s} \le \min\left\{8a, 2\left(1 - a - \sqrt{2(1-2(1-a)a)}\right)\right\}$,

$\pi_1 = (D_1^l + \frac{1}{2}D^s) \frac{2(8D_2^l D_1^l + 3(D_2^l + D_1^l)D^s + D^{s2})}{(2D_1^l + D^s)(8D_2^l + 3D^s)}(1-2a)2a$ *and*

$\pi_2 = (D_2^l + \frac{1}{2}D^s)\frac{2(4D_2^l + D^s)}{8D_2^l + 3D^s}(1-2a)2a;$

(ii)   *For* $\frac{D^s}{2D_2^l + D^s} > 2\left(1 - a - \sqrt{2(1-2(1-a)a)}\right)$ *and* $\frac{D^s}{2D_2^l + D^s} < 8a$,

$\pi_1 = \frac{32(1-2a)(1-D_1^l)^2(2D_1^l-1)+8(1-D_1^l)(10-13D_1^l-6a(3-4D_1^l))D^s-2(28-a(46-48D_1^l)-29D_1^l)D^{s2}+(11-16a)D^{s3}}{2(16(1-D_1^l)^2-12(1-D_1^l)D^s+D^{s2})}2a$ *and*

$\pi_2 = (D_2^l + \frac{1}{2}D^s)\frac{D^s(4D_2^l + D^s)}{16D_2^{l2} + 20D_2^l D^s + 5D^{s2}}(3-4a)2a$

(iii)   *For* $\frac{D^s}{2D_2^l + D^s} \ge 8a$, $\pi_1 = x_1(1 - \frac{4a}{x_1})2a$ *and* $\pi_2 = (1-x_1)(1-4a)2a$.

In each case, $\pi_1 / x_1 > \pi_2 / x_2$.

The asymmetric outlet case operates similarly to the symmetric outlet case but with an important difference: the 'larger' outlet in terms of readership share can command a premium for its ad space. This is a known puzzle in traditional media economics as it is usually thought that

consumers are equally valuable regardless of the outlet they are on. Here, because ads are tracked more effectively internally, placing ads on the larger outlet only involves less expected waste than when you place ads on the other outlet or spread them across outlets. Hence, the larger outlet can command a premium.

6.3   Incentives to compete for readers

We now turn to examine a simple game designed to illustrate the incentives to compete for readers under imperfect tracking versus perfect tracking. We suppose that prior to consumers and advertisers making any choices, outlets can invest an amount, $c(\sigma_i) = \frac{1}{2}\sigma_i^2$ which generates a probability $\sigma_i \in (0,1)$ of being a high rather than a low quality outlet. The probabilities are independent across outlets. Therefore, if outlets choose (σ_1, σ_2) then with probability $\sigma_1(1-\sigma_2)$ only outlet 1 has high quality and so $x_1 > x_2$ while with probability $\sigma_2(1-\sigma_1)$ the reverse is true. With probability $\sigma_1\sigma_2 + (1-\sigma_1)(1-\sigma_2)$ both outlets have the same quality (high or low as the case may be) and $x_1 = x_2$.

The outlet's choose their 'qualities' simultaneously. When outlets have different qualities, the high quality outlet earns π^H while the low quality outlet earns π^L. If they have the same quality an outlet earns π. The profits here are as given in Propositions 5 and 9 when there is imperfect tracking and (6) if there is perfect tracking. Thus, in each case, $\pi^H > \pi > \pi^L$. It is straightforward to determine that the unique equilibrium 'qualities' are:

$$\sigma_1 = \sigma_2 = \frac{\pi^H + \pi^L - \pi}{1 + \pi^H + \pi^L - 2\pi} \tag{32}$$

The following proposition characterizes the intensity of investments in quality as a function of the tracking technology adopted.

Proposition 10. σ_i *is higher under perfect tracking than under imperfect tracking if and only if* $\rho < \frac{4(1-a)x - 2\sqrt{2}\sqrt{(1-2(1-a)a)x_i^2}}{x_i^2}$ *for a given* x_i *achieved by a uniquely high quality outlet.*

The proof involves a simple comparison of equilibrium quality choices and is omitted. The condition in the proposition corresponds to the critical ρ above which, under imperfect tracking, one outlet sells additional impressions to multi-homing consumers (i.e., where $v_{12'} < 1$). Intuitively, the cost of being a low competing against a high quality outlet rises with the number of switchers. When the share of switchers becomes so high that multi-homers purchase additional impressions, that difference jumps upwards and beyond the difference that would arise under perfect tracking.

6.4   Magnet content

The analysis thusfar has assumed that outlets have sufficient content to attract attention of loyal consumers throughout the relevant attention period. Of course, on the Internet, much content is provided on a smaller scale. For providers of that content, there is no possibility of attracting loyal consumers. However, here we demonstrate how such providers may have a positional advantage in advertising markets; that is, what they lose in their inability to attract frequent visits from consumers, they can make up in terms of their reach across all consumers – acting as a magnet for attention in the relevant advertising period.

Suppose that outlet 2, in our current formulation, has only limited content; i.e., that consumers visiting that outlet will stay at most one period. To assist in identifying it notationally, let's rename it outlet f. Outlet 1 is unchanged. In this situation, the total expected traffic (over both periods) to outlet 1 is $x_1 + (1-\rho)x_1 + \rho x_1^2 + \rho x_f$ and to outlet 2 is $x_f + \rho x_1 x_f$. Using, this we can identify loyal and switching consumers in this context for any given period:

$$D_1^l = x_1 - \rho x_f x_1$$

$$D^s = \rho x_f (1 + x_1)$$

$$D_f^l = x_f - \rho x_f$$

Of course, there is an important sense in which the description 'loyal to outlet f' is a misnomer as consumers can consume one period of content. Consequently, this is more appropriately described as 'exclusive to outlet f.' Nonetheless, to focus on the impact of limited content, we will confine ourselves here to the case where $\rho = 1$. In this situation, $D_f^l = 0$ and outlet f only has consumers who are switchers. Thus, while outlet 1 supplies ad capacity of $D_1^l 2a + D^s a$ into the market, outlet f only supplies $D^s a$.

The following table identifies the surplus to an advertiser with value v from pursuing different choices.

Advertiser Choice	Frequency-Based Tracking
Single home on 1, 1 impression	$(D_1^l + \frac{1}{2} D^s)(v - p_1)$
Single home on 1, 2 impressions	$(D_1^l + D^s)v - (2D_1^l + D^s)p_1$
Single home on f, 1 impression	$\frac{1}{2} D^s(v - p_f)$
Single home on f, 2 impressions	$D^s(v - p_f)$
Multi-home, 1 impression each	$(D_1^l + \frac{3}{4} D^s)v - (D_1^l + \frac{1}{2} D^s)p_1 - \frac{1}{2} D^s p_f$
Multi-home, 2 on f and 1 on 1	$(D_1^l + D^s)v - (D_1^l + \frac{1}{2} D^s)p_1 - D^s p_f$
Multi-home, 2 on 1 and 1 on f	$(D_1^l + D^s)v - (2D_1^l + D^s)p_1 - \frac{1}{2} D^s p_f$

Notice that there are now three options for an advertiser to cover the entire consumer market – single homing on 1 with 2 impressions, and multi-homing with two impressions on at least one outlet. Of course, it is clear that multi-homing with 2 impressions on outlet 1 is dominated by single-homing on outlet 1 (as the former involves paying for impressions on f without any benefit). In addition, note that any advertiser who wants to single homing on outlet f will prefer to do so with two impressions as there is no waste from the additional impression. More subtly, we can always rule out multi-homing with one impression on each outlet. For this to be preferred to single-homing on outlet 1 (with one impression) it must be the case that $\frac{1}{4}D^s v > \frac{1}{2}D^s p_f$. However, this condition also means that by moving from multi-homing with single impressions to multi-homing on outlet f with 2 impressions is preferable. Consequently, if an advertiser wants to capture an additional $\frac{1}{4}D^s$ by purchasing an impression on outlet f, it will also want to do this by purchasing two additional impressions on outlet f.

This still leaves four choices that might be undertaken by advertisers. Importantly, as a means of covering the entire market, single-homing on outlet 1 with 2 impressions and multi-homing with 2 impressions on f are substitutes. Indeed, multi-homing will only be chosen if $(D_1^l + \frac{1}{2}D^s)p_1 > D^s p_f$; a condition that must hold if D^s is very small. Importantly, at any point in time, we will only observe one of these strategies being chosen. In each case, it will be the highest value advertisers who pursue them.

For the remaining choices, advertisers single homing on f (with 2 impressions) or on 1 (with 1 impression) are candidates to be the marginal advertiser in the market. If $\frac{1}{2}D^s > D_1^l$, higher value advertisers prefer (holding prices constant) purchasing impressions on f rather than 1. Under this condition, the marginal advertiser, with value p_1, would earn $D^s(p_1 - p_f)$ by

switching to outlet f which is negative if $p_1 < p_f$. Similarly, if the marginal advertiser has value,

p_f, it will earn $(D_1^l + \frac{1}{2}D^s)(p_f - p_1)$ by switching to outlet 1. This reduces its surplus if $p_f < p_1$.

Hence, the marginal advertiser will be on the lowest priced outlet.

Given this, we can prove the following proposition.

Proposition 11. *Suppose that $\rho = 1$. Equilibrium profits for outlets 1 and f are:*

$$\pi_1 = (D_1^l + \tfrac{1}{2}D^s)\frac{6D_1^l(1-2a)+D^s}{3(2D_1^l+D^s)}2a \text{ and } \pi_f = D^s\left(\tfrac{2}{3}-a\right)a$$

$$\pi_1 = (D_1^l + \tfrac{1}{2}D^s)\left(aD_1^l + D^s(1-2a)\right)2a \text{ and } \pi_f = D^s\left(1-2a-2(1-3a)\frac{D_1^{l2}}{D^s}\right)a$$

$$\text{if } \begin{array}{l} \tfrac{1}{2}D^s \leq D_1^l \\ \tfrac{1}{2}D^s > D_1^l \end{array}$$

The structure of the equilibrium is interesting. When f's share is low ($\frac{1}{2}D^s < D_1^l$) and begins to

rise, outlet 1, who was exclusively selling to single-homing advertisers (1 impression) continues

to do so but high valued advertisers also purchase 2 impressions on outlet f. The same is true of

low valued purchasers who now become the marginal advertisers in the market at a price of p_f.

Consequently, $p_f < p_1$ but as x_f rises outlet 1's profit falls as does total profits from advertising

in the industry. This changes when x_f reaches a critical level (i.e., 0.42265 so that $\frac{1}{2}D^s > D_1^l$).

At that point, marginal advertisers prefer to bid for 2 impressions on outlet f and so single-

homing advertisers with a single impression on outlet 1 become the marginal advertisers at a

price of p_1. This implies that $p_f > p_1$. In addition, the high valued advertisers no longer choose

to multi-home and become exclusive to outlet 1 with 2 impressions. Nonetheless, as x_f rises

outlet 1's profits continue to fall. In this case, however, industry profits rise again and indeed,

when $x_f \to 1$ they approach the same level as when $x_f = 0$. In this case, the profits are split

evenly between the two outlets rather than held entirely by outlet 1. Intuitively, at this point, all

consumers are switchers and so there is no longer any inefficiency resulting from wasted

impressions.

Where there is inefficiency at this limit is as a result of outlet 1's content. It now arguably too much as the small content outlet can earn exactly the same profits as it can with content sufficient to capture attention for only a single attention period. Indeed, when x_f is such that $\frac{1}{2}D^s > D_1^l$, outlet f earns more than half of outlet 1's profits. Thus, the rate of return for providing that additional content is lower for outlet 1 than for outlet f.

6.5   Paywalls

Paywalls have been proposed as a means by which outlets with falling advertising revenue may restore profitability. Of course, there are several different types of paywalls that may be employed. One possibility is a paywall – sometimes termed 'micropayments' – whereby consumers pay whenever they visit a website; similar to payments for physical newspapers at the newstand. Another type is a subscription whereby consumers pay once and can access a site for a length of time. Finally, some outlets have experimented with limited paywalls that permit limited reading on websites but if consumers want to consume more they have to subscribe. Here we analyze each of these types of strategies focusing on what it does to advertising revenue for each outlet. In so doing, we focus on a situation where one outlet, in this case outlet 1, introduces a paywall while the other outlet remains free.

The exploration here will be conducted within the context of the model thusfar to gain some insight on these issues. A full exploration would embed a proper model of consumer behavior in the consumer choice side of the market. Instead, we argue that one important effect of paywalls is to impact on switching behavior and through that on advertising markets. Specifically, we now propose that outlets are asymmetric in the probabilities that a consumer might have an opportunity to switch *away* from them. That is, we define ρ_{ij} as the probability

that a consumer who has visited outlet i, has an opportunity to switch from it. Consequently, the three consumer classes are now determined by:

$$D_1^l = x_1 - x_1(1-x_1)\rho_{12} \tag{33}$$

$$D_2^l = x_2 - x_2(1-x_2)\rho_{21} \tag{34}$$

$$D_{12}^s = (\rho_{21} + \rho_{12})x_1 x_2 \tag{35}$$

A higher ρ_{ij} may result from the consumer having a higher cost associated with remaining with outlet i. Of course, a paywall may impact upon x_i. However, for the most part, we will hold that effect fixed and comment on the impact of such movements below.

We begin by considering *micropayments* whereby outlet 1 charges consumers for each period they visit its website. Holding the impact on x_1 fixed, a micropayment makes it less likely that visitors to outlet 1 will stay on that outlet another period (increasing ρ_{12}) while making it less likely visitors to outlet 2 will switch to outlet 1 (decreasing ρ_{21}). This has two impacts on advertising markets. First, D_{12}^s could rise or fall depending upon what happens to $\rho_{21} + \rho_{12}$. If it falls, then this will put upward pressure on advertising prices if ad capacity is relatively low. Second, recall that when readership shares were asymmetric, an outlet commanded a positional advantage if its expected share of loyal consumers was relatively high. However, holding x_1 fixed and starting from a symmetric position prior to the paywall, micropayments on outlet 1 will cause $D_2^l > D_1^l$. Consequently, outlet 2 will be given a positional advantage in the advertising market so that $p_2 > p_1$. Add to that the likelihood that 1's paywall will reduce x_1 and this effect is only reinforced. Outlet 1 would have to not only make up for lost advertising revenues as a loss in visitors but also from the loss in positional advantage while outlet 2 clearly benefits on both of these dimensions from the paywall.

In contrast to a micropayment system, a *subscription* system will have a more directed impact. In such a system, a visitor to outlet 1 only pays on their first visit and not thereafter. This means that a subscriber to outlet 1 may be just as likely – should the opportunity and desire arise – to switch to outlet 2 (i.e., ρ_{12} will not change). However, a non-subscriber who had visited outlet 2 previously would be less likely to then subscribe to outlet 1 for what remained of the attention period (i.e., ρ_{21} would fall). Once again, starting from a position of symmetry, this implies that $D_2^I > D_1^I$ and so the paywall would not only lead to relatively more visitors to outlet 2 but a positional advantage for it in advertising markets. This is an interesting result as one of the claims associated with subscription paywalls is that they will increase consumer loyalty to an outlet. While it is true that such loyalty, if generated, would increase an outlet's advertising revenues per consumer, here a subscription generates increased loyalty for the rival outlet rather than the outlet imposing the paywall. Of course, this effect could be mitigated if, say because they are subscribers, consumers are more inclined to be loyal to outlet 1 thereby increasing ρ_{12}. The point here is that that outcome is not straightforward.

Finally, some outlets have proposed a *limited paywall*. In this case, outlets allow access to some content for free and then charge should a consumer wish to consume more. In the context of the model here, such a paywall would only be imposed, say, if a consumer chose to stay on outlet 1 for both attention periods. This type of paywall would be unlikely to have any impact on those who had previously visited outlet 2 as they could still freely switch to outlet 1 (i.e., ρ_{21} would be unchanged). However, this paywall would impose a penalty for staying on outlet 1 making consumers there more inclined to switch (i.e., ρ_{12} would rise). It is clear again, that other things being equal, the paywall would result in $D_2^I > D_1^I$.

The analysis here demonstrates that putting in a paywall may give an outlet a positional disadvantage in advertising markets. Of course if an outlet already has a positional advantage, the likelihood that this occurs is lower. Nonetheless, the impact of a paywall does confer benefits on rivals in advertising markets as well as increasing their readership. These consequences may explain the low use of paywalls for online news media.

7   Conclusions and Directions for Future Research

This paper resolves long-standing puzzles in media economics regarding the impact of competition by constructing a model where consumers can switch between media outlets and those outlets can only imperfectly track those consumers across outlets. This model generates a number of predictions including that as consumer switching increases total advertising revenue falls, that outlets with a larger readership share command premiums for advertisements, that greater switching may lead advertisers to increase the frequency of impressions purchased on outlets, that an increase in attention from non-advertising sources will increase advertising prices, that mergers may allow outlets to price discriminate in advertising markets, that ad platforms may not increase outlet profits, that investments in content quality will be associated with the frequency with which advertisers purchase impressions and that outlets that supply magnet content may be more profitable than outlets offering a deeper set of content. These predictions await thoughtful empirical testing but are thusfar consistent with stylized facts associated with the impact of the Internet on the newspaper industry.

While the model here has a wide set of predictions, extensions could deepen our understanding further. Firstly, the model involves two outlets usually modeled as symmetric with a distribution of advertisers with specific qualities. Generalizing these could assist in developing

more nuanced predictions for empirical analysis; specifically, understanding the impact of outlet heterogeneity on advertising prices, incentives to invest in quality and incentives to invest in tracking technology.

Related, in this paper, we focused on frequency-based tracking noting that other forms of tracking have been part of the news industry. An open question is what the incentives are for firms to unilaterally improve their internal tracking of consumers. As noted throughout this paper, the adoption of more efficient matching may increase marginal demand but reduce inframarginal demand from advertisers. When ad capacity is scarce, it is not clear that such moves will prove profitable for outlets.

Finally, throughout this paper we have assumed that advertisements were equally effective on both outlets. However, in some situations, it may be that the expected value from impressing a consumer on one outlet is higher than that from impressing consumers on another. For instance, consider (as in Athey and Gans, 2010), a situation where all advertisers are in a given local area. One outlet publishes in that local area only while the other is general and publishes across local areas.[27] Absent the ability to identify consumers based on their location, a consumer impressed on the local outlet will still generate an expected value of v to advertiser v whereas one impressed on the general outlet will only generate an expect value of θv with $\theta < 1$. In this situation, even if there are no switching consumers, advertisers on the general outlet will be paying for wasted impressions.

While this situation may be expected to generate outcomes similar to when readership shares are asymmetric, the effects can be subtle. A general outlet may have fewer consumers

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[27] Location is only one aspect upon which consumers and advertisers might sort according to common interests. Any specialized media content can perform this function and give an outlet a matching advantage over more general outlets.

who are of value to advertisers but also may have a larger readership.[28] Also, when consumers switch between outlets, the switching behavior is information on those hidden characteristics. Thus, switching behavior may actually increase match efficiency. Consequently, the effects of tailored content, self-selection and incentives to adopt targeting technologies that overcome these are not clear and likely to be an area where future developments can be fruitful.

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[28] Milgrom and Levin (2010) argue that targeting may be limited because it conflicts with goals of achieving market thickness (see also Athey and Gans, 2010).

8 Appendix

8.1 Proof of Proposition 1a,b and related results

Consider the following program: $\max_{\{n_1 \geq 0, n_2 \geq 0\}} \pi$, where

$$\pi := D_1^l(1-e^{-\frac{2n_1}{2D_1^l+D^s}})v + D_2^l(1-e^{-\frac{2n_2}{2D_2^l+D^s}})v + D^s(1-e^{-\frac{n_2}{2D_1^l+D^s}-\frac{n_1}{2D_2^l+D^s}})v - n_1 p_1 - n_2 p_2.$$

Let $n^* := (n_1^*, n_2^*)$ denote its solution, which depends, among other things, on v.

Result 1: $\frac{n_1^*}{2D_1^l+D^s} \leq \frac{n_2^*}{2D_2^l+D^s}$ *if and only if* $p_1 \geq p_2$.

In what follows we will use the above observation to derive a number of results. First, suppose that outlets are equally expensive: $p_1 = p_2 = p$ with $p > 0$.

Result 2: $n_1^* = \frac{2D_1^l+D^s}{2}\ln(v/p)$ *and* $n_2^* = \frac{2D_2^l+D^s}{2}\ln(v/p)$.

It follows that all active advertisers multi-home. Now suppose, without loss of generality, that $p_1 \leq p_2$, i.e. outlet two is the expensive one.

Result 3: If $p_2 > p_1$ *and* $\partial\pi/\partial n_2\big|_{n=n^*} = 0$ *then* $n_1^* > 0$.

Result 4: $n_2^* > 0$ *implies* $n_1^* > 0$ *(i.e. all advertisers active on the more expensive outlet multi-home).*

Result 5: $n_2^* = 0$ *and* $\partial\pi/\partial n_2\big|_{n=n^*} = 0$ *imply* $n_1^* > 0$.

Result 4 says that, in equilibrium, a sorting condition holds. High value advertisers will multi-home, intermediate value advertisers will single-home on the cheaper website. Note that the result holds regardless of the value of x_1 and ρ. This means that the equilibrium strategy of is pinned down by the relative price, not by asymmetries in readership share.

Result 6: $\frac{\partial n_1^*}{\partial \rho} \geq 0$ *and* $\frac{\partial n_2^*}{\partial \rho} \leq 0$ *if and only if* $p_1 \leq p_2$.

We shall now prove these results in turn.

Set $\pi := D_1^l(1-e^{-\frac{2n_1}{2D_1^l+D^s}})v + D_2^l(1-e^{-\frac{2n_2}{2D_2^l+D^s}})v + D^s(1-e^{-\frac{n_1}{2D_1^l+D^s}-\frac{n_1}{2D_2^l+D^s}})v - n_1 p_1 - n_2 p_2.$

Note that π is continuous and $\lim\limits_{\substack{n_1 \to 0 \\ n_2 \to 0}} \pi = 0, \lim\limits_{\substack{n_1 \to \infty \\ n_2 \to \infty}} \pi = -\infty$, so there exists a solution. Moreover $\pi : \circ^2 \to \circ$ is strictly concave in (n_1, n_2),[29] and since the set defined by the constraints is convex the solution denoted (n_1^*, n_2^*) is unique and characterized by the following necessary and sufficient conditions for maxima:

$$\frac{\partial \pi}{\partial n_1} \le 0 \Leftrightarrow \frac{2D_1^l}{2D_1^l + D^s} e^{-\frac{2n_1^*}{2D_1^l + D^s}} + \frac{D^s}{2D_1^l + D^s} e^{-\frac{n_1^*}{2D_1^l + D^s} - \frac{n_2^*}{2D_2^l + D^s}} \le \frac{p_1}{v}$$

$$\frac{\partial \pi}{\partial n_2} \le 0 \Leftrightarrow \frac{2D_2^l}{2D_2^l + D^s} e^{-\frac{2n_2^*}{2D_2^l + D^s}} + \frac{D^s}{2D_2^l + D^s} e^{-\frac{n_1^*}{2D_1^l + D^s} - \frac{n_2^*}{2D_2^l + D^s}} \le \frac{p_2}{v}$$

$$n_1^* \frac{\partial \pi}{\partial n_1} = 0$$

$$n_2^* \frac{\partial \pi}{\partial n_2} = 0$$

Note that $n_1^* = n_2^* = 0 \Leftrightarrow \min\{p_1, p_2\} \ge v$, so in what follows we shall assume $v > \min\{p_1, p_2\}$ to consider non-trivial solutions.

We have to consider three cases:

1. $\left.\dfrac{\partial \pi}{\partial n_1}\right|_{n=n^*} = \left.\dfrac{\partial \pi}{\partial n_2}\right|_{n=n^*} = 0 \Leftrightarrow n_1^*, n_2^* \ge 0$ (the "interior solution" case)

2. $\left.\dfrac{\partial \pi}{\partial n_1}\right|_{n=n^*} \le \left.\dfrac{\partial \pi}{\partial n_2}\right|_{n=n^*} = 0 \Leftrightarrow n_1^* = 0, n_2^* \ge 0$

3. $\left.\dfrac{\partial \pi}{\partial n_2}\right|_{n=n^*} \le \left.\dfrac{\partial \pi}{\partial n_1}\right|_{n=n^*} = 0 \Leftrightarrow n_1^* \ge 0, n_2^* = 0$

Let us consider case 1 first. Subtracting side by side and rearranging the FOCs we get:

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[29] Since π is twice differentiable and the Hessian matrix

$$\begin{pmatrix} -\dfrac{4D_1^e}{(2D_1^l + D^s)^2} e^{-\frac{2n_1}{2D_1^l + D^s}} - \dfrac{D^S}{(2D_1^l + D^s)^2} e^{-\frac{n_1}{2D_1^l + D^s} - \frac{n_2}{2D_2^l + D^s}} & -\dfrac{D^S}{(2D_1^l + D^s)(2D_2^l + D^s)} e^{-\frac{n_1}{2D_1^l + D^s} - \frac{n_2}{2D_2^l + D^s}} \\ -\dfrac{D^S}{(2D_1^l + D^s)(2D_2^l + D^s)} e^{-\frac{n_1}{2D_1^l + D^s} - \frac{n_2}{2D_2^l + D^s}} & -\dfrac{4D_2^e}{(2D_2^l + D^s)^2} e^{-\frac{2n_2}{2D_2^l + D^s}} - \dfrac{D^S}{(2D_2^l + D^s)^2} e^{-\frac{n_1}{2D_1^l + D^s} - \frac{n_2}{2D_2^l + D^s}} \end{pmatrix}$$

is definite negative.

$$e^{-\frac{2n_1^*}{2D_1^l+D^s}}\left(-\frac{2D_2^l}{2D_2^l+D^s}e^{-2\left(\frac{n_2^*}{2D_2^l+D^s}-\frac{n_1^*}{2D_1^l+D^s}\right)}+\left(\frac{D^s}{2D_1^l+D^s}-\frac{D^s}{2D_2^l+D^s}\right)e^{-\left(\frac{n_2^*}{2D_2^l+D^s}-\frac{n_1^*}{2D_1^l+D^s}\right)}+\frac{2D_1^l}{2D_1^l+D^s}\right)=\frac{p_1-p_2}{v}$$

Note that the sign of the left hand side is always equal to the sign of $\frac{n_2^*}{2D_2^l+D^s}-\frac{n_1^*}{2D_1^l+D^s}$. It follows that $p_1=p_2$ iff $\frac{n_2^*}{2D_2^l+D^s}-\frac{n_1^*}{2D_1^l+D^s}=0$, $p_1<p_2$ iff $\frac{n_2^*}{2D_2^l+D^s}-\frac{n_1^*}{2D_1^l+D^s}<0$ and $p_1>p_2$ iff $\frac{n_2^*}{2D_2^l+D^s}-\frac{n_1^*}{2D_1^l+D^s}>0$.

Consider case 2 when $p_1<p_2$. FOCs yield:

$$\frac{2D_1^l}{2D_1^l+D^s}+\frac{D^s}{2D_1^l+D^s}e^{-\frac{n_2^*}{2D_2^l+D^s}}\le\frac{p_1}{v}$$

$$\frac{2D_2^l}{2D_2^l+D^s}e^{-\frac{2n_2^*}{2D_2^l+D^s}}+\frac{D^s}{2D_2^l+D^s}e^{-\frac{n_2^*}{2D_2^l+D^s}}=\frac{p_2}{v}$$

Note that this should hold for all values of prices such that $p_1<p_2$ and parameters on which n_2^* depends. But if $n_2^*\approx0$ we get $p_1\ge v=p_2$ which is a contradiction. Thus case 2 cannot occur if $p_1<p_2$.

Finally, consider case 3 when $p_1<p_2$. As long as $n_1^*\ge0, n_2^*=0$, it is still true that $\frac{n_1^*}{2D_1^l+D^s}\ge\frac{n_2^*}{2D_2^l+D^s}=0$. Conversely if $p_1>p_2$, Case 3 cannot occur and in Case 2 as long as $n_1^*=0, n_2^*\ge0$ it is still true that $\frac{n_2^*}{2D_2^l+D^s}\ge\frac{n_1^*}{2D_1^l+D^s}=0$.

Now suppose $p_1=p_2=p>v$. Suppose $(n_1^*,n_2^*)=\left(\frac{2D_1^l+D^s}{2}\ln(v/p),\frac{2D_2^l+D^s}{2}\ln(v/p)\right)$. Substituting into the FOCs:

$$\frac{\partial \pi}{\partial n_1} = \frac{2D_1^l}{2D_1^l + D^s} e^{-\frac{2n_1}{2D_1^l + D^s}} + \frac{D^s}{2D_1^l + D^s} e^{-\frac{n_1}{2D_1^l + D^s} - \frac{n_2}{2D_2^l + D^s}} - \frac{p}{v}$$

$$= \frac{2D_1^l}{2D_1^l + D^s} e^{-\frac{2\left(\frac{2D_1^l + D^s}{2}\ln(v/p)\right)}{2D_1^l + D^s}} + \frac{D^s}{2D_1^l + D^s} e^{-\frac{\frac{2D_1^l + D^s}{2}\ln(v/p)}{2D_1^l + D^s} - \frac{\frac{2D_2^l + D^s}{2}\ln(v/p)}{2D_2^l + D^s}} - \frac{p}{v}$$

$$= \frac{2D_1^l}{2D_1^l + D^s}\frac{p}{v} + \frac{D^s}{2D_1^l + D^s}\frac{p}{v} - \frac{p}{v} = 0$$

$$\frac{\partial \pi}{\partial n_2} = \frac{2D_2^l}{2D_2^l + D^s} e^{-\frac{2n_2}{2D_2^l + D^s}} + \frac{D^s}{2D_2^l + D^s} e^{-\frac{n_1}{2D_1^l + D^s} - \frac{n_2}{2D_2^l + D^s}} - \frac{p}{v}$$

$$= \frac{2D_2^l}{2D_2^l + D^s} e^{-\frac{2\left(\frac{2D_2^l + D^s}{2}\ln(v/p)\right)}{2D_2^l + D^s}} + \frac{D^s}{2D_2^l + D^s} e^{-\frac{\frac{2D_1^l + D^s}{2}\ln(v/p)}{2D_1^l + D^s} - \frac{\frac{2D_2^l + D^s}{2}\ln(v/p)}{2D_2^l + D^s}} - \frac{p}{v}$$

$$= \frac{2D_1^l}{2D_2^l + D^s}\frac{p}{v} + \frac{D^s}{2D_2^l + D^s}\frac{p}{v} - \frac{p}{v} = 0$$

And, therefore, in this particular case $(n_1^*, n_2^*) = \left(\frac{2D_1^l + D^s}{2}\ln(v/p), \frac{2D_2^l + D^s}{2}\ln(v/p)\right)$ is the unique solution to the problem.

Proof of result 3, 4 and 5

If $p_2 > p_1$, suppose $\partial \pi / \partial n_2 \big|_{n=n^*} = 0$ and $n_1^* = 0$. Then FOCs become

$$\frac{2D_1^l}{2D_1^l + D^s} + \frac{D^s}{2D_1^l + D^s} e^{-\frac{n_2^*}{2D_2^l + D^s}} - \frac{p_1}{v} \leq 0$$

$$\frac{2D_2^l}{2D_2^l + D^s} e^{-\frac{2n_2^*}{2D_2^l + D^s}} + \frac{D^s}{2D_2^l + D^s} e^{-\frac{n_2^*}{2D_2^l + D^s}} - \frac{p_2}{v} = 0$$

Which can be rewritten as

$$\frac{2D_1^l}{2D_1^l + D^s} + \frac{D^s}{2D_1^l + D^s} e^{-\frac{n_2^*}{2D_2^l + D^s}} \leq \frac{p_1}{v} < \frac{p_2}{v} = \frac{2D_2^l}{2D_2^l + D^s} e^{-\frac{2n_2^*}{2D_2^l + D^s}} + \frac{D^s}{2D_2^l + D^s} e^{-\frac{n_2^*}{2D_2^l + D^s}}$$

Note that the right hand side is decreasing in n_2^*, and so the above inequality, which has to hold for all values of parameters and prices such that $p_2 > p_1$, is more easily satisfied for low values of n_2^*. But if $n_2^* \approx 0$ we get

$$\frac{2D_1^l}{2D_1^l + D^s} + \frac{D^s}{2D_1^l + D^s} < \frac{2D_2^l}{2D_2^l + D^s} + \frac{D^s}{2D_2^l + D^s}$$

which is a contradiction.

In particular, it must be that $n_1^* > 0$ both when $n_2^* > 0$, and when $n_2^* = 0$ and $\partial\pi/\partial n_2\big|_{n=n^*} = 0$

Proof of result 6

The Jacobian of the function $g(n_1, n_2, \rho): \circ^3 \to \circ^2$ defined by the first order conditions with respect to n_1 and n_2 corresponds to the Hessian matrix of π, which is definite negative and hence nonsingular. Therefore by the inverse function theorem the function g defines implicitly a function $n(\rho): \circ \to \circ^2$ around the point (n_1^*, n_2^*). Using the functional form of D_1^l, D_2^l and D^s we get

$$\frac{\partial n_1^*}{\partial\rho} = \frac{-(1 - e^{\frac{1}{2}(\frac{n_1^*}{x_1} - \frac{n_2^*}{1-x_1})})(\rho x_1 + e^{\frac{1}{2}(\frac{n_1^*}{x_1} - \frac{n_2^*}{1-x_1})}(2 - \rho x_1))(1 - x_1)}{\rho x_1(1 - \rho(1-x_1)) + (1 - \rho x_1)e^{\frac{1}{2}(\frac{n_1^*}{x_1} - \frac{n_2^*}{1-x_1})}(2 + \rho(1-x_1)(e^{\frac{1}{2}(\frac{n_1^*}{x_1} - \frac{n_2^*}{1-x_1})} - 2))}$$

$$\frac{\partial n_2^*}{\partial\rho} = \frac{(1 - e^{\frac{1}{2}(\frac{n_1^*}{x_1} - \frac{n_2^*}{1-x_1})})(2 - \rho(1-x_1) + \rho(1-x_1)e^{\frac{1}{2}(\frac{n_1^*}{x_1} - \frac{n_2^*}{1-x_1})})(1 - x_1)x_1}{\rho x_1(1 - \rho(1-x_1)) + e^{\frac{1}{2}(\frac{n_1^*}{x_1} - \frac{n_2^*}{1-x_1})}(1 - x_1)(2 + \rho(1-x_1)(e^{\frac{1}{2}(\frac{n_1^*}{x_1} - \frac{n_2^*}{1-x_1})} - 2))}$$

Note that if $\frac{n_1^*}{x_1} - \frac{n_2^*}{1-x_1} = 0$, then $\frac{\partial n_1^*}{\partial\rho} = \frac{\partial n_2^*}{\partial\rho} = 0$ and therefore $\frac{\partial n_1^*}{\partial\rho} = \frac{\partial n_2^*}{\partial\rho} = 0$ if and only if $p_1 = p_2$; on the other hand if $\frac{n_1^*}{x_1} - \frac{n_2^*}{1-x_1} > 0$, then $\frac{\partial n_1^*}{\partial\rho} > 0$ and $\frac{\partial n_2^*}{\partial\rho} < 0$, which implies that $\frac{\partial n_1^*}{\partial\rho} > 0$ and $\frac{\partial n_2^*}{\partial\rho} < 0$ if and only if $p_1 < p_2$. Conversely, $\frac{\partial n_1^*}{\partial\rho} < 0$ and $\frac{\partial n_2^*}{\partial\rho} > 0$ if and only if $p_1 > p_2$.

Proof of proposition 1b

We have two separate markets for impressions, one per outlet. An equilibrium price vector (p_1^*, p_2^*) solves:

$$a(2D_1^l + D^s) = \int_{v_1(p)}^{\infty} n_1^*(v, p_1, p_2, x_1, \rho)f(v)dv$$

$$a(2D_2^l + D^s) = \int_{v_2(p)}^{\infty} n_2^*(v, p_1, p_2, x_1, \rho)f(v)dv$$

where $n_i^*(v, p_1, p_2)$ is type v's demand of impressions of outlet i at prices (p_1, p_2) and the indifferent types are left unspecified but clearly depend on the price vector. We want to show that a solution to the above system exists and is unique under our assumptions.

Here I will just prove that a symmetric solution with $p_1^* = p_2^*$ exists and is unique.

The candidate price vectors lie in $\Re^2$. Let's restrict our search on the diagonal: $p_1^* = p_2^* = p$. From the above analysis we know what the aggregate demand is on the diagonal for both outlets:

$$n_1^* = \frac{2D_1^l + D^s}{2}\ln(v/p) \text{ and } n_2^* = \frac{2D_2^l + D^s}{2}\ln(v/p).$$

Furthermore we know that all advertisers multi-home when prices are equal: $v_1 = v_2 = p$. Hence we can rewrite the market clearing conditions as:

$$a(2D_1^l + D^s) = \int_p^\infty \frac{2D_1^l + D^s}{2}\ln(v/p)f(v)dv$$

$$a(2D_2^l + D^s) = \int_p^\infty \frac{2D_2^l + D^s}{2}\ln(v/p)f(v)dv$$

And hence:

$$2a = \int_p^\infty \ln(v/p)f(v)dv$$

$$2a = \int_p^\infty \ln(v/p)f(v)dv$$

It follows that if there is a price that solves $2a = \int_p^\infty \ln(v/p)f(v)dv$, it must be a market clearing price. Since the right hand side is strictly decreasing in p, and satisfies the following boundary conditions: $\lim_{p \to 0}\log(v/p) = \infty, \lim_{p \to \infty}\int_p^\infty \ln(v/p)f(v)dv = 0$, then a solution to $2a = \int_p^\infty \ln(v/p)f(v)dv$ exists and is unique. Call it p^* and notice that such price does not depend on x_1 or on ρ. Plugging this into total advertising revenues (that is, the sum of the outlets' profits) we get: $\pi_1^* + \pi_2^* = a(2D_1^l + D^s)p^* + a(2D_2^l + D^s)p^* = 2ap^*$

To exclude asymmetric market clearing vectors we shall use the following result:

Claim 1 define $v_1(p) := \left\{ v \in \mathbb{°}^+ : n_1^*(p,v) = 0, \partial\pi/\partial n_1\big|_{n=n^*} = 0 \right\}$ and $v_2(p) := \left\{ v \in \mathbb{°}^+ : n_2^*(p,v) = 0, \partial\pi/\partial n_1\big|_{n=n^*} = 0 \right\}$. If $p_1 > p_2$ then $v_1(p) < v_2(p)$, and if $p_1 < p_2$ then $v_1(p) > v_2(p)$.

PROOF: consider the case $p_1 > p_2$. We have already shown that if $\partial\pi/\partial n_1\big|_{n=n^*} = 0$ then $n_2^* > 0$. Two corollaries of this are:

1. if $\partial\pi/\partial n_1\big|_{n=n^*} = 0$ and $n_1^* = 0$, then $n_2^* > 0$.

2. if $\partial\pi/\partial n_2\big|_{n=n^*}=0$ and $n_2{}^*=0$ then $n_1{}^*=0$, since $\frac{n_2{}^*}{2D_2^l+D^s}\geq\frac{n_1{}^*}{2D_1^l+D^s}$ holds.

Note that $\partial\pi/\partial n_1\big|_{n=n^*}=0$ and $n_1{}^*(p,v)=0$ define $v_1(p)$ while $\partial\pi/\partial n_2\big|_{n=n^*}=0$ and $n_2{}^*(p,v)=0$ define $v_2(p)$. Now suppose that $v_1(p)<v_2(p)$. This implies that there exists a range of values of $v\in\big[v_1(p),v_2(p)\big]$ in which $\partial\pi/\partial n_1\big|_{n=n^*}=0$ and $n_2{}^*=0$, which is a contradiction. Conversely, if $p_1<p_2$, then it must be the case that $v_1(p)>v_2(p)$.

Moreover note that when $p_1>p_2$, by result 2 we also know from $\partial\pi/\partial n_2\big|_{n=n^*}=0$ that $v_2(p)=p_2$, while $v_1(p)$ depends on $n_1{}^*$ which in turn depends on both prices. Vice-versa, if $p_1<p_2$

$v_1(p)=p_1$ while $v_2(p)$ will depend on both prices.

Claim 2. The system

$$a(2D_1^l+D^s)=\int_{v_1(p)}^{\infty}n_1{}^*(v,p_1,p_2,x_1,\rho)f(v)dv$$

$$a(2D_2^l+D^s)=\int_{v_2(p)}^{\infty}n_2{}^*(v,p_1,p_2,x_1,\rho)f(v)dv$$

has no solution in which $p_1\neq p_2$.

PROOF: Suppose $p_1>p_2$. Then we know that $\frac{n_2{}^*}{2D_2^l+D^s}<\frac{n_1{}^*}{2D_1^l+D^s}$, which implies $n_1{}^*<\frac{2D_1^l+D^s}{2D_2^l+D^s}n_2{}^*$. Substituting into the system:

$$a(2D_1^l+D^s)<\int_{v_1(p)}^{\infty}(2D_1^l+D^s)\frac{n_2{}^*}{2D_2^l+D^s}(v,p_1,p_2,x_1,\rho)f(v)dv$$

$$a(2D_2^l+D^s)=\int_{v_2(p)}^{\infty}n_2{}^*(v,p_1,p_2,x_1,\rho)f(v)dv$$

Now, since $v_2(p_1,p_2)>v_1(p_1,p_2)$ the interval over which $n_2{}^*(v,p_1,p_2,x_1,\rho)f(v)$ is integrated is such that

$$\int_{v_2(p)}^{\infty}n_2{}^*(v,p_1,p_2,x_1,\rho)f(v)dv>\int_{v_1(p)}^{\infty}n_2{}^*(v,p_1,p_2,x_1,\rho)f(v)dv$$

Therefore plugging into the system we get

$$a(2D_1^l + D^s) < \int_{v_1(p)}^{\infty} n_2^{*}(v, p_1, p_2, x_1, \rho) f(v) dv < \int_{v_2(p)}^{\infty} n_2^{*}(v, p_1, p_2, x_1, \rho) f(v) dv = a(2D_1^l + D^s)$$

which is a contradiction. Similarly, suppose $p_1 < p_2$. Then $\frac{n_2^{*}}{2D_2^l + D^s} < \frac{n_1^{*}}{2D_1^l + D^s}$, and the system becomes

$$a(2D_1^l + D^s) > \int_{v_1(p)}^{\infty} (2D_1^l + D^s) \frac{n_2^{*}}{2D_2^l + D^s}(v, p_1, p_2, x_1, \rho) f(v) dv$$

$$a(2D_2^l + D^s) = \int_{v_2(p)}^{\infty} n_2^{*}(v, p_1, p_2, x_1, \rho) f(v) dv$$

Since $v_1(p_1, p_2) > v_2(p_1, p_2)$ the interval over which $n_2^{*}(v, p_1, p_2, x_1, \rho) f(v)$ is integrated is such that

$$\int_{v_2(p)}^{\infty} n_2^{*}(v, p_1, p_2, x_1, \rho) f(v) dv < \int_{v_1(p)}^{\infty} n_2^{*}(v, p_1, p_2, x_1, \rho) f(v) dv$$

and therefore

$$a(2D_2^l + D^s) = \int_{v_2(p)}^{\infty} n_2^{*}(v, p_1, p_2, x_1, \rho) f(v) dv < \int_{v_1(p)}^{\infty} n_2^{*}(v, p_1, p_2, x_1, \rho) f(v) dv < a(2D_1^l + D^s)$$

A contradiction. Therefore, it must be $p_1^{*} = p_2^{*} = p$.

8.2 Proof of Proposition 6

When D^s is low, outlet 1's profits under no tracking are $\frac{2(2-D^s)}{4-D^s}(1-(a_1+a_2))a_1$ whereas outlet 1's profits under perfect tracking are $(1-a_1-a_2)a_1 D^s + (1-2a_1)2a_1 D^l$. Profits under perfect tracking exceed those under no tracking if: $(a_1-a_2)D^s(4-D^s) + (1-2a_1)(4-D^s) > 2(2-D^s)(1-(a_1+a_2))$. With $a_1 = a_2$, this becomes: $D^s > 0$.

When D^s is high, outlet 1's profits under no tracking may be $\frac{D^s(2-D^s)}{4+D^s(2-D^s)}(3-2(a_1+a_2))a_1$. Comparing these to the profits under perfect tracking and imposing $a_1 = a_2 = a$, perfect tracking will yield higher profits if: $\frac{2(1-2a)}{1-a} > D^s(2-D^s)$. Examining the case where $D^s = \frac{1}{2}$, note that these profits will be an equilibrium if the equilibrium price they are based on $\frac{2(2-D^s)}{4-D^s}(1-2a)$ is less than ¼. That is, if $\frac{6}{7}(1-2a) < \frac{1}{4} \Rightarrow a > \frac{17}{48}$. At $D^s = \frac{1}{2}$, we have $\frac{2(1-2a)}{1-a} > \frac{3}{4} \Rightarrow a < \frac{5}{13}$ so for $a \in [\frac{17}{48}, \frac{5}{13}]$, perfect tracking yields superior profits but for $a > \frac{5}{13}$, profits are higher under no tracking.

8.3   Proof of Proposition 8

Suppose that $a_1 < a_2$ and that $\sigma_1 = 0$. Also, assume for the moment that $v_{12'} > 1$. In this case, the conditions for outlet supply to equal outlet demand become:

$$2a_1 = 1 - v_{12} \tag{36}$$

$$2a_2 = 1 - v_{12} + v_{12} - v_2 \tag{37}$$

as outlet 1 only sells to multi-homers while outlet 2 sells to all of the single-homers. For this to be an equilibrium, prices in each outlet (which may be different) must be at a level where the marginal multi-homer is indifferent between multi-homing and single-homing on outlet 2.[30]

$$(D_1^l + \tfrac{1}{4}D^s)v_{12} - (D_1^l + \tfrac{1}{2}D^s)p_1 > (D_1^l + \tfrac{1}{2}D^s)(v_2 - p_1) \tag{38}$$

$$(D_2^l + \tfrac{1}{4}D^s)v_{12} - (D_2^l + \tfrac{1}{2}D^s)p_2 \geq (D_2^l + \tfrac{1}{2}D^s)(v_2 - p_2) \tag{39}$$

Note, first, that this requires that $p_1 \geq p_2$, otherwise, as we demonstrated above (38) could not hold, as single-homers would successful bid for impressions on 1. Instead, if $p_1 < p_2$, $(D_1^l + \tfrac{1}{4}D^s)v_{12} - (D_1^l + \tfrac{1}{2}D^s)p_1 = 0$ as multi-homers will bid up 1's impression price. Given this and (36), we can determine that in any equilibrium of this kind,

$$p_1 = \frac{D_1^l + \tfrac{1}{4}D^s}{D_1^l + \tfrac{1}{2}D^s}(1 - 2a_1) \tag{40}$$

Hence, $v_{12} = 1 - 2a_1$. Note also, that single-homers will set the impression price on outlet 2 (so that $p_2 = v_2$) and hence, the RHS of (39) will equal zero. Substituting in $v_{12} = 1 - 2a_1$ on the LHS we have:

$$p_2 \leq \frac{D_2^l + \tfrac{1}{4}D^s}{D_2^l + \tfrac{1}{2}D^s}(1 - 2a_1) \tag{41}$$

Note, however, we also have from (37) that $p_2 = 1 - 2a_2$. Thus, for this to be an equilibrium outcome requires:

$$1 - 2a_2 \leq \frac{D_2^l + \tfrac{1}{4}D^s}{D_2^l + \tfrac{1}{2}D^s}(1 - 2a_1) \tag{42}$$

Note that if $a_1 \approx a_2$ and $D^s > 0$ this cannot hold. Thus, 2's ad capacity must be significantly greater than 1's. Thus, with symmetric readerships, the asymmetric equilibrium will occur for $a_i \in [0, \frac{4a_j - D^s}{2(2 - D^s)}]$ and $a_j \in [\frac{1}{4}(2a_i(2 - D^s) + D^s, 1]$. Note that if $a_j = \frac{1}{2}$, $a_i \in [0, \frac{1}{2}]$ while if $a_i = \frac{1}{2}$, then $a_j \in [\frac{1}{2}, 1]$. Thus, if each outlet has capacity of $\frac{1}{2}$, any asymmetry will generate the asymmetric equilibrium.

This derivation assumes that $v_{12'} > 1$. If this was not the case and if $p_1 > p_2$ then the market clearing conditions for the asymmetric equilibrium would become:

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[30] With symmetric readership shares, the marginal multi-homer would not choose to single-home on outlet 1 if $p_1 > p_2$ which will turn out to be the case.

$$2a_1 = 1 - v_{12} \tag{43}$$

$$2a_2 = 2(1 - v_{12'}) + v_{12'} - v_2 \tag{44}$$

as only outlet 2 sells additional impressions to some multi-homers. Thus, outlet 1's price would remain as in (40) while outlet 2's pricing condition would satisfy (substituting $v_{12'}$ into (44)):

$$p_2 = \frac{2D^s}{2 + D^s}(1 - a_2) \tag{45}$$

This would be an equilibrium so long as $v_{12'}(p_2) < 1$ or $a_2 > \frac{2 - D^s}{4}$ in addition to the ad capacity asymmetries as identified earlier. It is easy to confirm in this case that $p_1 > p_2$.

8.4　Proof of Proposition 11

Case 1: $\frac{1}{2}D^s > D_1^l$. Suppose that $(D_1^l + \frac{1}{2}D^s)p_1 < D^s p_f$. Then consider a candidate equilibrium where high value advertisers sort as single-homers (2 impressions) on 1, then single-homers (2 impressions) on f and finally as single-homers (1 impression) on 1. In this case, equilibrium prices will be the solution to:

$$D_1^l 2a + D^s a = (D_1^l + \tfrac{1}{2}D^s)\left(2(1 - v_{1f}) + (v_f - p_1)\right) \tag{46}$$

$$\tfrac{1}{2}D^s 2a = \tfrac{1}{2}D^s 2(1 - v_{1f}) \tag{47}$$

where $v_{1f} = \frac{(2D_1^l + D^s)p_1 - D^s p_f}{D_1^l}$ and $v_f = \frac{2D^s p_f - (2D_1^l + D^s)p_1}{D^s - 2D_1^l}$. Solving this gives:

$$p_1 = \frac{aD_1^l + D^s(1 - 2a)}{D_1^l + D^s} \tag{48}$$

$$p_f = 1 - 2a - 2(1 - 3a)\frac{D_1^{l2}}{D^s} \tag{49}$$

(recalling that we assume that $a \le \frac{1}{4}$). It is easy to demonstrate that $p_f > p_1$ and that $(D_1^l + \frac{1}{2}D^s)p_1 < D^s p_f$. This confirms the equilibrium.

Is it possible that $(D_1^l + \frac{1}{2}D^s)p_1 > D^s p_f$? In this case, a candidate equilibrium would have high value advertisers sort as multi-homers (2 impressions) on f and then single-homers (2 impressions) on f. In this case, no advertiser will choose single-homing on 1. Thus, equilibrium prices will be the solution to:

$$D_1^l 2a + D^s a = (D_1^l + \tfrac{1}{2}D^s)(1 - v_{1f}) \tag{50}$$

$$\tfrac{1}{2}D^s 2a = \tfrac{1}{2}D^s 2(1 - p_f) \tag{51}$$

where $v_{1f} = \frac{(D_1^l + \frac{1}{2}D^s)p_1}{D_1^l}$. Solving this gives:

$$p_1 = \frac{D_1^l(1 - 2a)}{2D_1^l + D^s} \tag{52}$$

$$p_f = 1 - a \tag{53}$$

It is easy to demonstrate that $p_f > p_1$ but that $(D_1^l + \frac{1}{2}D^s)p_1 - D^s p_f = (\frac{1}{2}-a)D_1^l - D^s(1-a) > 0 \Rightarrow \frac{D^s}{D_1^l} < \frac{\frac{1}{2}-a}{1-a}$ which cannot hold as the LHS is greater than 2 while the RHS is less than 2. Thus, this cannot be an equilibrium.

Case 2: $\frac{1}{2}D^s < D_1^l$. Suppose that $(D_1^l + \frac{1}{2}D^s)p_1 > D^s p_f$. Then consider a candidate equilibrium where high value advertisers sort as multi-homers (2 impressions) on f, then single-homers (1 impression) on 1 and finally single-homers (2 impressions) on f. In this case, equilibrium prices will be the solution to:

$$D_1^l 2a + D^s a = (D_1^l + \frac{1}{2}D^s)(1-v_1) \tag{54}$$

$$\frac{1}{2}D^s 2a = \frac{1}{2}D^s 2(1-v_{1f}+v_1-p_f) \tag{55}$$

where $v_{1f} = 2p_f$ and $v_1 = \frac{(2D_1^l+D^s)p_1-2D^s p_f}{2D_1^l-D^s}$. Solving this gives:

$$p_1 = \frac{6D_1^l(1-2a)+D^s}{3(2D_1^l+D^s)} \tag{56}$$

$$p_f = \frac{2}{3}-a \tag{57}$$

(recalling that we assume that $a \leq \frac{1}{4}$). It is easy to demonstrate that $p_f < p_1$ and that $(D_1^l + \frac{1}{2}D^s)p_1 > D^s p_f$. This confirms the equilibrium.

9 Appendix B: Endogenous Advertising Capacity

9.1 Perfect Tracking

The previous results highlight the importance of relative advertising capacity in determining which outlets may gain from the Internet in the future. We now endogenize capacity choice, so that outlets can commit to smaller capacity levels than could be potentially supplied, focusing on how it relates to both readership share and the share of multi-homing consumers. Observe that the choice here for outlets is capacity *per consumer per unit of attention.* We do *not* allow outlets to sell different quantities of advertising to different types of consumers.

We assume that there are only two outlets to focus on the impact of outlet asymmetry.[31] This means that an outlet will face demands for two sets of consumers – one set that it has monopoly control over and the other for which it competes with its rival a la Cournot. We now consider an analysis of the comparative statics of competition in this set-up.

We can write profits as a function of capacity, readership share and ρ:

$$\pi_i(a_i,a_j;x_i,\rho)=P(a_i+a_j)a_iD_{ij}^s(x_i,1-x_i,\rho)+P(2a_i)2a_iD_i^l(x_i,\rho)$$

Let $MR_i^D(a_i,a_T)=\left(a_iP'(a_T)+P(a_T)\right)$ and $MR_i^M(a_i)=2\left(2a_iP'(2a_i)+P(2a_i)\right)$. The first-order conditions for outlet i imply:

$$MR_i^D(a_i,a_i+a_j)D_{ij}^s(x_i,x_j,\rho)+MR_i^M(a_i)D_i^l(x_i,\rho)=0.$$

This shows that the outlet considers the relative proportion of switchers and loyals when choosing output, and it will select capacity so that one of the marginal revenue terms is positive while the other is negative. Note that if $a_i>a_j$, then if P is decreasing and concave, $MR_i^D(a_i,a_i+a_j)<0$ implies that $MR_i^M(a_i)<0$. Thus, for the outlet with the larger equilibrium capacity, we must have $MR_i^D(a_i,a_i+a_j)\geq 0$ in equilibrium: capacity is chosen lower than the Cournot best response, but higher than the monopoly level for that outlet. The converse is not necessarily true, however; the outlet with small equilibrium capacity may also have $MR_i^D(a_i,a_i+a_j)\geq 0$ (and indeed, this holds in the case of uniformly distributed advertiser valuation).

The impact of an increased readership share on the incentive to expand capacity is:

$$\tfrac{\partial^2}{\partial a_i\partial x_i}\pi_i=MR_i^D(a_i,a_i+a_j)\tfrac{\partial}{\partial x_i}D_{ij}^s(x_i,1-x_i,\rho)+MR_i^M(a_i)\tfrac{\partial}{\partial x_i}D_i^l(x_i,\rho)$$

$$=MR_i^D(a_i,a_i+a_j)2\rho(1-x_i)+MR_i^M(a_i)(1-\rho+2x_i\rho)$$

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[31] All of the qualitative predictions in this subsection apply for a general $F(.)$ assumed to be log-concave. (Proofs available from the authors).

At an equilibrium choice of capacity, the ratio of the marginal revenue terms is equal to the ratio of switchers to loyal users, so that we will have (where $\hat{a}_i$ is the equilibrium capacity for i):

$$\frac{\partial^2}{\partial a_i \partial x_i} \pi_i \Big|_{(a_i,a_j)=(\hat{a}_i,\hat{a}_j)} = MR_i^D(\hat{a}_i, \hat{a}_i + \hat{a}_j)\left(\frac{\partial}{\partial x_i} D_{ij}^s(x_i, 1-x_i, \rho) - \frac{D_{ij}^s(x_i, x_j, \rho)}{D_i^l(x_i, \rho)} \frac{\partial}{\partial x_i} D_i^l(x_i, \rho) \right)$$

$$= -MR_i^D(\hat{a}_i, \hat{a}_i + \hat{a}_j) 2\rho(1-x_i)\left(\frac{x_i \rho}{1-(1-x_i)\rho} \right)$$

Since higher readership share increases the proportion of loyal users, its direct effect on capacity is negative if and only if $MR_i^D(\hat{a}_i, \hat{a}_i + \hat{a}_j) \geq 0$. Intuitively, becoming larger causes a firm to put more weight on loyal users, giving it the incentive to reduce output. However, clear equilibrium comparative statics are complicated by the fact that Cournot outputs are strategic substitutes.

We can also consider the impact of switching on capacity choice:

$$\frac{\partial^2}{\partial a_i \partial \rho} \pi_i = MR_i^D(a_i, a_i + a_j)\frac{\partial}{\partial \rho} D_{ij}^s(x_i, 1-x_i, \rho) + MR_i^M(a_i)\frac{\partial}{\partial \rho} D_i^l(x_i, \rho)$$

$$= MR_i^D(a_i, a_i + a_j) 2x_i(1-x_i) - MR_i^M(a_i)x_i(1-x_i)$$

At an equilibrium capacity choice, we will have

$$\frac{\partial^2}{\partial a_i \partial \rho} \pi_i \Big|_{(a_i,a_j)=(\hat{a}_i,\hat{a}_j)} = MR_i^D(\hat{a}_i, \hat{a}_i + \hat{a}_j) 2x_i^2(1-x_i)\left(\frac{1 - 2\rho(1-x_i)}{1 - \rho(1-x_i)} \right)$$

So long as switching is not too prevalent and outlets are not too asymmetric, switching decreases the share of loyal users, so that the direct effect of switching on capacity is positive if and only if $MR_i^D(\hat{a}_i, \hat{a}_i + \hat{a}_j)(1 - 2\rho(1-x_i)) \geq 0$. Thus, the direct effect is unambiguously positive for the outlet with the larger share.

Using the envelope theorem, we can write the impact of ρ on profits as follows:

$$\frac{d}{d\rho} \pi_i(a_i^*(x_i,\rho), a_j^*(x_i,\rho); x_i, \rho) = P'(a_i^* + a_j^*)a_i^* D_{ij}^s(x_i, 1-x_i, \rho)\frac{\partial}{\partial \rho} a_j^*(x_i, \rho)$$

$$+ P(a_i^* + a_j^*)a_i^* \frac{\partial}{\partial \rho} D_{ij}^s(x_i, 1-x_i, \rho) + P(2a_i^*)2a_i^* \frac{\partial}{\partial \rho} D_i^l(x_i, \rho)$$

$$= P'(a_i^* + a_j^*)a_i^* D_{ij}^s(x_i, 1-x_i, \rho)\frac{\partial}{\partial \rho} a_j^*(x_i, \rho)$$

$$+ 2P(a_i^* + a_j^*)a_i^* x_i(1-x_i) - 2P(2a_i^*)a_i^* x_i(1-x_i)$$

Switching has an indirect effect through increasing the opponent's output, which (if it increases opponent capacity) lowers price and thus profits. It also has a direct effect of increasing the proportion of switchers and decreasing the proportion of loyals. The sum of the last two terms is negative if and only if $a_i^* \leq a_j^*$: for the lower-capacity outlet, switchers are less profitable. The analysis for the outlet with the higher equilibrium output appears ambiguous if its competitor's output is increasing in ρ, as the price effect and the switcher/loyal effect move in opposite directions.

Summarizing the discussion so far, we can gain some intuition about the direct effects of parameter changes on outlet capacity choices and profits, but some additional structure on demand is required to obtain unambiguous comparative statics results. To do so, we focus on the

case of linear demand (uniformly distributed advertiser valuations). The following proposition demonstrates that the larger outlet will provide the lowest advertising capacity.

Proposition A1. *Suppose that there are two outlets and that $F(v) = v$. Equilibrium advertising for each outlet, $\hat{a}_i$ are non-increasing in readership share, x_i. Equilibrium advertising $\hat{a}_i$ is non-decreasing in ρ if $x_i \leq (21 - \sqrt{249})/6 \approx .87$ or $\rho \leq (2/3)(3 - \sqrt{3}) \approx .84$. Total ad capacity, $\hat{a}_i + \hat{a}_j$, is non-decreasing in ρ. For sufficiently symmetric firms ($.33 \leq x_i \leq .67$), profits of both firms are decreasing in ρ, while for sufficiently asymmetric firms, profits are decreasing (increasing) in ρ for $x_i > (<)x_j$. $\pi_i^{PT}/x_i < \pi_j^{PT}/x_j$ when $x_i > x_j$. $\pi_i^{PT} - \pi_j^{PT}$ is decreasing in ρ for $x_i > x_j$.*

PROOF: Solving for the unique Nash equilibrium with the uniform distribution we have:

$$\hat{a}_i = \frac{16 D_i^l D_j^l + 6 D_j^l D^s + 4 D_i^l D^s + D^{s2}}{64 D_j^l D_i^l + 16(D_j^l + D_i^l)D^s + 3 D^{s2}} \tag{58}$$

$$\pi_i^{PT} = \frac{(4 D_i^l + D^s)(16 D_i^l D_j^l + 6 D_j^l D^s + 4 D_i^l D^s + D^{s2})^2}{(64 D_j^l D_i^l + 16(D_j^l + D_i^l)D^s + 3 D^{s2})^2} \tag{59}$$

The rest of the proposition follows from manipulating these expressions.

We have already developed some intuition for these results, but the uniform distribution gives us more definitive conclusions. Consider the comparative statics of switching on profits. The increase in capacity of an opponent's outlet has a negative impact on each outlet. However, the increase in the share of switchers has a positive (resp. negative) effect on the smaller (larger) outlet, as the share of consumers coming from the switchers goes up. Switchers are more (less) profitable than loyals for the smaller (larger) outlet, because the larger outlet serves less capacity than the smaller outlet. With the uniform distribution, for the small outlet the latter effect dominates the negative effect of increase in capacity and small outlet profits go up.

Note that switching also affects the impact of an increase in readership share on profits. Under the benchmark single-homing consumer case, more readers simply improved profits in a linear fashion; that is π_i^{PT}/x_i was independent of x_i. With perfect tracking, an additional reader attracted from a rival outlet not only causes an outlet to restrict advertising capacity but for that capacity to increase elsewhere (since capacities are strategic substitutes in our Cournot setup), decreasing impression prices for switchers. Thus, outlets with a lower readership share have a higher incentive to attract marginal readers.

It is also useful to note that if the two outlets were commonly owned, their owner would maximize joint outlet profits by setting $a_1 = a_2 = \frac{1}{4}$. By Proposition 2, in this case, realized profits in this case will be the same as those generated when there are no switchers. Thus, under perfect tracking with $D^s > 0$ there will be an incentive for outlets to merge.

In the absence of common ownership, multi-homing consumers cause outlets to compete for advertisers and a greater proportion of them increases available advertising space and decreases overall profits. However, the question of interest is what this does to the marginal incentive to attract an additional reader at the expense of rivals. What we can demonstrate is that as $x_i \to 0$ or $x_i \to 1$, then $\frac{\partial \pi_i^{PT}}{\partial x_i} > \frac{\partial \pi_i^{NS}}{\partial x_i} = \frac{1}{4}$. It is useful to note that if both outlets are commonly

owned (i.e., in a monopoly), then profits under perfect tracking are the same as profits earned for each outlet in the no switching case. Thus, competition is the source of any reduction in profits as a result of switching but this competition can, in turn, promote higher incentives to attract readership when there are asymmetric readership shares.

9.2   Imperfect tracking

Suppose that competition comprises two stages (as in the perfect tracking case). In stage 1, both outlets simultaneously choose their ad capacities. In stage 2, the market clears based on those capacities and prices and profits are realized. It turns out that, in this situation, a pure strategy equilibrium in the Stage 1 (Cournot) game does not exist for a non-trivial rage of D^s.

Proposition A2. *With endogenous capacity, $F(v) = v$ and symmetric readership shares, the pure strategy equilibrium outcomes are:*

(i) For $D^s = 0$, $a_i = \frac{1}{4}$ with per consumer profits of $\pi_i = \frac{1}{4}$ for all i.

(ii) For $D^s \geq \frac{4}{9}$, $a_i = \frac{1}{3}$ with per consumer profits of $\pi_i = \frac{2(2-D^s)}{4-D^s}\frac{2}{9}$ for all i.

Otherwise no pure strategy equilibrium exists.

PROOF: Note that for $D^s = 0$, $v_{12} = p$ and the asymmetric equilibrium holds for any $(a_1, a_2) \notin (\frac{1}{4}, \frac{1}{4})$. In any asymmetric equilibrium, per consumer profits equal $(1 - 2a_i)2a_i$ for each outlet; which is maximized at a capacity of ¼. Hence, by deviating, each would receive no greater profits than they do under the equilibrium as specified in (i).

To check that outcome (ii) is an equilibrium, observe that if each outlet plays a local best response, they each choose capacity equal to $\frac{1}{3}$. Now consider a choice $a_1 >> \frac{1}{3}$ so that $p(a_1, a_2) \leq \frac{1}{2}D^s$. In this case, the highest profits outlet 1 could earn are: $\max_{a_1} \frac{D^s(2-D^s)}{4+D^s(2-D^s)}(3 - 2(a_1 + \frac{1}{3}))2a_1$ which is maximized at $a_1 = \frac{7}{12}$; which would create the asymmetric equilibrium. Thus, the maximum capacity 1 would chose would be $\frac{1}{12}(4 + D^s)$ resulting in profits of $\frac{1}{36}(2 - D^s)(4 + D^s) < \frac{2(2-D^s)}{4-D^s}\frac{2}{9}$. Now consider a choice $a_1 << \frac{1}{3}$ so that $\sigma_1 = 0$; specifically, $a_1 \leq \frac{4 - \frac{1}{3}D^s}{6(2-D^s)}$. In this case, outlet 1 maximizes profits with a choice of $a_1 = \frac{1}{4}$ earning profits of $p_1 2a_1 = \frac{D_1^l + \frac{1}{4}D^s}{D_1^l + \frac{1}{2}D^s}(1 - 2a_1)2a_1 = \frac{1}{8}(2 - D^s)$ which is greater than $\frac{2(2-D^s)}{4-D^s}\frac{2}{9}$ for $D^s \leq \frac{4}{9}$. When $D^s > \frac{4}{9}$, this deviation is not profitable. Finally, we need to check that, in fact, $p(a_1, a_2) \geq \frac{1}{2}D^s$. This implies that $\frac{1}{2}D^s < \frac{2(2-D^s)}{4-D^s}\frac{1}{3} \Rightarrow D^s < \frac{2}{3}(4 - \sqrt{10})$ which always holds for $D^s \leq \frac{1}{2}$.

We now turn to establish that there are no other pure strategy equilibria. First, note when $p < \frac{1}{2}D^s$, it is easy to see that $a_1 = \frac{1}{2}$ is a local best response to $a_2 = \frac{1}{2}$. At this point, each outlet earns profits of $\frac{D^s(2-D^s)}{4+D^s(2-D^s)}$. Note, however, that any deviation from these capacities generates the asymmetric equilibrium. Thus, setting $a_1 >> \frac{1}{2}$ would earn that

outlet profits of $\frac{2D^s}{2+D^s}(1-a_1)2a_1$ which are maximized at ½ and exceed $\frac{D^s(2-D^s)}{4+D^s(2-D^s)}$ at this point. A reduction in capacity would involve maximum profits at $a_1 = \frac{1}{4}$. In this case, it is easy to establish that $\frac{D^s(2-D^s)}{4+D^s(2-D^s)} < \frac{1}{8}(2-D^s)$ and so a large reduction in ad capacity is a profitable deviation for outlet 1. Thus, no equilibria of this type exists.

What about an asymmetric equilibrium? Any equilibrium would involve the outlet with the smaller capacity, say 1, choosing $a_1 = \frac{1}{4}$ while the other outlet chooses $a_2 = \frac{1}{8}(2+D^s)$. Note that this is consistent with $v_{12'} > 1$ and it is straightforward to establish that outlet 2 would not want to choose a higher ad capacity to change this. In this case, outlet 2 earns per consumer profits of $(1-2a_2)2a_2$ and it is easy to determine that these are decreasing in a_2 at $a_2 = \frac{1}{8}(2+D^s) > \frac{1}{4}$. Therefore, given 1's choice, 2 would not find it profitable to expand output. Contracting it would generate profits of $\frac{2(2-D^s)}{4-D^s}(1-\frac{1}{4}-a_2)2a_2$; maximized at 3/8 which would involve too much asymmetry to generate that outcome. Thus, any contraction involves profits less than $\frac{1}{16}(4-D^{s2})$. For 1, $a_1 = \frac{1}{4}$ is a local best response, but by choosing a higher ad capacity, it may earn different profits depending upon the resulting impression price. For $p \geq \frac{1}{2}D^s$, outlet 1 would earn per consumer profits of $\frac{2(2-D^s)}{4-D^s}(1-\frac{1}{8}(2+D^s)-a_1)2a_1$ which is maximized at $a_1 = \frac{1}{16}(6-D^s)$. However at this capacity, ad capacities would be sufficiently asymmetric that this would not be feasible. Instead, outlet 1 is constrained to a capacity no more than $\frac{1}{16}(4+4D^s-D^{s2})$. Note that this results in a price $p = \frac{2(2-D^s)}{4-D^s}(1-\frac{1}{8}(2+D^s)-\frac{1}{16}(4+4D^s-D^{s2})) \geq \frac{1}{2}D^s$. It is straightforward to demonstrate that this deviation is profitable for 1. A similar reasoning holds for the case where $p < \frac{1}{2}D^s$. Thus, there is no pure strategy equilibrium involving asymmetric capacity choices.

Intuitively, for smaller levels of D^s, each outlet would prefer to be the outlet with the larger capacity so long as the required asymmetry is not too large. When that occurs, their preferences switch. Consequently, there is a (downwards) discontinuity in the best response functions of each outlet for $D^s \in (0, \frac{4}{9})$ and no pure strategy equilibrium exists.

Given the lack of a pure strategy equilibrium for a non-trivial set of parameters, we might consider a mixed strategy equilibrium. However, given this application, it is unclear whether mixing in its strict form is something that we would expect to see; specifically, because ad capacity may be a design decision for web pages.[32] As an alternative, the following proposition characterizes the Stackelberg outcome where one outlet chooses its ad capacity prior to the other.

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[32] Frankly, we have also been unable to identify the mixed strategy equilibrium although we know the set that contains its support and that that set converges to (¼, ¼) as D^s goes to 0.

Proposition A3. *In a sequential move game where outlet 1 chooses a_1 before outlet 2 chooses a_2, the unique equilibrium outcome involves $a_1 = \frac{2+\sqrt{2D^s}-2D^s}{4(2-D^s)}$ and $a_2 = \frac{1}{4}$ with per consumer profits of $\pi_1 = \frac{2-3D^s+\sqrt{2D^{s3}}}{2(2-D^s)^2}$ and $\pi_2 = \frac{1}{8}(2-D^s)$.*

> PROOF: If $a_1 = \frac{2+\sqrt{2D^s}-2D^s}{4(2-D^s)}$, then outlet 2 is different between $a_2 = \frac{1}{4}$ or setting its capacity high enough to ensure that outlet 1 only has multi-homers; that is, $a_2 \geq \frac{1}{4}(2a_1(2-D^s)+D^s = \frac{1}{8}(2+\sqrt{2D^s})$. So 2 has no incentive to deviate. Outlet 1 has no incentive to increase capacity as this lowers its asymmetric equilibrium profits. It could, however, decrease capacity. This would result in 2 no longer being indifferent between a high and low capacity and choosing a high capacity, $\frac{1}{4}(2a_1(2-D^s)+D^s$. This would result in profits for 1 as the low capacity outlet in the asymmetric equilibrium which are maximized at ¼ yielding $\frac{1}{8}(2-D^s)$. These are less than the equilibrium profits and hence, there is no profitable deviation for 1.

Notice that the equilibrium profits of both outlets is decreasing in D^s from a starting point of ¼ where $D^s = 0$.

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