From perception to action: an economic model of brain processes [*]

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Abstract

Building on evidence from neurobiology and neuroscience, we model the physiological limitations faced by individuals in the process of decision-making that starts with sensory perception and ends in action selection. The brain sets a neuronal threshold, observes whether the neuronal cell firing activity reaches the threshold or not, and takes the optimal action conditional on that (limited) information. We show that the optimal threshold is set in a way that existing beliefs are likely to be confirmed. The conclusion holds in static and dynamic settings, and with linear and quadratic loss functions. We then relate our result to the somatic marker theory, and argue that it provides support for the hypothesis that emotions help decision-making. Last, we discuss the implications for choices in concrete vs. abstract situations, for interactions in cooperative vs. competitive activities, for reactions to expected vs. unexpected events, and for the choice of cognitive vs. affective encoding channels.

Keywords: Neuroeconomic theory, neurobiology, neuroscience, reason, emotion, information processing, Bayesian learning.

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1 Introduction

Economic theory has traditionally been interested in the analysis of choices. In particular and with some exceptions (Bernheim and Rangel, 2004), the *processes* by which individuals reach decisions have been overlooked, mainly because we had little knowledge of the pathways going from perception to action. With the development of increasingly sophisticated designs and techniques to measure brain activity, the neurobiology and neuroscience literatures have substantially improved our understanding of the biological mechanisms that transform sensory perceptions into voluntary actions. These results can now be incorporated into formal economic models of decision-making.

Building theoretical models of brain processes is an important step both for economics and neuroscience. For economics, incorporating *physiological costs and constraints* in the capacity of individuals to evaluate situations, process information and reach conclusions has two advantages. First, it provides guidelines on which assumptions seem most plausible when we try and model "bounded rationality." Second, it can help provide microfoundations for some well-documented errors and biases in choices (see Brocas and Carrillo (2008) for a more detailed exposition of these arguments).[1] For neuroscience, formal models of the brain can provide testable implications about the functionality of different brain systems and their interactions in decision-making.

Since our theory builds on literatures that, in principle, are distant from economics, we start with a brief overview of the recent research relevant for our study.

1.1 Some background from the brain sciences

The basic premises for our theory come from two overlapping literatures.

1. Neurobiology. Researchers in neurobiology have studied the neural mechanisms underlying the transformation of sensory signals into decisions. One of the early theories, the "Efficient Coding Hypothesis" postulates that neurons encode information as compactly as possible, so as to use resources efficiently (Barlow (2001), Simoncelli (2003)). This theory has recently led to a myriad of sophisticated statistical models that describe bayesian stochastic processing of information by neurons in visual, auditory and haptic perception tasks (see e.g. Schwartz and Simoncelli (2001), Ernst and Banks (2002), Körding and Wolpert (2004) and Ma et al. (2006)).

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[1] One could draw a parallel with the theory of organizations, where a more accurate modelling of organizational constraints (agency problems, restricted information channels, limited resources) has helped understanding the tendency of organizations to take certain decisions and avoid some others.

In a classical study, Hanes and Schall (1996) use single cell recording to analyze the neural processes responsible for the duration and variability of reaction times in monkeys. The authors find that movements are initiated when *neural activity reaches a certain threshold activation level*, in a winner-takes-all type of contest.[2] Also, *stochastic variability in cell firing rates* is responsible for the observed differences in reaction times. Building on this work, Shadlen et al. (1996) and Gold and Shadlen (2001) study a motion discrimination task, where monkeys must decide whether the net direction of dots that appear on a monitor is upward or downward. The authors develop a theory of how information is processed. Neurons "compute" approximately the log-likelihood ratio of the alternatives in order to determine which hypothesis should be supported by the evidence. Thus, according to this result, neurons incorporate the two major ingredients of bayesian theory: prior probabilities, and stochastic information processing (see also Deneve et al. (1999) for a numerical simulation model).

The work by Shadlen et al. (1996) has led other researchers to study whether neural circuits have a similar way to encode information in more complex situations. Ditterich et al. (2003) show that when the task is more difficult (fewer dots move in synchrony), monkeys make more mistakes, a result also consistent with a stochastic information accumulation theory. Also, using fMRI studies, Heekeren et al. (2004) find that the mechanism by which the brain of a monkey computes perceptual decisions is also at work for humans and for more sophisticated choices, such as image recognition. Last, Platt and Glimcher (1999) demonstrate that neurons react not only to probabilities of gains (as already discussed) but also to magnitudes of gains, hence computing approximately the "expected value" associated to each alternative (see also Glimcher et al. (2005) and the review by Glimcher and Rustichini (2004)).

2. Affective neuroscience. The increased interest during the 1990s in understanding the neural basis of emotions led to the development of a new subdiscipline called "affective neuroscience" (Davidson and Sutton, 1995). Numerous studies using both PET scan and fMRI techniques have been conducted on different emotions and reached two general conclusions. First, emotions are crucial for decision-making, as they guide actions towards salient goals (Davidson and Irwin, 1999). Second, abnormalities in the functioning of some brain regions are responsible for emotional disorders, including social phobias (Reiman, 1997) and depression (Drevets et al., 1997). These pathologies result in poor choices.[3]

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[2] Nichols and Newsome (2002) provide a further analysis of the type of situations where information processing is likely to follow a winner-takes-all vs. a vector averaging pattern.

[3] The primary goal in most of these studies is not so much to discuss these well-established facts but, instead, to identify the brain circuitry behind each particular emotion or emotion disorder. These details, however, are less crucial for the purpose of our study.

In his review of the brain areas involved in reward processing, Schultz (2000) concludes that neurons adapt their activity according to ongoing experiences. Of special importance are neurons in the orbitofrontal cortex (Tremblay and Schultz, 2000), an area that plays a major function in the experience of emotions. Inspired by this research, and in many cases preceding it, Damasio (1994) developed the "somatic marker hypothesis", a neural theory that rests on two ingredients. First, emotions affect decisions by modifying neuronal thresholds in a precise direction: the individual becomes more receptive to information that supports current beliefs and less receptive to information that contradicts current beliefs. As summarized by Bechara and Damasio (2005):

> [P]re-existing somatic states influence the threshold of neuronal cell firing in trigger structures (e.g., VM cortex) so that subsequent somatic states from thoughts (secondary inducers) are triggered more or less easily. [...] While pre-existing negative somatic states reinforce subsequent negative states, they may impede the effectiveness of positive ones. Similarly, pre-existing positive states reinforce positive states, but they may impede negative ones (p. 363-4).

Second, this threshold modification induced by somatic dispositions improves decision-making. Again, in the words of Bechara and Damasio (2005):

> These somatic states are indeed beneficial, because they consciously or non-consciously bias the decision in an *advantageous* manner (p. 351, italics added).

Rustichini et al. (2005) provide further evidence that the emotional circuitry is active in the task of processing information. In our view, one weakness of the somatic marker theory is that it does not explain why a modification in that direction is beneficial. One objective of our paper is to build a formal framework that can address this issue.

1.2 Implications for decision-making and overview of the results

According to the neurobiology literature reviewed above, there are three basic principles in the physiological mechanism of information processing. *First*, neurons carry information from the sensory to the decision-making system, using an imperfect encoding technology: the level of neuronal cell firing depends stochastically on the information obtained. *Second*, the motor cortex triggers an action whenever the cell firing activity in favor of one alternative reaches a certain threshold. *Third*, the individual has the ability to modify the triggering threshold. By acting on the threshold, the individual affects the likelihood of interpreting evidence for and against each option.

Assume now that the brain has been developed in a way that it optimizes the acquisition of information, in order to take the best possible action.[4] Due to the biological constraints described above, it is only able to choose a neuronal threshold and learn whether it is reached or not. The first objective of our study is to find the optimal threshold. That is, we determine what is the best way to process information given the documented physiological limitations, and then discuss its implications for decision-making. We study how the thresholds vary over time, across activities, under different priors, and for different objectives. Finally, we analyze the effect on choices of an impairment in the capacity of an individual to modulate thresholds. The second (complementary) objective of the paper is to relate our findings to somatic dispositions. According to the affective neuroscience literature also reviewed above, emotions play an important role in the regulation of neuronal activity.[5] The somatic marker theory goes one step beyond in arguing that: (i) emotions operate on neuronal thresholds, (ii) they modify thresholds in a way that existing beliefs are likely to be confirmed, and (iii) this modulation of thresholds improves decision-making. However, it does not provide a compelling argument for the optimality of this threshold modulation. Since our model determines the optimal threshold, it is equipped to discuss whether and why the threshold modifications postulated by the somatic marker theory are beneficial. In other words, if we accept (i) and (ii) as a premise, our model can determine whether (iii) is indeed its logical consequence.

To formalize the neurobiological principles described above, we consider a simple model with two states, A and B. We assume that cell firing stochastically depends on the state, with high cell firing being more likely in state A and low cell firing in state B. The brain sets a threshold and determines only whether the cell firing surpasses it or not. Given the information obtained, an action is undertaken. Payoffs depend on the combination of action and state. The first and probably less surprising contribution of the paper is to show that the threshold is optimally set in a way that beliefs are likely to be supported. That is, if the agent becomes more confident that the state is A, the threshold is decreased. Thus, the new threshold is more likely to be surpassed *whether the state is indeed A or not* and, as a result, the agent is more likely to take the action which is optimal in that state but suboptimal in the other (Proposition 1). The logic for this property is simple. As the likelihood of A increases, stronger contradictory information is required to reverse that belief. From standard Bayesian theory, we know that stronger information

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[4] Optimization in this context is a working assumption which may be defended on evolutionary grounds. It is obviously debatable. However, we feel that any other assumption would have been less satisfactory.

[5] As documented in psychology and acknowledged in neuroscience, emotions affect behavior in many other ways. To better focus the discussion, here we concentrate on its biological role and ignore the other components.

towards state B can only be obtained if *a lower* threshold is not reached, hence the result. This result matches the findings obtained in the classical theory of organizations literature (Calvert (1985), Sah and Stiglitz (1986), Meyer (1991)) using related models. Importantly, this simple result generalizes in a number of dimensions, such as different payoff formulations (e.g., linear and quadratic loss functions) and a continuum of states. Under some conditions, it also holds when multiple thresholds can be set sequentially, which implies that the ability to modify neuronal thresholds has a snowball effect on decision-making: a stronger belief towards one state implies a greater threshold variation in its favor, therefore a higher probability that new information supports it, and so on (Propositions 2 to 6).

The second contribution is more directly tied to the neurobiology of emotions and the somatic marker theory. According to Bechara and Damasio (2005), the role of emotions is to act on neuronal thresholds by rendering the individual relatively more receptive to information that supports current beliefs. If we accept this as a premise, then our previous results demonstrate that the somatic marker's claim –namely, that emotions improve decisions– is indeed correct. In other words, a person with an emotional deficit will not modulate thresholds in this direction and, as a result, will take suboptimal actions more frequently. Furthermore, the effect of emotions on thresholds is all the more important in dynamic settings: as time passes, emotionally balanced individuals are most likely to maintain the beliefs initially favored (Corollaries 1 and 2).

The third contribution is to connect more tightly the economic model with the neuroscience evidence, and discuss some comparative statics and implications of the theory. We show that the limited ability of the individual to process information does not prevent efficient decision-making if the environment is static and there are only two relevant alternatives, which we label as "basic" situations. We also argue that cognitive encoding channels with multiple thresholds are relatively more valuable in complex environments whereas affective encoding channels with one or few thresholds are more appropriate in simple environments (Propositions 7 and 8). As for the implications, we prove that optimal thresholds are more sensitive to initial beliefs the weaker the correlation between state and cell firing. This suggests that threshold modulation is most important in activities where information is subjective and difficult to interpret, an argument that has received support in experimental neuroscience. We also show that thresholds are more sensitive to beliefs when agents benefit if their peers also take the correct action (positive externalities) than when they benefit if they alone take the correct action (negative externalities). Thus, optimal threshold modulation seems more important in cooperative environments than in competitive ones. Last, we highlight the importance of the correlation between action and payoff when discussing the reaction of individuals to expected and unexpected events. If the correlation is high, an agent who sets thresholds optimally will exhibit a low (positive) reaction to success and high (negative) reaction to failures, as emphasized in the neuroscience literature. However, if the correlation is weak, that agent understands the importance of luck on outcomes and has a low reaction to both successes and failures. (Propositions 9 to 11).

1.3 Related literature

The existing neuroeconomic literature that models the interplay between reason and emotion (Bernheim and Rangel (2004), Benhabib and Bisin (2005), Loewenstein and O'Donoghue (2005)) assumes competition between a rational / cognitive system and an impulsive / affective system as well as a specific cost-benefit tradeoff between the two (see Zak (2004) or Camerer et al. (2005) for reviews).[6] Our work departs substantially from this literature in that we do not presuppose any tradeoff between reason and emotion. In fact, we do not even assume separation and competition between cognitive and affective systems. Instead, we propose a more primitive model that incorporates the *physiological constraints faced by the brain in the decision-making process*, and discuss the role of emotions in this constrained optimization problem. Since we focus on the physiological mechanisms behind the choice process, our paper is closer to the "physiological expected utility" theory developed by Glimcher et al. (2005).[7] It is worth noting that the dichotomy between rational/cognitive and automatic/affective systems has been very successful in neuroeconomic circles, mainly because it captures in a parsimonious way the tension between reason and passion. However, it should also be acknowledged that most neuroscientists strongly disagree with a literal interpretation of this dichotomy. According to Glimcher et al. (2005, p. 251-2): "[T]here is no evidence that hidden inside the brain are two fully independent systems, one rational and one irrational. [...] What we cannot stress strongly enough is that the vast majority of evolutionary biologists and neurobiologists reject this view." Similarly, Phelps (2006, p. 27) argues "that the classic division between emotion and cognition may be unrealistic and that an understanding of human cognition

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[6] Brocas and Carrillo (2007) and Fudenberg and Levine (2006, 2007) adopt the model by Thaler and Shefrin (1981), where the two competing systems have different temporal horizons. In those works, the myopic system is not necessarily irrational; it simply has a different (in this case, more immediate) objective. There is also a literature where the individual makes choices anticipating that emotions –fear, anxiety or rejoicing– affect utility (Caplin and Leahy (2001), Palacios-Huerta (2004)) and some other works where the individual has imperfect self-knowledge resulting in other types of intrapersonal conflicts (Carrillo and Mariotti (2000), Benabou and Tirole (2002, 2004), Bodner and Prelec (2003)).

[7] There is also a literature that explores the neurobiological foundations for social behavior. Again, it analyzes the problem from an experimental viewpoint (see for e.g., Zak et al. (2004) on the effect of oxytocin on trust).

requires the consideration of emotion."[8] There is no doubt that the metaphor is adopted mainly for modelling purposes, and therefore can be a reasonable first approximation in many contexts. Nonetheless, the objections raised by neuroscientists suggest that it can be complementary and interesting to explore other ways (maybe more directly connected to evidence from the brain sciences) to model and understand the role of emotions on decision-making. The present paper offers such an alternative approach.

Finally, there are several areas outside economics that study a related problem, although from a different angle. First, there is a literature on information processing that adopts a purely statistical approach. Neurobiologists have worked on complex statistical algorithms that mimic what neurons in the brain do (see the references in section 1.1). Theoretical neuroscientists have constructed mathematical and computational models of the brain based on the underlying biological mechanisms (see Dayan and Abbott (2005) for an introduction). Psychophysicists have developed a "signal-detection theory" to study the likelihood of finding a weak signal in a noisy environment, depending on the statistical properties of the noise and signal random variables (see McNicol (1972) or Wickens (2002) for an introduction). Second, there is also a literature on neural networks and artificial intelligence which builds models inspired by the architecture of the brain in order to solve specific tasks like data processing or filtering (see Abdi (1996) for a review).

2 A model of neuronal cell firing activity

2.1 The brain: a constrained processor of information

An individual (he) obtain some evidence and takes an action. His objective is to process the information as efficiently as possible, given the physiological limitations of his brain. As reviewed above, the neurobiology literature highlights three key aspects of signal processing. First, information is scarce and imperfect: neuronal cell firing is stochastically correlated with the state. Second, the brain can only determine whether the neuronal cell firing activity surpasses a given threshold or not. Third, the brain (and, more specifically, the soma according to the somatic marker theory) has the ability to choose the neuronal threshold.

To understand the effects of this limited capacity to process information, we consider a basic choice situation. The individual must decide how far from the cave to go hunting. There are two states, $S \in \{A, B\}$: leaving the cave is either dangerous (A) or safe (B), and the individual assigns probability $p \in (0,1)$ to state A. We denote by $\gamma \in [0,1]$ the action, where lower values of γ denote going farther away from the cave to hunt. The payoff of

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[8] See also LeDoux (1996, ch. 4) for an non-technical, historical perspective of the misconceptions about the physical location of functions in the brain.

the individual is $\pi_A \, l(\gamma - 1)$ if $S = A$ and $\pi_B \, l(\gamma - 0)$ if $S = B$, where $l(z) = l(-z)$ for all z and $l'(z) < 0$ for all $z > 0$. Thus, if hunting is dangerous ($S = A$), the individual should stay as close as possible to the cave ($\gamma = 1$), since each mile traveled increases the probability of encountering a predator. If hunting is safe ($S = B$), then he should go as far away as possible to find the best preys ($\gamma = 0$).[9] Note that π_S captures the marginal cost of taking a wrong action given that the true state. So, $\pi_A > \pi_B$ would reflect the idea that hunting on a dangerous day is more costly than staying in the cave on a safe day. Given a belief p, the expected payoff function that the individual maximizes is:

$$L(\gamma; p) = p \Big[\pi_A \, l(\gamma - 1)\Big] + (1 - p) \Big[\pi_B \, l(\gamma)\Big] \tag{1}$$

Before making his choice, the individual takes one step out of the cave and looks outside. The information transmitted by the sensory system is modeled in a way to incorporate the three premises reviewed above. First, neuronal cell firing c is stochastic and depends on the state. Formally, the likelihood of a cell firing level $c \in [0, 1]$ is $g(c)$ if the state is A and $f(c)$ if the state is B, with $G(c) = \int_0^c g(y)dy$ and $F(c) = \int_0^c f(y)dy$ representing the probability of a cell firing activity not greater than c when the state is A and B, respectively. Furthermore, high cell firing is relatively more likely when $S = A$ and low cell firing is relatively more likely when $S = B$. Said differently, on dangerous days there are stochastically more neurons in the amygdala carrying a message of fear to the brain than on safe days. Formally, the functions $g(\cdot)$ and $f(\cdot)$ satisfy the standard Monotone Likelihood Ratio Property (MLRP):[10]

Assumption 1 (MLRP) $\left(\dfrac{f(c)}{g(c)}\right)' < 0$ *for all* c. **(A1)**

This assumption states that the probability of state B (hunting is safe) rather than A (hunting is dangerous) monotonically decreases with the level of cell firing. In other words, an increase in the number of neurotransmitters carrying the signal "fear" indicates a higher likelihood that a predator is present. Figure 1 provides a graphical representation of cell firing probabilities for symmetric functions $f(\cdot)$ and $g(\cdot)$.

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[9] We deliberately chose an example encountered by a primitive individual because it illustrates reasonably well evolutionary relevant situations. For current applications, it makes more sense to think of the states as investment A or B being most profitable, relationship A or B being more gratifying, etc.

[10] MLRP implies: (i) $\frac{1-G(c)}{g(c)} > \frac{1-F(c)}{f(c)}$, (ii) $\frac{G(c)}{g(c)} < \frac{F(c)}{f(c)}$, and (iii) $G(c) < F(c) \; \forall \; c \in (0, 1)$.

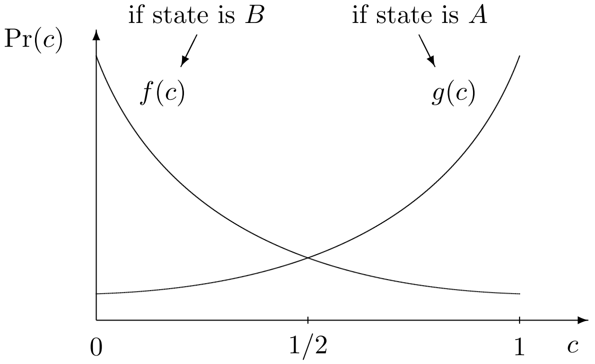

Figure 1. Stochastic cell firing level.

Second, the brain can only determine whether the neuronal activity is above or below a threshold x. This means that, for *any* given threshold x, a cell firing activity above it will suggest (more or less strongly) that $S = A$, whereas a cell firing activity below it will indicate (more or less strongly) that $S = B$. Third, we allow the brain to modify the threshold, that is, to select x. In the next section, we determine how the threshold should be set in order to optimize learning.

The timing of the game can thus be summarized as follows. In stage 0, nature picks a state $S \in \{A, B\}$. In stage 1, the brain sets a threshold $x \in (0, 1)$, there is neuronal activity $c \in [0, 1]$, and the brain determines whether the cutoff is reached or not ($c \gtrless x$). In stage 2, the individual updates his beliefs and chooses an action $\gamma \in [0, 1]$. Final payoffs depend on the action γ and the state S. Figure 2 depicts this sequence of events.

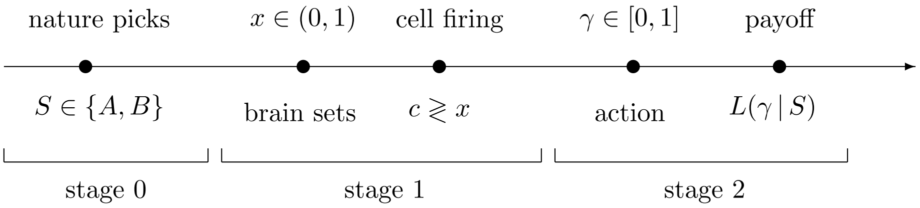

Figure 2. Timing.

2.2 Optimal cell firing threshold

Our first objective is to determine the optimal threshold. We solve this game by backward induction. In stage 2, the optimal action depends on the shape of the payoff $l(\cdot)$. We first assume that $l(z)$ is weakly convex on both sides of its bliss point $z = 0$: $l''(z) \geqslant 0$ for all z, so that departures from the optimal action are decreasingly costly. A special case of this class of functions is the linear loss function $l(z) = -|z|$. Note that when $l(z)$ is weakly convex, then the expected payoff function, $L(\gamma; p)$, is weakly convex in γ and differentiable in $(0, 1)$, so corner solutions are optimal. If we denote by $\gamma^*(p) = \arg\max_\gamma \; L(\gamma; p)$, necessary and sufficient conditions for $\gamma^*(p) = 1$ and $\gamma^*(p) = 0$ are, respectively, $L(1; p) > L(0; p)$ and $L(0; p) > L(1; p)$. Using (1) and given that $l(0) > l(1)$, we have:

$$\gamma^*(p) = 1 \;\text{ if }\; p > p^* \equiv \frac{\pi_B}{\pi_A + \pi_B} \qquad \text{and} \qquad \gamma^*(p) = 0 \;\text{ if }\; p < p^* \equiv \frac{\pi_B}{\pi_A + \pi_B}$$

Not surprisingly, if the marginal cost of an incorrect action in a given state increases, then the individual is more willing to take the action optimal in that state even at the increased risk of erring in the other state. Formally, $dp^*/d\pi_A < 0$ and $dp^*/d\pi_B > 0$. In our example, as predators become smarter and more dangerous, the individual is more likely to decide to stay in the cave, even on days that are apparently safe.

In stage 1, the threshold x is selected. Before proceeding, three properties must be noted. First, for any p and x, the belief about state A is revised upwards if x is surpassed and downwards if x is not reached:

$$\overline{p}(x) \equiv \Pr(A \,|\, c > x) \quad = \quad \frac{p(1 - G(x))}{p(1 - G(x)) + (1 - p)(1 - F(x))} \; > \; p \quad \forall p,\, x \in (0, 1)^2 \tag{2}$$

$$\underline{p}(x) \equiv \Pr(A \,|\, c < x) \quad = \quad \frac{p\, G(x)}{p\, G(x) + (1 - p) F(x)} \; < \; p \quad \forall p,\, x \in (0, 1)^2 \tag{3}$$

It captures the idea that low cell firing is an (imperfect) indicator of state B and high cell firing an (imperfect) indicator of state A. Second, a necessary condition for a threshold x to be optimal is that it must prescribe different actions depending on whether $c \gtrless x$, otherwise stage 1 would be uninformative. Third, when evaluating the likelihood of A, surpassing a stringent cutoff is more unlikely than surpassing a weak cutoff, but it is also a stronger indicator that this state is correct: $\partial \Pr(A \,|\, c > x)/\partial x > 0$. Also, not reaching a stringent cutoff is more likely than not reaching a weak cutoff, but it is also a weaker indicator that this state is incorrect: $\partial \Pr(A \,|\, c < x)/\partial x > 0$. A similar argument holds for state B. This property captures the negative relation in Bayesian learning contexts between likelihood and impact of information.

Properties one and two imply that the individual will set a threshold x such that $\gamma^* = 1$ if the threshold is exceeded and $\gamma^* = 0$ if the threshold is not met. Together with property three, it means that the optimal threshold maximizes the following value function:

$$\begin{aligned} V(x;p) &= \Pr(c > x)\, L(1;\overline{p}(x)) + \Pr(c < x)\, L(0;\underline{p}(x)) \\ &= p\,\pi_A \Big[(1-G(x))l(0) + G(x)l(1)\Big] + (1-p)\,\pi_B \Big[(1-F(x))l(1) + F(x)l(0)\Big] \end{aligned} \tag{4}$$

Denote by $x^*(p) = \arg\max_x\ V(x;p)$ the optimal threshold as a function of the belief p, and assume that the parameters of the model are such that $\frac{f(0)}{g(0)} > \frac{p}{1-p}\frac{\pi_A}{\pi_B} > \frac{f(1)}{g(1)}$.[11] Our first result is the following.

Proposition 1 *When $l''(z) \geqslant 0$, the optimal threshold $x^*(p)$ is unique and given by:*

$$\frac{f(x^*(p))}{g(x^*(p))} = \frac{p}{1-p}\,\frac{\pi_A}{\pi_B} \tag{5}$$

which, in particular, implies that $dx^*/dp < 0$.

Consider two individuals who differ only in their belief about the likelihood of state A. According to Proposition 1, the individual with strongest prior in favor of A sets the lowest threshold. As a result, he is more likely to receive evidence that endorses A and less likely to receive evidence that endorses B than the other agent, *both if the true state is A and if the true state is B*. In other words, the ability to set optimal thresholds increases the likelihood that an individual be reaffirmed in his belief.

This result is due to the trade-off between the likelihood of information and its impact. To see this, suppose that the individual believes that A is more likely than B and, consequently, prefers to choose action $\gamma = 1$ given his current prior. Assume also that he sets a high threshold. If the threshold is surpassed, he will be extremely confident that the state is A, whereas if the threshold is not reached, he will be only moderately convinced about state B. In both cases, his posterior belief hinges towards A, and he takes the same action he would have taken without this information. Suppose instead that the individual sets a low threshold. If the threshold is surpassed, he will slightly increase his confidence in state A, whereas if the threshold is not reached, he will become quite convinced that the state is B. The individual ends up taking different actions and, in both cases, he is quite confident about his choice. Overall, the optimal threshold must balance the belief in favor of A conditional on the threshold being surpassed and the belief in favor of B conditional on the threshold not being reached. In order to achieve this balance, the threshold should be low whenever A is a priori more probable than B In other words, when one state is a priori more likely than the other, the individual will require strong information against it to change his beliefs.

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[11] This condition ensures that the optimal solution is interior.

Notice that **(A1)** ensures the uniqueness of the local (hence, global) maximum. If, instead, we assumed stochastic dominance (a substantially weaker condition than **(A1)** formally defined as $F(c) > G(c)$ for all c), x^* would not necessarily be unique. And yet, since $V(x;p)$ is always submodular, the monotonic relation between threshold and likelihood of state, $dx^*/dp < 0$, would be preserved in all the local maxima.[12] Finally, note that the threshold is set in such a way that, in equilibrium, the most costly mistakes are most likely to be avoided: $dx^*/d\pi_A < 0$ and $dx^*/d\pi_B > 0$.

2.3 The role of emotions

The model presented so far relies exclusively on our three neurobiological premises. According to the somatic marker theory (Bechara and Damasio, 2005), there is a tight physiological link between these findings and the somatic dispositions of individuals (see also the consistent findings of Rustichini et al. (2005)). More precisely, the soma modifies the neuronal threshold by increasing the likelihood of interpreting evidence in favor of the currently supported hypothesis. They also conclude that this threshold modulation is beneficial for decision-making. Our model clarifies this issue.

Corollary 1 *If we accept as a premise that somatic dispositions play the biological role described in the somatic marker theory, then emotions do improve decision-making.*

Proposition 1 states that it is optimal to set a threshold so that the likelihood of confirming a belief is greater the stronger the prior in its favor. The somatic marker theory argues that emotions affect the threshold in that precise direction. Corollary 1 simply puts these two results together to prove that the somatic marker's claim regarding the desirability of favoring the current belief is correct. Conversely, consider an individual with an abnormal activation of somatic signals who sets the threshold independently of his belief ($dx^*/dp = 0$) or who is most receptive to information that contradicts his current belief ($dx^*/dp > 0$). According to our theory, this person will make suboptimal choices more frequently. Needless to say, the model does not argue that the soma is an intelligent system that performs the sophisticated trade-off described in Proposition 1. It simply suggests that the development through evolution of a biological mechanism (in this case, governed by emotions) with these qualitative properties is indeed "advantageous."

3 Optimal threshold in other environments

The model presented in section 2 makes a number of simplifying assumptions. It is only natural to discuss whether the properties of the mechanism hold in more general settings.

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[12] We thank Guofu Tan for pointing this out.

In this section, we extend the model in a number of directions: a concave payoff function (section 3.1), a continuum of states (section 3.2), and two stages of cell firing (section 3.3). To avoid unnecessary complications, we assume without loss of generality $\pi_A = \pi_B = 1$.

3.1 Increasingly costly departures from the optimal action

Suppose that hunting is increasingly dangerous as the individual travels farther away from the cave. This situation can be formalized with a strictly concave, continuous and twice-differentiable payoff function: $l''(z) < 0$ for all z and $l'(0) = 0$. Denote by $\gamma^{**}(p) = \arg\max_{\gamma} L(\gamma; p)$. Taking the first-order condition in (1), we have:

$$\frac{l'(\gamma^{**}(p))}{l'(1-\gamma^{**}(p))} = \frac{p}{1-p} \tag{6}$$

Note that $\left.\frac{\partial L(\gamma;p)}{\partial \gamma}\right|_{\gamma=0} = -p\,l'(1) > 0$, $\left.\frac{\partial L(\gamma;p)}{\partial \gamma}\right|_{\gamma=1} = (1-p)l'(1) < 0$, and $\frac{\partial^2 L(\gamma,p)}{\partial \gamma^2} < 0$. Thus, contrary to the previous case, extreme choices are never optimal when departures are increasingly costly: $\gamma^{**}(p) \in (0,1)$ for all $p \in (0,1)$. Furthermore:

$$\frac{d\gamma^{**}(p)}{dp} \propto \frac{\partial^2 L(\gamma;p)}{\partial \gamma \partial p} = -l'(1-\gamma) - l'(\gamma) > 0$$

This means that, as the likelihood of danger increases, the individual decides to stay closer to the cave. Suppose the brain sets a threshold x. Given (2), (3) and (6), the optimal actions when $c > x$ and $c < x$ are, respectively:

$$\frac{l'(\gamma^{**}(\overline{p}(x)))}{l'(1-\gamma^{**}(\overline{p}(x)))} = \frac{\overline{p}(x)}{1-\overline{p}(x)} = \frac{p}{1-p}\,\frac{1-G(x)}{1-F(x)} \tag{7}$$

$$\frac{l'(\gamma^{**}(\underline{p}(x)))}{l'(1-\gamma^{**}(\underline{p}(x)))} = \frac{\underline{p}(x)}{1-\underline{p}(x)} = \frac{p}{1-p}\,\frac{G(x)}{F(x)} \tag{8}$$

where $\gamma^{**}(\overline{p}(x)) > \gamma^{**}(\underline{p}(x))$. Differentiating (7) and (8) and using **(A1)**, we obtain:

$$\frac{d\gamma^{**}(\overline{p}(x))}{dx} > 0 \quad \text{and} \quad \frac{d\gamma^{**}(\underline{p}(x))}{dx} > 0 \tag{9}$$

According to (9), the individual reacts to an increase in the threshold by choosing always a higher action. Indeed, if the higher threshold is surpassed, then the evidence in favor of state A is stronger. Conversely, if the higher threshold is not reached, then the evidence in favor of B is weaker. In both cases, higher actions follow. We can now define the value function of the individual. Adapting (4) to the present case, we get:

$$\begin{aligned} V(x;p) &= \Pr(c > x)\, L(\gamma^{**}(\overline{p}(x)); \overline{p}(x)) + \Pr(c < x)\, L(\gamma^{**}(\underline{p}(x)); \underline{p}(x)) \\ &= p(1-G(x))l(1-\gamma^{**}(\overline{p}(x))) + (1-p)(1-F(x))l(\gamma^{**}(\overline{p}(x))) \\ &\quad + p\,G(x)\,l(1-\gamma^{**}(\underline{p}(x))) + (1-p)F(x)\,l(\gamma^{**}(\underline{p}(x))) \end{aligned} \tag{10}$$

To solve this value function, we first need to introduce a strengthened version of MLRP:

Assumption 2 (s-MLRP) $\frac{f(c)}{1-F(c)} > \frac{g(c)}{1-G(c)}$, $\left(\frac{f(c)}{g(c)}\frac{1-G(c)}{1-F(c)}\right)' \leqslant 0$, $\left(\frac{f(c)}{g(c)}\frac{G(c)}{F(c)}\right)' \leqslant 0$ **(A2)**

Note that the first and second or the first and third parts in **(A2)** imply **(A1)**, but the converse is not true. Denote by $x^{**}(p) = \arg\max_x \; V(x;p)$. The optimal threshold maximizes (10) given (7) and (8). The first-order condition is:

$$\frac{\partial V(x;p)}{\partial x}\bigg|_{x=x^{**}} = 0 \;\Rightarrow\; \frac{f(x^{**})}{g(x^{**})} = \frac{p}{1-p}\,\frac{l(1-\gamma^{**}(\overline{p}(x^{**}))) - l(1-\gamma^{**}(\underline{p}(x^{**})))}{l(\gamma^{**}(\underline{p}(x^{**}))) - l(\gamma^{**}(\overline{p}(x^{**})))} \tag{11}$$

and we can now state our next result.

Proposition 2 *Under* **(A2)** *and when* $l''(z) < 0$*, a sufficient condition to have a unique optimal threshold* x^{**} *given by (11) and such that* $dx^{**}/dp < 0$ *is:*

$$\frac{d}{dx}\Big[F(x)l(\gamma^{**}(\underline{p}(x))) + (1-F(x))l(\gamma^{**}(\overline{p}(x)))\Big]_{x=x^{**}} > 0 \qquad \textbf{(C1)}$$

Under **(A1)***, condition* **(C1)** *guarantees* $dx^{**}/dp < 0$ *in every locally optimal threshold but not uniqueness.*

When departures from the optimal action are increasingly costly, the quasi-concavity of the value function $V(x;p)$ is not guaranteed for generic values of the densities and payoff functions $f(\cdot)$, $g(\cdot)$ and $l(\cdot)$. In fact, there are two countervailing forces at play when choosing x. First, a higher threshold is less likely to be surpassed and therefore more likely to induce the low action. Second, either outcome is a weaker indicator that the state is B. Therefore, the final action will be higher both when the threshold is surpassed and when it is not reached. Proposition 2 provides a sufficient condition, **(C1)**, such that $V(\cdot)$ is well-behaved. The interpretation of this condition is simple: starting from the optimal threshold, setting a higher x increases the payoff of the individual if and only if the state is B. In other words, as x increases, the direct effect of increasing the likelihood of choosing the low action must dominate the indirect effect of choosing relatively higher actions. It is also clear why the condition is automatically satisfied when payoffs are weakly convex: since only $\gamma^* = 0$ or $\gamma^* = 1$ are optimal, a marginal change in the threshold does not affect the choice of action (indirect effect) but it does change the likelihood of surpassing it (direct effect).

In order to provide a full characterization of the equilibrium, we will now restrict attention to quadratic payoffs, $l(z) = \alpha - \beta z^2$ with $\beta > 0$. Under this restriction, the optimal action coincides with the belief of the individual. Formally, (7) and (8) become:

$$\gamma^{**}(\overline{p}(x)) = \overline{p}(x) \quad \text{and} \quad \gamma^{**}(\underline{p}(x)) = \underline{p}(x) \tag{12}$$

Replacing (12) into (11), we obtain the following result.

Proposition 3 *Under* **(A2)** *and with quadratic payoffs, the optimal threshold* x^{**} *is unique, it satisfies* $dx^{**}/dp < 0$*, and it solves:*

$$\frac{f(x^{**})}{g(x^{**})} = \frac{p}{1-p} \, \frac{(1-\overline{p}(x^{**})) + (1-\underline{p}(x^{**}))}{\overline{p}(x^{**}) + \underline{p}(x^{**})} \tag{13}$$

Under **(A1)***, there might be multiple locally optimal thresholds, but they all satisfy* $dx^{**}/dp < 0$*.*

The main conclusion in Proposition 1 is that thresholds are set in such a way that existing beliefs are likely to be reaffirmed. According to Propositions 2 and 3, the result extends to quadratic payoffs. It also extends to other concave loss functions as long as **(C1)** is satisfied. It is important to notice that the results are consistent with the experimental evidence according to which individuals can and often do generate reports of beliefs that vary continuously. Indeed, the ability to perform marginal changes in thresholds results in continuous changes in expected beliefs, both when these are surpassed and when they are not. Furthermore, under concave loss functions, these continuous changes in beliefs imply also continuous changes in reports. The following example illustrates some other properties of the equilibrium.

Example 1. Suppose that the cell firing distribution functions are $G(c) = c^2$ and $F(c) = c$. From (5) and (13) and after some algebra, the optimal thresholds with linear ($l(z) = -|z|$) and quadratic ($l(z) = -z^2$) payoffs are respectively:

$$x^*(p) = \tfrac{1-p}{2p} \quad \text{and} \quad x^{**}(p) = \tfrac{\sqrt{1-p}}{\sqrt{1-p}+\sqrt{1+p}}$$

where x^* and x^{**} are interior if $p > 1/3$. In this example, the optimal threshold is always less extreme with quadratic than with linear payoffs: $x^* \gtreqqless x^{**} \gtreqqless 1/3$ for all $p \lesseqqgtr 3/5$.

3.2 Enlarging the state space

In this section, we are interested in situations in which making a decision requires to assess the likelihood of many states, so that information is more complex to evaluate. To better isolate this new dimension, we consider the model presented in section 2.1 and assume that there is a continuum of states $s \in [0, 1]$, but only two possible actions $\gamma \in \{0, 1\}$. In our example, there are many predators, and s captures the proportion of predators who are currently in the neighborhood. The individual can only go hunting ($\gamma = 0$) or stay in the cave ($\gamma = 1$). We order the states by the increasing degree of danger, from safest

$(s = 0)$ to most dangerous $(s = 1)$. The probability of a cell firing level c given state s is now $f(c \,|\, s)$. The generalization of MLRP to the continuous case is:

Assumption 1' (continuous MLRP) $\frac{d}{dc}\left(\frac{f_s(c \,|\, s)}{f(c \,|\, s)}\right) \geqslant 0$ for all c and s. **(A1')**

The individual initially believes that the state is s with probability $p(s)$, where $\int_0^1 p(s)ds = 1$. The expected payoff function described in (1) can then be generalized as:

$$L(\gamma; p(s)) = \int_0^1 p(s) l(\gamma - s) ds$$

Since we restrict the action space to $\gamma \in \{0, 1\}$, the optimal action is:

$$\hat{\gamma} = 1 \text{ if } \int_0^1 p(s)\Big(l(1-s) - l(s)\Big)ds > 0 \quad \text{and} \quad \hat{\gamma} = 0 \text{ if } \int_0^1 p(s)\Big(l(1-s) - l(s)\Big)ds < 0$$

Given a prior distribution $p(s)$ and a threshold x, the individual will put more weight in states closer to 1 if $c > x$ and in states closer to 0 if $c < x$. Thus, in equilibrium, the individual will choose $\hat{\gamma} = 1$ if the threshold is surpassed and $\hat{\gamma} = 0$ if the threshold is not reached (this property will be checked ex-post). The value function is then:

$$\begin{aligned} V(x; p(s)) &= \Pr(c > x)\, L(1; p(s \,|\, c > x)) + \Pr(c < x)\, L(0; p(s \,|\, c < x)) \\ &= \int_0^1 p(s)\Big((1 - F(c \,|\, s)) l(1-s) + F(c \,|\, s) l(s)\Big) ds \end{aligned} \tag{14}$$

Denote by $\hat{x}(p(s)) = \arg\max_x \; V(x; p(s))$. We have:

Proposition 4 *With a continuum of states $s \in [0, 1]$ and only two actions $\gamma \in \{0, 1\}$, the optimal threshold $\hat{x}$ is unique and given by:*

$$-\int_0^1 p(s) f(\hat{x} \,|\, s)\Big(l(1-s) - l(s)\Big) ds = 0 \tag{15}$$

If $\left(\frac{q(s)}{p(s)}\right)' \geqslant 0$, then $\hat{x}(q(s)) < \hat{x}(p(s))$.

The main conclusions stated in Proposition 1 extend to the case of two actions and a continuum of states: (i) the optimal threshold is unique and (ii) if one individual puts more weight in higher states than another in a MLRP sense, $\left(\frac{q(s)}{p(s)}\right)' \geqslant 0$, then he also sets a lower threshold. This property is simply a generalization of the comparative statics on p to the case of a continuous distribution of beliefs. It thus strengthens the idea that, under an optimal threshold, existing beliefs are more likely to be supported and less likely to be refuted than opposite beliefs.

3.3 Dynamic thresholds

So far, we have assumed that the individual takes one look out of the cave before choosing whether to go hunting. Another natural extension is to consider the situation in which he can take a second look before deciding what to do. This case is interesting only if the individual can re-optimize the threshold after having looked for the first time (otherwise, it boils down to a model with one stage of cell firing and a more accurate information). We deal with this situation by adding one stage to the basic model described in section 2.1. In stage 1, the individual has a prior belief p. He sets a threshold y, learns whether $c_1 \gtrless y$, and updates his belief. In stage 2 and given his new belief, he sets a new threshold x, learns whether $c_2 \gtrless x$ and, again, updates his belief. The action (and payoff) is contingent on his posterior belief which depends on $c_1 \gtrless y$ and $c_2 \gtrless x$. We assume that c_t is independently drawn from distribution $G_t(c_t)$ if $S = A$ and distribution $F_t(c_t)$ if $S = B$, with $t \in \{1, 2\}$. Distributions may be different across stages but both $\frac{f_1(c)}{g_1(c)}$ and $\frac{f_2(c)}{g_2(c)}$ satisfy **(A1)**.

The game is solved by backward induction. The second cell firing stage is identical to that described in section 2.2. The second stage value function is then given by (4). If the first threshold y is surpassed, the posterior is $\overline{p}(y)$ and the optimal second period threshold is $x^*(\overline{p}(y))$. If the first threshold y is not reached, the posterior is $\underline{p}(y)$ and the optimal second period threshold is $x^*(\underline{p}(y))$. Combining (2), (3) and (5), we get:

$$\frac{f_2(x^*(\overline{p}(y)))}{g_2(x^*(\overline{p}(y)))} = \frac{\overline{p}(y)}{1-\overline{p}(y)} = \frac{p}{1-p}\,\frac{1-G_1(y)}{1-F_1(y)} \tag{16}$$

$$\frac{f_2(x^*(\underline{p}(y)))}{g_2(x^*(\underline{p}(y)))} = \frac{\underline{p}(y)}{1-\underline{p}(y)} = \frac{p}{1-p}\,\frac{G_1(y)}{F_1(y)} \tag{17}$$

The value function that the individual maximizes in the first cell firing stage is:

$$W(y;p) = \Pr(c_1 > y)\Big[V(x^*(\overline{p}(y)); \overline{p}(y))\Big] + \Pr(c_1 < y)\Big[V(x^*(\underline{p}(y)); \underline{p}(y))\Big] \tag{18}$$

The first term is the likelihood of surpassing a cutoff y, in which case the posterior becomes $\overline{p}(y)$, multiplied by the second-stage value function given this posterior (see (4)), and under the anticipation of an optimal second-stage threshold $x^*(\overline{p}(y))$ (see (5)). The same logic applies to the second term. Notice that threshold y affects the utility of the individual only through its effect in the posterior belief transmitted to stage 2. Denote by $y^*(p) = \arg\max_y \; W(y;p)$ the optimal stage 1 threshold. It maximizes (18) under (16) and (17). Taking the first-order condition in (18) and applying the envelope theorem:

$$\left.\frac{\partial W(y;p)}{\partial y}\right|_{y=y^*} = 0 \quad \Rightarrow \quad \frac{f_1(y^*)}{g_1(y^*)} = \frac{p}{1-p}\,\frac{G_2(x^*(\underline{p}(y^*))) - G_2(x^*(\overline{p}(y^*)))}{F_2(x^*(\underline{p}(y^*))) - F_2(x^*(\overline{p}(y^*)))} \tag{19}$$

and we get the analogue of Proposition 2 to the dynamic threshold case.

Proposition 5 *Under* **(A2)** *and with two cell firing stages, a sufficient condition to have a unique optimal threshold* y^* *in stage 1 given by (19) and such that* $dy^*/dp < 0$ *is:*

$$\frac{d}{dy}\Big[F_1(y)\,F_2(x^*(\underline{p}(y))) + (1 - F_1(y))F_2(x^*(\overline{p}(y)))\Big]_{y=y^*} > 0 \qquad \textbf{(C2)}$$

Under **(A1)***, condition* **(C2)** *guarantees* $dy^*/dp < 0$ *in every locally optimal threshold but not uniqueness.*

It is well-known that two-stage optimization problems are easily plagued by non-convexities in the overall maximand, hence the possibility of multiple local optima. Proposition 5 determines a sufficient condition for uniqueness of the maximum. Under this condition, the familiar comparative statics with respect to p is also preserved. In fact, the two-stage model with decreasingly costly departures is technically similar to the one-stage model with increasingly costly departures. In particular, the same two effects operate when the threshold is increased. First, a direct effect: the new threshold is less likely to be surpassed. Second, an indirect effect: because surpassing a higher threshold is a stronger indicator of state A whereas not reaching a higher threshold is a weaker indicator of state B, an increase in stage 1 threshold is always followed by a decrease in stage 2 threshold ($dx^*(\overline{p})/dy < 0$ and $dx^*(\underline{p})/dy < 0$). As before, the condition stated in the proposition ensures that the direct effect dominates the indirect one.

Following a similar procedure as in section 3.1, we impose specific functional forms to be able to characterize the equilibrium. We assume that the second stage cell firing densities are linear and symmetric, $g_2(c) = 2c$ and $f_2(c) = 2(1 - c)$, and keep a general formulation for the first stage cell firing densities. It turns out that, under this restriction, the optimal first stage threshold takes the same form as in the one-stage quadratic case.

Proposition 6 *Under* **(A2)** *and with second stage linear and symmetric cell firing densities, the optimal first stage threshold* y^* *is unique, it satisfies* $dy^*/dp < 0$*, and it solves:*

$$\frac{f_1(y^*)}{g_1(y^*)} = \frac{p}{1-p}\,\frac{(1 - \overline{p}(y^*)) + (1 - \underline{p}(y^*))}{\overline{p}(y^*) + \underline{p}(y^*)} \qquad (20)$$

Under **(A1)***, there might be multiple locally optimal thresholds, but they all satisfy* $dy^*/dp < 0$*.*

Proposition 6 shows that, under some conditions, being more receptive to information that confirms rather than contradicts existing beliefs is optimal also when further news are subsequently available. The intuition relies, just as before, on the balance between the likelihood of the information and its impact: the individual must sacrifice either quality or probability of obtaining information, and quality is relatively more valuable for the state that is a priori less favored. Interestingly, with an increased number of information processing stages, the partition of beliefs becomes finer. This means that a larger number of posterior beliefs can be reached depending on how many thresholds are surpassed and how many are not. Note also that, with three or more stages of cell firing, the thresholds at all but the last one only affect the belief inherited at the following stage (just like y only affects the belief at stage 2). Thus, we conjecture that the main properties of the thresholds emphasized in Propositions 1 and 6 should, under reasonable conditions, be preserved in a situation involving more than two cell firing stages. Unfortunately, when we increase the number stages, the problem becomes too challenging to be solved analytically.

Propositions 1 and 6 taken together have an interesting implication for the neurobiological role of emotions on decision-making.

Corollary 2 *Under the premises of the somatic marker theory, emotions improve decision-making at every stage of the information gathering process. Furthermore, an emotional individual is most likely to develop beliefs that are maintained and least likely to develop beliefs that are abandoned.*

Because beliefs are developed on the basis of relevant information, it is obvious that, other things being equal, further evidence is more likely to reaffirm the individual in his beliefs than contradict him. Corollary 2 makes a stronger argument: compared to an individual with an abnormal activation of somatic signals, an emotional individual will modulate thresholds in a way that he is most likely to stick with the same action as his prior intention was. Indeed, an increase in the belief that the true state is A implies a decrease in the first period cutoff, so a higher likelihood of surpassing it. Furthermore, if it is surpassed, the belief is again updated upwards and the second stage threshold is decreased even further, resulting in a snowball effect.[13] We conclude this section with a simple example that nicely illustrates these and other effects of optimal threshold selection on the process of information acquisition

Example 2. Consider a two-stage cell firing model and suppose that the density functions are identical in both stages, symmetric and linear: $g_t(c) = 2c$ and $f_t(c) = g(1-c) = 2(1-c)$, with $t \in \{1, 2\}$. From (5) and (20) and after some algebra, the optimal thresholds in the

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[13] Naturally and using a basic property of martingales, if the emotional individual receives contradictory evidence, he will revise his beliefs more strongly in the other direction (see section 5.3). Also, as signals tend to infinity, the individual always ends up learning the true state independently of whether thresholds are set optimally or not. The point here is that by optimally modulating thresholds, the total probability of taking the action favored by the prior belief is increased.

first and second stage are:

$$\left(\frac{1-y^*}{y^*}\right)^2 = \frac{p}{1-p} \;\Leftrightarrow\; y^*(p) = \frac{\sqrt{1-p}}{\sqrt{1-p}+\sqrt{p}} \quad \text{and} \quad \frac{1-x^*}{x^*} = \frac{p}{1-p} \;\Leftrightarrow\; x^*(p) = 1-p$$

The threshold is always less sensitive to the belief in the first stage than in the second stage, $x^*(p) \gtreqless y^*(p) \gtreqless 1/2$ for all $p \lesseqgtr 1/2$, which seems quite natural. The individual anticipates that, after the first cell firing stage, there is still more information to come. Therefore, maximizing the amount of information acquired in the first stage is relatively more important than learning whether, at this point, it supports or contradicts current beliefs, and more information is on average obtained with a less extreme threshold. More generally, the result implies that for some extreme priors, the belief about which state is more probable will not be reversed after the first cell firing stage.[14] This sharply contrasts with the second cell firing stage, where a necessary condition for a threshold to be optimal is that it must prescribe different actions depending on the result of cell firing.

4 The effect of physiological limitations on decision-making

The information processing mechanism by which the individual sets a threshold and learns only whether it is surpassed necessarily implies some information loss, relative to a situation where the individual could determine the exact level of cell firing. In this section, we discuss the cost of this physiological constraint as a function of the environment considered: two or more states, one or two cell firing stages, and concave or convex loss functions (section 4.1). We then propose a simple extension where the individual can choose between a low cost but coarse information partition and a high cost but precise information partition, and study in which cases one of them dominates the other (section 4.2).

4.1 Costs of a coarse information partition

Suppose that the brain could determine the exact level of cell firing. Interestingly, in the environment described in section 2.2 with two states and a weakly convex utility loss, physiological limitations *do not prevent efficient decision-making.* The key is that $l''(z) \geqslant 0$ implies that only extreme choices are optimal. The threshold x^* is set in such a way that if the individual learned that $c = x^*$ (a zero-probability event), then he would

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[14] Formally, $\Pr(A|c_1 > y^*(p)) = 1 - [y^*(p)]^2 < \frac{1}{2}$ for all $p < \frac{2-\sqrt{2}}{4}$. Symmetrically, $\Pr(A|c_1 < y^*(p)) = [1 - y^*(p)]^2 > \frac{1}{2}$ for all $p > \frac{2+\sqrt{2}}{4}$. If $p \in (0, \frac{2-\sqrt{2}}{4})$, the individual will believe at the end of the first stage that B is more likely than A independently of whether $c_1 \gtrless y^*$. If $p \in (\frac{2+\sqrt{2}}{4}, 1)$, the individual will believe at the end of the first stage that A is more likely than B independently of whether $c_1 \gtrless y^*$.

be indifferent between actions $\gamma^* = 0$ and $\gamma^* = 1$. Formally and using (5):

$$\Pr(A \,|\, c = x^*) = \frac{p\, g(x^*)}{p\, g(x^*) + (1-p)\, f(x^*)} = \frac{\pi_B}{\pi_A + \pi_B}$$

As a result, $\gamma^* = 1$ dominates $\gamma^* = 0$ *for all* $c > x^*$ and $\gamma^* = 0$ dominates $\gamma^* = 1$ *for all* $c < x^*$ which, in turn, means that learning whether x^* is surpassed or not is sufficient for the purpose of determining which action to take. Even though the assumptions in section 2.2 are restrictive, it is worth thinking in which circumstances they are met. Convexity of the payoff function reflects the fact that marginal departures from the ideal choice are the most costly ones. Therefore, it is suitable to model environments where life-threatening events occur as soon as the optimal action is not taken. In our example, it can capture a high probability of both fatal injury when a predator is encountered and death by starvation if no food is collected.

The same conclusion may extend to more complex environments. In the continuum of states extension presented in section 3.2, the individual does not suffer a utility loss form a coarse information partition, simply because the action space is binary. Overall, as long as (i) there is one stage of cell firing and (ii) only two actions are relevant (in equilibrium as in section 2.2 or by assumption as in section 3.2), setting one threshold will be sufficient. We label these environments as "basic" or "primitive" decision-making situations. We can also see why the two conditions are necessary. Consider a situation with increasingly costly departures and a large action space. As discussed in section 3.1, the optimal action in that environment is different for every belief. Therefore, if the individual is only able to set a threshold, he will have only one of two posterior beliefs (depending on whether the threshold is surpassed or not), which immediately results in a strictly positive utility loss. A similar argument applies to the dynamic environment considered in section 3.3. Finally, notice that under a coarse information partition, the order in which evidence is received matters. For example, under the sequence c^* 'small' followed by c^{**} 'high', $\gamma = 1$ is more likely to be chosen (i.e., stage 2 threshold is more likely to be surpassed) than under the sequence c^{**} followed by c^*. The results of this section are summarized as follows.

Proposition 7 *Physiological limitations in information processing may not prevent efficient decision-making but only in basic situations: a static environment with a choice between two relevant alternatives.*

We provide a simple analytical example characterizing the utility loss due to a coarse information partition in a two-stage cell firing environment.

Example 3. Consider the same setting as in Example 2. Let $l(0) = 1$, $l(1) = 0$ and $p = 1/2$, which implies that $y^* = \frac{1}{2}$, $\overline{p}(\frac{1}{2}) = \frac{3}{4}$, $x^*(\overline{p}(\frac{1}{2})) = \frac{1}{4}$, $\underline{p}(\frac{1}{2}) = \frac{1}{4}$, and $x^*(\underline{p}(\frac{1}{2})) = \frac{3}{4}$.

Given $p(A|c_1) = c_1$, the expected payoff if the exact level of cell firing c_1 is observed is:

$$\tilde{W} = p\int_0^1 \Pr(c_1|A)\Big[1 - G(x^*(p(A|c_1)))\Big]dc_1 + (1-p)\int_0^1 \Pr(c_1|B)\Big[F(x^*(p(A|c_1)))\Big]dc_1 = \frac{5}{6}$$

The payoff if the individual learns only whether the threshold $y^* = 1/2$ is surpassed is:

$$\begin{aligned} W \;=\; & \Pr(A)\Big[\Pr(c_1 > \tfrac{1}{2}\,|\,A)\ \Pr(c_1 > \tfrac{1}{4}\,|\,A) + \Pr(c_1 < \tfrac{1}{2}\,|\,A)\Pr(c_1 > \tfrac{3}{4}\,|\,A)\Big] \\ & + \Pr(B)\Big[\Pr(c_1 < \tfrac{1}{2}\,|\,B)\ \Pr(c_1 < \tfrac{3}{4}\,|\,B) + \Pr(c_1 > \tfrac{1}{2}\,|\,B)\Pr(c_1 < \tfrac{1}{4}\,|\,B)\Big] \;=\; \frac{13}{16} \;<\; \tilde{W} \end{aligned}$$

which, in this case, results in a utility loss of only 2.5%.

4.2 The affective and cognitive encoding channels

As discussed in section 1.3, the main objective of the paper is not to analyze the interplay between a rational and an emotional system in the brain. Nevertheless, understanding whether neuronal thresholds are modulated by the soma (and, if so, why) remains a challenge in neuroeconomics. In that respect, comparing the behavior of an individual supposedly using an emotional process with the behavior of an individual using a "more sophisticated" process might shed some light on this puzzle.

In this section, we postulate that, for each decision, the brain may have to choose between different encoding channels: an affective process where the partition of information is coarse but fast vs. a cognitive process where the partition is fine but slow and/or energy demanding.[15] The affective process is the process analyzed so far; it only determines whether the neuronal activity is above or below a threshold. The cognitive process is a sophisticated channel capable of interpreting neuronal activity in more detail (for example, by setting more than one threshold). It is crucial to note that this approach does not presuppose that the cognitive channel leads to correct choices whereas the affective channel makes mistakes. Rather, both channels process the information received in an optimal way. However, the latter channel faces tighter constraints than the former.

According to Proposition 7, the affective channel is sufficiently precise in basic situations. When the environment is more complex, the utility loss of a coarse partition becomes positive, and the cognitive channel becomes relatively more valuable. To study in more detail the marginal gain of using the cognitive channel, we consider the simplest extension of the basic model in which the affective channel is not fully efficient. Formally,

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[15] Contrary to the rest of the paper (and to the general methodology advocated in it) this subsection is based more on an "intuitive trade-off" than on evidence from neurobiology. The conclusions should then be taken with extra caution.

we add a third state, $S = O$, to the two-state and a continuum of actions model of section 2. State O is intermediate and captures, for example, a case where hunting is mildly dangerous. The payoff of the individual in that state is $l(\gamma - \frac{1}{2})$ and the probability of a cell firing level c is $h(c)$, with $\left(\frac{f(c)}{h(c)}\right)' < 0$ and $\left(\frac{h(c)}{g(c)}\right)' < 0$ for all c. Given a linear loss function, $l(z) = -|z|$, a straightforward extension of the argument in Proposition 1 implies that only three actions can be optimal, $\tilde{\gamma} \in \{0, \frac{1}{2}, 1\}$. This has two implications. First, there is a utility loss whenever the affective channel is used. Second, there is no gain in utility by being able to set more than two cutoffs. Therefore, without loss of generality, we assume in what follows that the cognitive process sets exactly two thresholds.

Denote by p_S the probability of state S (with $\sum_S p_S = 1$). For the same reason as in section 2.2, a necessary condition for cutoffs x_1 and x_2 ($> x_1$) to be optimal is that $\tilde{\gamma} = 0$ if $c \in [0, x_1)$, $\tilde{\gamma} = \frac{1}{2}$ if $c \in [x_1, x_2]$ and $\tilde{\gamma} = 1$ if $c \in (x_2, 1]$. The value function that the individual maximizes is then:

$$\begin{aligned} V(x_1, x_2) &= \Pr(c < x_1) L(0; p(\cdot \mid c < x_1)) + \Pr(c \in [x_1, x_2]) L(\tfrac{1}{2}; p(\cdot \mid c \in [x_1, x_2]) \\ &\quad + \Pr(c > x_2) L(1; p(\cdot \mid c > x_2)) \\ &= -p_B \Big[(1 - F(x_2)) + \tfrac{1}{2}(F(x_2) - F(x_1))\Big] - p_O \Big[\tfrac{1}{2}(1 - H(x_2)) + \tfrac{1}{2} H(x_1)\Big] \qquad (21) \\ &\quad -p_A \Big[\tfrac{1}{2}(G(x_2) - G(x_1)) + G(x_1)\Big] \end{aligned}$$

Taking first-order conditions in (21), we obtain that the optimal cutoffs under the cognitive information processing channel, $\underline{x}$ and $\overline{x}$, solve:

$$\frac{f(\underline{x})}{g(\underline{x})} = \frac{p_A}{p_B} + \frac{p_O}{p_B} \frac{h(\underline{x})}{g(\underline{x})} \qquad \text{and} \qquad \frac{f(\overline{x})}{g(\overline{x})} = \frac{p_A}{p_B} - \frac{p_O}{p_B} \frac{h(\overline{x})}{g(\overline{x})}$$

Notice that $\underline{x} < \overline{x}$ for all $p_A, p_O, p_B \in (0,1)^3$ and $\underline{x} = x^* = \overline{x}$ when $p_O = 0$. If an information partition in three regions is cognitively too demanding, the individual can activate an affective channel, characterized by one cutoff only. Information processing is then faster but partitioned in only two regions. Let $\breve{x}$ be the cutoff that solves:

$$\frac{f(\breve{x})}{g(\breve{x})} = \frac{p_A}{p_B}$$

It is immediate to see that $\breve{x} \in (\underline{x}, \overline{x})$, and we can state the following result.

Proposition 8 *With three states, the cognitive channel sets two thresholds* $(\underline{x}, \overline{x})$ *whereas the affective channel sets one threshold which, depending on* (p_A, p_O, p_B)*, is either* $\underline{x}$*,* $\breve{x}$ *or* $\overline{x}$*. The utility loss under the affective channel is highest when all states are equally likely and lowest when one of the states is highly unlikely.*

When the individual sets only one threshold, he realizes that this excessively coarse information partition results in some utility loss, since he will be able to discriminate only between two actions. The question is to decide which action is not worth pursuing. This depends on the *relative likelihood* of the different states. When A is the most unlikely state (p_B/p_O and p_O/p_A are high), the individual sets threshold $\underline{x}$, and chooses $\gamma = 0$ if $c < \underline{x}$ and $\gamma = \frac{1}{2}$ if $c > \underline{x}$. Thus, the affective channel discriminates *optimally* between the low and medium actions and fully disregards the high one. When B is most unlikely, the individual sets $\overline{x}$ and disregards the low action, and when O is most unlikely, the threshold is $\check{x}$ and only extreme actions (0 or 1) are undertaken. Overall, the affective channel sacrifices the action which is optimal in the state most unlikely to occur, and discriminates optimally between the other two. Therefore, even though by definition the cognitive channel always outperforms the affective channel, the utility difference between the two depends on the probability distribution of states: the difference is greatest when all states are equally likely and smallest when one state occurs with very low probability.

5 Implications of the theory

We now analyze some implications of our theory. We also discuss and evaluate some arguments developed recently in neuroscience regarding the effect of emotional deficits in decision-making. Again, emotions are viewed here as brain mechanisms that channel threshold modulation. The skeptical reader can focus on the general implications of the mechanisms and ignore the discussion about the role of the soma. For tractability, we consider the basic model presented in section 2, which is simple enough to be extended in a number of directions, and we assume that $\pi_A = \pi_B = 1$.

5.1 Emotional choices in simple and complex activities

If emotions are responsible for modulating thresholds, a natural question is to determine for which activities the cost of an emotional impairment is greatest. Activities differ in several dimensions, such as the temporal horizon (near vs. distant), the likelihood of occurrence (common vs. exceptional) and the difficulty to mentally represent the consequences of outcomes (concrete vs. abstract). Based on studies with brain lesion patients, the somatic marker theory argues that impairments in the emotional system have more dramatic effects in choices related to abstract, exceptional and temporally distant situations than in choices related to concrete, common and temporally close situations (Anderson et al. (1999), Bechara and Damasio (2005)).

An extension of our theory can help understanding this issue. We divide activities into two categories: simple (concrete, common, temporally close) vs. complex (abstract, exceptional, temporally distant) and assume that activities in the former category are easier to evaluate than in the latter one. Formally, we assume that information obtained through cell firing is *uniformly more accurate* in simple than in complex activities. This means for example that it is easier to process news about the relative value of two goods available for immediate purchase than about the relative desirability of two future sentimental relationships. We use subscript $k \in \{\alpha, \beta\}$ to denote the two categories of activities, where "α" refers to simple activities and "β" refers to complex activities. The probability of cell firing c in activity k is denoted by $f_k(c)$ if $S = B$ and by $g_k(c)$ if $S = A$. We assume that the complex activity satisfies **(A1)**, that is, $\left(\frac{f_\beta(c)}{g_\beta(c)}\right)' < 0$. The greater accuracy of cell firing conditional on state in simple than in complex activities is also captured with a standard MLRP:

$$\left(\frac{g_\alpha(c)}{g_\beta(c)}\right)' > 0 \quad \text{and} \quad \left(\frac{f_\alpha(c)}{f_\beta(c)}\right)' < 0$$

The idea is simply that "neuronal mistakes", defined as low cell firing when $S = A$ or high cell firing when $S = B$, are uniformly less frequent in α-activities than in β-activities. Taken together, these conditions imply that the simple activity also satisfies **(A1)**, that is, $\left(\frac{f_\alpha(c)}{g_\alpha(c)}\right)' < 0$. Denote by $x_k^*(p)$ the optimal threshold in activity k as a function of p. We can show the existence of a belief $\hat{p} \in (0, 1)$ such that the optimal threshold coincides in both types of activities ($x_\beta^*(\hat{p}) = x_\alpha^*(\hat{p}) = \hat{x}$). Our next result is as follows.

Proposition 9 *Optimal thresholds are more sensitive to initial beliefs in complex than in simple activities, that is,* $x_\beta^*(p) \lesseqqgtr x_\alpha^*(p) \lesseqqgtr \hat{x}$ *for all* $p \gtreqqless \hat{p}$.

We showed in section 2 that the optimal threshold x^* is set in such a way that the ex-post confidence in the true state is balanced. When the evidence is accurate and therefore the correlation between state and cell firing is high (simple activities), small variations in the threshold are sufficient to achieve the desired balance. Conversely, when the evidence is noisy and therefore the correlation is low (complex activities), the individual is forced to resort to more extreme variations in thresholds. The implications of this simple argument are interesting. It suggests that the inability to modulate neuronal thresholds, due for example to a somatic deficit, results in relatively poorer choices and more mistakes when the situation is subtle (e.g., an action that may cause a moral harm on someone) than when the situation is straightforward to evaluate (e.g., an action that may cause a physical harm on someone). This hypothesis is largely favored in the somatic marker theory (Damasio (1994), Bechara and Damasio (2005)).

The result is also in accordance with the evidence in animals regarding the correlation between task difficulty, belief, and probability of a choice error (Ditterich et al., 2003).

Indeed, as briefly discussed in section 1.1, when monkeys choose between two options (net direction of random dots), the likelihood of mistakes is higher the smaller the fraction of dots moving in synchrony.[16] A testable implication of our theory is that their confidence in the action selected should increase with the degree of synchrony of dots.

5.2 Emotional interactions in cooperative and competitive environments

Given that the optimal threshold depends on prior beliefs and payoffs derived from ensuing decisions, we can reasonably anticipate that any element that modifies the structure of individual payoffs should impact the threshold modulation. Natural environments with this property are strategic interactions.

Consider the following extension. Two agents, i and j with $i, j \in \{1, 2\}$, are either in competition or cooperation for an activity. We adopt a reduced form model of the positive (cooperation) and negative (competition) externalities exerted by individuals on one another: in cooperative situations, agents benefit if their colleague also takes the correct action whereas in competitive situations, agents benefit if they alone take the correct action. This, for example, captures the cases where individuals hunt together and compete for preys, respectively.

Agents cannot exchange information. They each set a threshold x_i, learn whether the cell firing activity is below or above the threshold, and choose an action. The state is common to both agents but, conditional on it, cell firing is independent across individuals. Denote by λ the agent's utility of taking the correct action. If, in the cooperation setting, the other agent also takes the correct action or if, in the competition setting, the other agent takes the incorrect action, the utility is increased by μ. Using the superscript "+" and "−" to denote the cooperation and competition settings respectively, we can rewrite the value function of agent i as follows:

$$\begin{aligned} V_i^+(x_i, x_j) &= \mathrm{P}(A)\mathrm{P}(c > x_i|A)\Big[\lambda + \mu \mathrm{P}(c > x_j|A)\Big] + \mathrm{P}(B)\mathrm{P}(c < x_i|B)\Big[\lambda + \mu \mathrm{P}(c < x_j|B)\Big] \\ &= p(1 - G(x_i))\Big[\lambda + \mu(1 - G(x_j))\Big] + (1 - p)F(x_i)\Big[\lambda + \mu\, F(x_j)\Big] \qquad (22) \\ V_i^-(x_i, x_j) &= \mathrm{P}(A)\mathrm{P}(c > x_i|A)\Big[\lambda + \mu \mathrm{P}(c < x_j|A)\Big] + \mathrm{P}(B)\mathrm{P}(c < x_i|B)\Big[\lambda + \mu \mathrm{P}(c > x_j|B)\Big] \\ &= p(1 - G(x_i))\Big[\lambda + \mu\, G(x_j)\Big] + (1 - p)F(x_i)\Big[\lambda + \mu(1 - F(x_j))\Big] \qquad (23) \end{aligned}$$

Note that $\mu = 0$ corresponds to the case of independent activities analyzed in section 2.2. Denote by $x_i^+ = \arg\max_{x_i} V_i^+(x_i, x_j^+)$ and by $x_i^- = \arg\max_{x_i} V_i^-(x_i, x_j^-)$. Taking

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[16] The paper also shows that monkeys require more time to take a decision, an issue not modelled or discussed in this paper.

first-order conditions in (22) an (23), if interior solutions exist, they will be such that:

$$\frac{f(x_i^+)}{g(x_i^+)} = \frac{p}{1-p}\,\frac{\lambda + \mu(1 - G(x_j^+))}{\lambda + \mu\, F(x_j^+)} \tag{24}$$

$$\frac{f(x_i^-)}{g(x_i^-)} = \frac{p}{1-p}\,\frac{\lambda + \mu\, G(x_j^-)}{\lambda + \mu(1 - F(x_j^-))} \tag{25}$$

According to this formulation, thresholds are strategic complements in cooperative activities ($dx_i^+/dx_j > 0$) and strategic substitutes in competitive activities ($dx_i^-/dx_j < 0$). The reason is simple. Suppose that agent 1 decreases his threshold. This makes him more likely to surpass it, and therefore to take action $\gamma^* = 1$. In cooperative activities, it also increases the expected benefit for agent 2 of taking action $\gamma^* = 1$ when $S = A$ relative to the expected benefit of taking action $\gamma^* = 0$ when $S = B$, since the extra cooperative payoff is more likely to be enjoyed in the first than in the second case. The cost of either wrong choice does not change. Therefore, agent 2 has an incentive to move the threshold in the same direction as agent 1. The opposite is true in competitive activities.

In what follows, we look for interior symmetric equilibria, where $x_i^+ = x_j^+ = x^+$ in the cooperative case and $x_i^- = x_j^- = x^-$ in the competitive one. We denote by $\kappa \equiv \mu/\lambda$ the relative importance of the other agent's choice on one's payoff. In order to have interior solutions, the following technical restriction needs to be imposed.

Assumption 3 (Interior stability) *The parameter κ is such that:*

$$\frac{\kappa\, g(x^+)}{1 + \kappa(1 - G(x^+))} + \frac{\kappa\, f(x^+)}{1 + \kappa\, F(x^+)} < \frac{g'(x^+)}{g(x^+)} - \frac{f'(x^+)}{f(x^+)} \tag{\textbf{A3.1}}$$

$$\frac{\kappa\, g(x^-)}{1 + \kappa\, G(x^-)} + \frac{\kappa\, f(x^-)}{1 + \kappa(1 - F(x^-))} < \frac{g'(x^-)}{g(x^-)} - \frac{f'(x^-)}{f(x^-)} \tag{\textbf{A3.2}}$$

If assumptions **(A3.1)** and **(A3.2)** are violated, then the solutions in the cooperative and competitive settings respectively may not be interior. For some distributions, these inequalities are satisfied for all κ. Otherwise, they require κ to be small enough.[17] The reason is straightforward. As κ increases, the decision of each agent becomes closer to a

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[17] Technically, these assumptions ensure that the slopes of the reaction functions are smaller than 1 in absolute value at the symmetric intersection. Note that the L.H.S. in **(A3.1)** and **(A3.2)** are increasing in κ. Therefore, sufficient conditions for the inequalities to hold are:

$$\frac{g(x^+)}{1-G(x^+)} + \frac{f(x^+)}{F(x^+)} < \frac{g'(x^+)}{g(x^+)} - \frac{f'(x^+)}{f(x^+)} \quad \text{and} \quad \frac{g(x^-)}{G(x^-)} + \frac{f(x^-)}{1-F(x^-)} < \frac{g'(x^-)}{g(x^-)} - \frac{f'(x^-)}{f(x^-)}$$

The first inequality always holds for instance if $f(\cdot)$ and $g(\cdot)$ are linear and symmetric, as in Example 2.

coordination problem. Suppose that one agent disregards information and always takes action $\gamma^* = 1$. Then, if κ is high, the other agent also has an incentive to disregard information and take action $\gamma^* = 1$ (in the cooperative setting) and action $\gamma^* = 0$ (in the competitive setting). Under these conditions, we can prove that there exists $\tilde{p} \in (0, 1)$ and $\tilde{x}$ such that $x^+(\tilde{p}) = x^*(\tilde{p}) = x^-(\tilde{p}) = \tilde{x}$. We have the following result.

Proposition 10 *Optimal thresholds are more sensitive to initial beliefs in cooperative than in independent activities, and less sensitive in competitive than in independent activities. Formally,* $x^+(p) \lesseqqgtr x^*(p) \lesseqqgtr x^-(p) \lesseqqgtr \tilde{x}$ *for all* $p \gtreqqless \tilde{p}$.

Proposition 10 states that, from a pure information processing perspective, threshold modulation is more decisive in cooperative than in competitive situations. Because in cooperative activities there is an extra payoff when both agents undertake the same correct action, the benefit of favoring the current belief (the state a priori most likely to be correct) is increased relative to the case of independent activities. Conversely, in competitive situations, each agent is more interested in learning that the least likely state is the correct one. As a result, they both set the threshold in a way that existing beliefs are less favored than under independent activities. Overall, the result of this simple extension suggests that an emotional deficit that prevents the individual from modulating neuronal thresholds (for example, keeping it always at $\tilde{x}$) is more costly in environments with positive externalities than in environments with negative externalities. That is, emotions may be more helpful in cooperative than in competitive situations.

5.3 Emotional reactions to expected and unexpected events

Neuroscientists have been interested in the reaction of individuals to unexpected events. Schultz et al. (2000) claim that somatic states induce an overreaction to unanticipated outcomes, and determine in a series of experiments the biological mechanism behind this behavior, labeled the "Dopaminergic Reward Prediction Error" (DRPE) hypothesis. This theory argues that the response to rewards materializes in changes in dopamine release, which reflects the discrepancy between the reward itself and its prediction.[18] Bechara and Damasio (2005) integrate this idea in the somatic marker theory: "[T]he very quick, almost instantaneous, switch from one somatic state to another when an unexpected event occurs, can exert a disproportional impact on somatic state activation. Thus people's decision may get reversed completely if a disappointment was encountered" (p.365).

From a Bayesian perspective, beliefs are revised more strongly the greater the distance between prior and observation. So, without a formal benchmark for comparison, it is

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[18] Caplin and Dean (2007) provide an axiomatic approach of the DRPE hypothesis.

difficult to determine whether there is "overreaction" and a "disproportional" response, or just a natural belief reversal due to the influx of contradictory evidence.

We study this issue within the framework of our model by considering the following extension. In stage 1, the brain sets a threshold x and determines whether cell firing is above or below it. In stage 2, the individual updates his beliefs and chooses an action. In stage 3, he observes the outcome and assesses his ex post confidence in the choice he made. If the payoff is perfectly correlated with the true state, the individual learns with certainty whether he made the correct choice. That is, the ex-post probability of each state is either 0 or 1. More interestingly, under imperfect correlation, the (positive) reaction after a success and (negative) reaction after a failure may depend on the threshold selected. Our objective is to analyze this case in more detail. Imperfect correlation is modeled in a simple way. If $S = A$, the payoff of action γ is $l(\gamma - 1)$ with probability θ and $l(\gamma)$ with probability $1 - \theta$, with $\theta \in (1/2, 1]$. If $S = B$, the payoff is $l(\gamma)$ with probability θ and $l(\gamma - 1)$ with probability $1 - \theta$.[19] In our example, on a dangerous day, the individual may be lucky, avoid all predators, and go back to the cave believing it was a safe day.

We first need to characterize the optimal strategy of the individual in this modified version of the problem. In stage 2, the expected payoff of the individual is:

$$L(\gamma; p, \theta) = \Big[p\,\theta + (1-p)(1-\theta)\Big]\,l(\gamma - 1) + \Big[p\,(1-\theta) + (1-p)\,\theta\Big]\,l(\gamma) \tag{26}$$

Let $p' = p\,\theta + (1-p)(1-\theta)$ be the probability of encountering a predator. It is immediate that the strategy of the individual is the same as in section 2.2: $\gamma^* = 1$ if $p' > 1/2$ and $\gamma^* = 0$ if $p' < 1/2$, where $p' \gtrless 1/2$ if and only if $p \gtrless 1/2$. In words, the correlation between the ex post payoff and the state does not affect the optimal action.

In stage 1, the brain sets a threshold x which may or may not coincide with $x^*(p)$, the optimal one. We restrict the attention to the non-trivial case where the threshold is such that the result of the cell firing activity is informative ($\Pr(A \,|\, c < x) < 1/2 < \Pr(A \,|\, c > x)$), because otherwise this stage is ignored. The question we ask is: for any given x, how will the individual revise his beliefs after observing the outcome? Note that there are only two possible outcomes in stage 3: $l_H \equiv l(0)$ and $l_L \equiv l(1)$ $(< l(0))$. By construction, the individual expects to be correct (otherwise he would choose the other action) and get l_H. We thus call l_H the "expected outcome" and l_L the "unexpected outcome." The confidence of the individual is boosted when followed by a high payoff and reduced when followed by a low payoff. If the level of cell-firing is high, he takes action $\gamma^* = 1$ and believes this action is correct with probability:

$$\Pr(A \,|\, c > x) = \overline{p}(x)$$

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[19] Section 2 thus corresponds to $\theta = 1$, where the true state is known ex post.

If the outcome is the expected one, his revised belief that the action was indeed correct is:

$$\Pr(A \,|\, c > x, l_H) = \frac{p(1-G(x))\theta}{p(1-G(x))\theta + (1-p)(1-F(x))(1-\theta)} = \overline{p}(x, l_H) \quad (> \overline{p}(x))$$

Otherwise, his belief is revised downwards to:

$$\Pr(A \,|\, c > x, l_L) = \frac{p(1-G(x))(1-\theta)}{p(1-G(x))(1-\theta) + (1-p)(1-F(x))\theta} = \overline{p}(x, l_L) \quad (< \overline{p}(x))$$

Let $\overline{p}(x, l_H) - \overline{p}(x)$ be the increase in confidence after a high payoff, and $\overline{p}(x) - \overline{p}(x, l_L)$ the decrease in confidence after a low payoff. These differences measure the positive reaction to an expected event and the negative reaction to an unexpected event, respectively.

Similarly, if the level of cell firing is low, the individual takes action $\gamma^* = 0$ anticipating it is correct with probability:

$$\Pr(B \,|\, c < x) = 1 - \underline{p}(x)$$

Again, if the action is confirmed with a high payoff, his revised belief becomes:

$$\Pr(B \,|\, c < x, l_H) = \frac{(1-p)F(x)\theta}{(1-p)F(x)\theta + pG(x)(1-\theta)} = 1 - \underline{p}(x, l_H) \quad (> 1 - \underline{p}(x))$$

Otherwise, he believes the action was correct with probability:

$$\Pr(B \,|\, c < x, l_L) = \frac{(1-p)F(x)(1-\theta)}{(1-p)F(x)(1-\theta) + pG(x)\theta} = 1 - \underline{p}(x, l_L) \quad (< 1 - \underline{p}(x))$$

In that case, $\underline{p}(x) - \underline{p}(x, l_H)$ measures the reaction to the expected event, and $\underline{p}(x, l_L) - \underline{p}(x)$ measures the reaction to the unexpected event. It can be easily shown that, for all x, the reaction to unexpected events is always greater than the reaction to expected events. Also, as the environment becomes more stochastic, observing an event is less informative and beliefs are less drastically revised in either direction. We also have the following result.

Proposition 11 *More extreme cutoffs lead always to lower reactions to expected events. They lead to higher reactions to unexpected events if θ is high but also to lower reactions to unexpected events if θ is low.*

Thresholds affect the magnitude of the reaction to different events. An individual who surpasses an extreme threshold is more confident in his action (here $\gamma^* = 1$) than an individual who surpasses a weak threshold. Therefore, he also experiences less (positive) surprise when his decision is confirmed by a high payoff. More surprising is the case of an individual who obtains a low payoff. If the environment is deterministic (θ high), he becomes almost sure that his decision was incorrect. Given that extreme thresholds lead to stronger beliefs, they also lead to a higher (negative) surprise. By contrast, if the environment is stochastic (θ low), there is little information in the payoff and the posterior belief hinges towards the prior. Here, given that extreme thresholds lead to stronger beliefs, they also lead to a smaller surprise.

We can put this result in the context of the discussion regarding the reaction of individuals to anticipated and unanticipated events. The informal arguments in the neuroscience literature briefly reviewed above emphasize that an individual with emotional stability exhibits a small positive surprise to expected events and a large negative surprise to unexpected events. This is only partly consistent with our findings. Indeed, the ability to set extreme cutoffs always results in lower reaction to expected events. However, our model predicts a high negative reaction to unexpected events only if outcomes are sufficiently deterministic. This literature thus ignores the fact that, in highly stochastic environments, individuals realize the negligible informational content of outcomes, and therefore react mildly to realized payoffs, whether positive or negative.

6 Concluding remarks

We have modeled the physiological limitations faced by individuals when processing information. We have shown that, under such constraints, it is optimal to set neuronal thresholds in a way that initial beliefs are favored. The conclusion holds in a wide array of settings and has interesting implications for the somatic marker theory.

As discussed in detail, the general methodology used in the paper consists in building an economic model of decision processes based on evidence from neuroscience and neurobiology. This "neuroeconomic theory" approach to decision-making has two main advantages. First, by modeling the underlying mechanisms that lead to choices, it may be possible to predict behavior more accurately. Second, the models provide testable predictions concerning the effects of pathologies on choices. We have extensively discussed the first point but only briefly mentioned how emotional deficiencies that translate into suboptimal thresholds affect decisions. We have therefore not exploited the full potential of the methodology in the second dimension. A natural alley for future research would be to test some of our implications in patients with brain lesions.

The model has also implications that extend beyond the realm of brain information processing. First, we can apply it to a standard individual learning problem, under the assumption that news obtained is coarse (much on teh liens of the theory of organizations literature). Under that interpretation, our model provides “a rationale for stubbornness”: agents are less likely to change their mind as time passes, not only because they are more confident about which alternative they prefer, but also because they modify thresholds in a way that existing beliefs are most likely to be reinforced. At the same time, when they change their mind, they do it more drastically. It also suggests that stubbornness should be more prevalent in complex issues, where information is more difficult to interpret. Second and related, two individuals with opposite beliefs will set thresholds at opposite ends and therefore may interpret the same evidence in opposite directions. In other words, in a world of different priors, common information may increase polarization of opinions, at least in the short run.[20] Third, we have seen that the information sequence matters in dynamic settings. This suggests that behavior can be influenced by manipulating the order in which news is revealed. One could possibly develop a theory of framing based on this approach. Last, the way information is interpreted in strategic settings seems to affect behavior substantially. The multi-agent application we offered is introductory. It would be interesting to analyze interactions in more general environments.

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[20] See Sobel (2007) for a different study of polarization in group decision-making.

Appendix

A1. Proof of Proposition 1

Taking the first-order condition in (4), we find that $x^*(p)$ satisfies (5). Given **(A1)**, $x^*(p)$ is unique and the local second-order condition is satisfied:

$$\begin{aligned} \left.\frac{\partial^2 V}{\partial x^2}\right|_{x^*} &= -p\, g'(x^*)\pi_A(l(0)-l(1)) + (1-p)f'(x^*)\pi_B(l(0)-l(1)) \\ &= (1-p)\pi_B\, g(x^*)(l(0)-l(1))\left(\tfrac{f(x^*)}{g(x^*)}\right)' < 0. \end{aligned}$$

$dx^*/dp < 0$ is immediate from **(A1)**. For $V(x^*;p)$ to be the value function, we must check ex-post that it is optimal to select $\gamma^* = 1$ when $c > x^*$ and $\gamma^* = 0$ when $c < x^*$. We have:

$\Pr(A|c > x^*) > \frac{\pi_B}{\pi_A+\pi_B} \;\Leftrightarrow\; p\,\pi_A(1-G(x^*)) > (1-p)\pi_B(1-F(x^*)) \;\Leftrightarrow\; \frac{1-G(x^*)}{g(x^*)} > \frac{1-F(x^*)}{f(x^*)}$

$\Pr(A|c < x^*) < \frac{\pi_B}{\pi_A+\pi_B} \;\Leftrightarrow\; p\,\pi_A\, G(x^*) > (1-p)\pi_B F(x^*) \;\Leftrightarrow\; \frac{G(x^*)}{g(x^*)} < \frac{F(x^*)}{f(x^*)}$

Both inequalities are satisfied given **(A1)**. This completes the proof.

A2. Proof of Proposition 2

From (11) and using (7) and (8), we get:

$$\begin{aligned} \left.\frac{\partial^2 V(x;p)}{\partial x \partial p}\right|_{x=x^{**}} &= -\frac{1}{p}\Bigg[f(x^{**})\Big(l(\gamma^{**}(\underline{p})) - l(\gamma^{**}(\overline{p}))\Big) + F(x^{**})l'(\gamma^{**}(\underline{p}))\left.\frac{d\gamma^{**}(\underline{p})}{dx}\right|_{x^{**}} \\ &\qquad + (1-F(x^{**}))l'(\gamma^{**}(\overline{p}))\left.\frac{d\gamma^{**}(\overline{p})}{dx}\right|_{x^{**}}\Bigg] \\ &= -\frac{1}{p} \times \frac{d}{dx}\Big[F(x)l(\gamma^{**}(\underline{p})) + (1-F(x))l(\gamma^{**}(\overline{p}))\Big]_{x=x^{**}} \end{aligned}$$

Similarly,

$$\begin{aligned} \left.\frac{\partial^2 V(x;p)}{\partial x^2}\right|_{x=x^{**}} = (1-p)\frac{\left(\frac{f(x)}{g(x)}\right)'}{\frac{f(x)}{g(x)}}\Bigg[& f(x)\Big(l(\gamma^{**}(\underline{p})) - l(\gamma^{**}(\overline{p}))\Big) + F(x)l'(\gamma^{**}(\underline{p}))\frac{d\gamma^{**}(\underline{p})}{dx}\frac{\frac{f(x)}{g(x)}}{\left(\frac{f(x)}{g(x)}\right)'}\frac{\left(\frac{F(x)}{G(x)}\right)'}{\frac{F(x)}{G(x)}} \\ &+ (1-F(x))l'(\gamma^{**}(\overline{p}))\frac{d\gamma^{**}(\overline{p})}{dx}\frac{\frac{f(x)}{g(x)}}{\left(\frac{f(x)}{g(x)}\right)'}\frac{\left(\frac{1-F(x)}{1-G(x)}\right)'}{\frac{1-F(x)}{1-G(x)}}\Bigg]_{x=x^{**}} \end{aligned}$$

By **(A2)**, $\frac{\frac{f(x)}{g(x)}}{\left(\frac{f(x)}{g(x)}\right)'}\frac{\left(\frac{F(x)}{G(x)}\right)'}{\frac{F(x)}{G(x)}} \leqslant 1$ and $\frac{\frac{f(x)}{g(x)}}{\left(\frac{f(x)}{g(x)}\right)'}\frac{\left(\frac{1-F(x)}{1-G(x)}\right)'}{\frac{1-F(x)}{1-G(x)}} \leqslant 1$. Therefore, $\left.\frac{\partial^2 V(x;p)}{\partial x \partial p}\right|_{x=x^{**}} < 0 \Rightarrow \left.\frac{\partial^2 V(x;p)}{\partial x^2}\right|_{x=x^{**}} < 0$ and the proposition follows.

A3. Proof of Proposition 3

Let $P \equiv \frac{p}{1-p}$. The F.O.C. in the quadratic case can be rewritten as:

$$\frac{1-p}{p^2}\left.\frac{\partial V(x;p)}{\partial x}\right|_{x^{**}} = k(x^{**},P)$$

where $k(x,P) \equiv \left(\frac{1-G(x)}{(1-G(x))P+(1-F(x))} - \frac{G(x)}{G(x)\,P+F(x)}\right)\Big[f(x)\left(\frac{1-G(x)}{(1-G(x))P+(1-F(x))} + \frac{G(x)}{G(x)\,P+F(x)}\right)$ $-g(x)\left(\frac{1-F(x)}{(1-G(x))P+(1-F(x))} + \frac{F(x)}{G(x)\,P+F(x)}\right)\Big]$. Differentiating the F.O.C., we get

$$\left.\frac{\partial^2 V(x;p)}{\partial x \partial p}\right|_{x^{**}} \propto \frac{\partial k(x^{**},P)}{\partial P}$$

After some tedious algebra, and using (13), we get:

$$\frac{\partial k(x^{**},P)}{\partial P} = -g(x^{**})\frac{\left(\frac{F(x^{**})}{G(x^{**})} - \frac{1-F(x^{**})}{1-G(x^{**})}\right)^3}{\left(2P+\frac{F(x^{**})}{G(x^{**})}+\frac{1-F(x^{**})}{1-G(x^{**})}\right)\left(P+\frac{F(x^{**})}{G(x^{**})}\right)^2\left(P+\frac{1-F(x^{**})}{1-G(x^{**})}\right)^2} < 0$$

and the result follows.

A4. Proof of Proposition 4

Taking the F.O.C. in (14), we obtain (15). The local S.O.C. is:

$$\begin{aligned}\left.\frac{\partial^2 V}{\partial x^2}\right|_{\hat{x}} &= -\int_0^1 p(s)\, f_x(\hat{x}\,|\,s)\Big(l(1-s)-l(s)\Big)ds \\ &= \int_0^1 \left(-\frac{f_x(\hat{x}\,|\,s)}{f(\hat{x}\,|\,s)}\right) p(s)\, f(\hat{x}\,|\,s)\Big(l(1-s)-l(s)\Big)ds\end{aligned}$$

Let $h(s) \equiv -\dfrac{f_x(\hat{x}\,|\,s)}{f(\hat{x}\,|\,s)}$. By **(A1')**, $h'(s) \leqslant 0$. We can then rewrite the local S.O.C. as:

$$\begin{aligned}\left.\frac{\partial^2 V}{\partial x^2}\right|_{\hat{x}} &= \int_0^{1/2} h(s)\, p(s)\, f(\hat{x}\,|\,s)\Big(l(1-s)-l(s)\Big)ds + \int_{1/2}^1 h(s)\, p(s)\, f(\hat{x}\,|\,s)\Big(l(1-s)-l(s)\Big)ds \\ &< h(1/2)\left[\int_0^{1/2} p(s)\, f(\hat{x}\,|\,s)\Big(l(1-s)-l(s)\Big)ds + \int_{1/2}^1 p(s)\, f(\hat{x}\,|\,s)\Big(l(1-s)-l(s)\Big)ds\right] = 0\end{aligned}$$

Regarding the comparative statics, if $\left(\dfrac{p(s)}{q(s)}\right)' \leqslant 0$, then:

$$\begin{aligned}\left.\frac{\partial V(x;p(s))}{\partial x}\right|_{\hat{x}(p(s))} &= -\int_0^1 \left(\frac{p(s)}{q(s)}\right) q(s)\, f(\hat{x}(p(s))\,|\,s)\Big(l(1-s)-l(s)\Big)ds \\ &> -\left(\frac{p(1/2)}{q(1/2)}\right)\int_0^1 q(s)\;\, f(\hat{x}(p(s))\,|\,s)\Big(l(1-s)-l(s)\Big)ds\end{aligned}$$

Therefore,

$$\left.\frac{\partial V(x;p(s))}{\partial x}\right|_{\hat{x}(p(s))} = 0 \; > \; \left(\frac{p(1/2)}{q(1/2)}\right) \left.\frac{\partial V(x;q(s))}{\partial x}\right|_{\hat{x}(p(s))} \;\; \Rightarrow \; \hat{x}(p(s)) > \hat{x}(q(s))$$

Last, we need to check that it is indeed optimal to choose $\hat{\gamma} = 1$ when $c > \hat{x}$ and $\hat{\gamma} = 0$ when $c < \hat{x}$. Let $\mathcal{L}(x) \equiv L(1; p(s \,|\, c = x)) - L(0; p(s \,|\, c = x))$, also $p(s \,|\, c = x) \equiv j(s \,|\, x) = \frac{p(s) f(x \,|\, s)}{\int_0^1 p(s) f(x \,|\, s) ds}$ and $J(s \,|\, x) = \int_0^s j(\tilde{s} \,|\, x) d\tilde{s}$. Integrating by parts:

$$\begin{aligned} \mathcal{L}(x) &= \int_0^1 j(s \,|\, x)\Big(l(1-s) - l(s)\Big) ds \\ &= l(0) - l(1) + \int_0^1 J(s \,|\, x)\Big(l'(1-s) + l'(s)\Big) ds \end{aligned}$$

Therefore

$$\frac{d\mathcal{L}(x)}{dx} = \int_0^1 J_x(s \,|\, x)\Big(l'(1-s) + l'(s)\Big) ds > 0$$

since, by **(A1')**, we know that $F_s(x \,|\, s) < 0$ and therefore $J_x(s \,|\, x) < 0$. From (15), $\mathcal{L}(\hat{x}) = 0$, so $\mathcal{L}(x) \gtreqless 0$ for all $x \gtreqless \hat{x}$. This also proves that, for the purpose of the action to be taken, it is equivalent to learn c or to learn whether c is greater or smaller than $\hat{x}$.

A5. Proof of Proposition 5 and 6

They follow the exact same steps as the proofs of Propositions 2 and 3, and are therefore omitted for the sake of brevity.

A6. Proof of Proposition 8

Cognitive channel. Taking F.O.C. in (21), we obtain $\underline{x}$ and $\overline{x}$. We also have $\left.\frac{\partial^2 V(x_1,x_2)}{\partial x_1^2}\right|_{\underline{x}} = -\frac{1}{2} p_O f(\underline{x}) \left(\frac{h(\underline{x})}{f(\underline{x})}\right)' - \frac{1}{2} p_A f(\underline{x}) \left(\frac{g(\underline{x})}{f(\underline{x})}\right)' < 0$, $\left.\frac{\partial^2 V(x_1,x_2)}{\partial x_2^2}\right|_{\overline{x}} = \frac{1}{2} p_O g(\overline{x}) \left(\frac{h(\overline{x})}{g(\overline{x})}\right)' + \frac{1}{2} p_B g(\overline{x}) \left(\frac{f(\overline{x})}{g(\overline{x})}\right)' < 0$, and $\frac{\partial^2 V(x_1,x_2)}{\partial x_1 \partial x_2} = 0$. Therefore $\underline{x}$ and $\overline{x}$ are maxima. Last, it can be easily checked that $\Big(\Pr(A \,|\, c \in \mathcal{X}), \Pr(O \,|\, c \in \mathcal{X}), \Pr(B \,|\, c \in \mathcal{X})\Big)$ are such that it is indeed optimal to have $\tilde{\gamma} = 0$ if $\mathcal{X} = [0, \underline{x})$, $\tilde{\gamma} = \frac{1}{2}$ if $\mathcal{X} = [\underline{x}, \overline{x}]$, and $\tilde{\gamma} = 1$ if $\mathcal{X} = (\overline{x}, 1]$.

Affective channel. The three candidates for optimal cutoffs are:

$$\begin{cases} x_a & \text{so that} \quad \tilde{\gamma} = 0 \quad \text{if} \quad c < x_a \quad \text{and} \quad \tilde{\gamma} = 1 \quad \text{if} \quad c > x_a \\ x_b & \text{so that} \quad \tilde{\gamma} = 0 \quad \text{if} \quad c < x_b \quad \text{and} \quad \tilde{\gamma} = 1/2 \quad \text{if} \quad c > x_b \\ x_c & \text{so that} \quad \tilde{\gamma} = 1/2 \quad \text{if} \quad c < x_c \quad \text{and} \quad \tilde{\gamma} = 1 \quad \text{if} \quad c > x_c \end{cases}$$

These cutoffs are formally defined by:

$$\begin{cases} x_a &= \arg\max_x \quad V^a(x) \equiv \Pr(c<x)\, L(0; p(\cdot\,|c<x)) + \Pr(c>x)\, L(1; p(\cdot\,|c<x)) \\ x_b &= \arg\max_x \quad V^b(x) \equiv \Pr(c<x)\, L(0; p(\cdot\,|c<x)) + \Pr(c>x)\, L(\tfrac{1}{2}; p(\cdot\,|c<x)) \\ x_c &= \arg\max_x \quad V^c(x) \equiv \Pr(c<x)\, L(\tfrac{1}{2}; p(\cdot\,|c<x)) + \Pr(c>x)\, L(1; p(\cdot\,|c<x)) \end{cases}$$

It is straightforward to check that $x_a = \check{x}$, $x_b = \underline{x}$, $x_c = \overline{x}$. Now, fix p_O. Differentiating each first-order condition with respect to p_B, we get:

$$\frac{dx_a}{dp_B} > 0, \quad \frac{dx_b}{dp_B} > 0, \quad \frac{dx_c}{dp_B} > 0$$

Furthermore:

$$\frac{dV^a(x^a)}{dp_B} = F(x^a) + G(x^a) - 1 \gtrless 0, \quad \frac{d^2V^a(x^a)}{dp_B^2} = \Big[f(x^a) + g(x^a)\Big]\frac{dx_a}{dp_B} \geqslant 0,$$

$$\frac{dV^b(x^b)}{dp_B} = \frac{F(x^b) + G(x^b)}{2} \geqslant 0, \quad \frac{dV^c(x^c)}{dp_B} = \frac{F(x^c) + G(x^c)}{2} - 1 \leqslant 0$$

Also, $\lim_{p_B \to 0} V^a(x^a) = -\frac{p_O}{2} < \lim_{p_B \to 0} V^c(x^c)$ and $\lim_{p_B \to 1-p_O} V^a(x^a) = -\frac{p_O}{2} < \lim_{p_B \to 1-p_O} V^b(x^b)$.

Combining these results, we have that there exist p^* such that x^c dominates x^b if $p_B < p^*$ and x^b dominates x^c if $p_B > p^*$. Also, there exist p^{**} and p^{***} such that x^c dominates x^a if $p_B < p^{**}$ and x^b dominates x^a if $p_B > p^{***}$. The ranking between p^*, p^{**} and p^{***} will depend on the relative values of p_O and p_A.

A7. Proof of Proposition 9

$\left(\frac{g_\alpha(c)}{g_\beta(c)}\right)' > 0$, $\left(\frac{f_\beta(c)}{g_\beta(c)}\right)' < 0$, $\left(\frac{f_\alpha(c)}{f_\beta(c)}\right)' < 0 \;\Rightarrow\; \frac{g'_\alpha(c)}{g_\alpha(c)} > \frac{g'_\beta(c)}{g_\beta(c)} > \frac{f'_\beta(c)}{f_\beta(c)} > \frac{f'_\alpha(c)}{f_\alpha(c)} \;\Rightarrow\; \left(\frac{f_\alpha(c)}{g_\alpha(c)}\right)' < 0$. Now, suppose there exists $\hat{c} \in (0,1)$ such that $\frac{f_\alpha(\hat{c})}{g_\alpha(\hat{c})} = \frac{f_\beta(\hat{c})}{g_\beta(\hat{c})}$. Then,

$$\frac{d}{dc}\left[\frac{f_\alpha(c)}{g_\alpha(c)} - \frac{f_\beta(c)}{g_\beta(c)}\right]_{c=\hat{c}} = \frac{f_\alpha(\hat{c})}{g_\alpha(\hat{c})}\left(\frac{f'_\alpha(\hat{c})}{f_\alpha(\hat{c})} - \frac{g'_\alpha(\hat{c})}{g_\alpha(\hat{c})}\right) - \frac{f_\beta(\hat{c})}{g_\beta(\hat{c})}\left(\frac{f'_\beta(\hat{c})}{f_\beta(\hat{c})} - \frac{g'_\beta(\hat{c})}{g_\beta(\hat{c})}\right) < 0$$

so $\frac{f_\alpha(c)}{g_\alpha(c)}$ and $\frac{f_\beta(c)}{g_\beta(c)}$ cross at most once. Furthermore, $\left(\frac{g_\alpha(c)}{g_\beta(c)}\right)' > 0$ and $\left(\frac{f_\alpha(c)}{f_\beta(c)}\right)' < 0 \;\Rightarrow\; \frac{f_\alpha(0)}{g_\alpha(0)} > \frac{f_\beta(0)}{g_\beta(0)}$ and $\frac{f_\alpha(1)}{g_\alpha(1)} < \frac{f_\beta(1)}{g_\beta(1)}$. Together with the previous result, it means that there exists $\hat{x} \in (0,1)$ such that $\frac{f_\alpha(x)}{g_\alpha(x)} \underset{<}{\overset{>}{=}} \frac{f_\beta(x)}{g_\beta(x)}$ for all $x \underset{>}{\overset{<}{=}} \hat{x}$. Finally, given (5), there exists $\hat{p}$ such that $\frac{f_\alpha(x^*_\alpha(\hat{p}))}{g_\alpha(x^*_\alpha(\hat{p}))} = \frac{f_\beta(x^*_\beta(\hat{p}))}{g_\beta(x^*_\beta(\hat{p}))} = \frac{\hat{p}}{1-\hat{p}}$, that is, $x^*_\alpha(\hat{p}) = x^*_\beta(\hat{p}) = x^*(\hat{p}) \equiv \hat{x}$. For all $p \gtrless \hat{p}$, $\frac{f_\alpha(x^*_\alpha(p))}{g_\alpha(x^*_\alpha(p))} = \frac{f_\beta(x^*_\beta(p))}{g_\beta(x^*_\beta(p))} = \frac{p}{1-p} \;\Rightarrow\; x^*_\beta(p) \lessgtr x^*_\alpha(p) \lessgtr x^*(\hat{p}) \lessgtr \hat{x}$.

A8. Proof of Proposition 10

Cooperation. Taking the FOC in (22) and then differentiating it with respect to the other agent's threshold, we get:

$$\frac{\partial x_i^+}{\partial x_j} = -\frac{\partial^2 V_i^+/\partial x_i^+ \partial x_j}{\partial^2 V_i^+/\partial (x_i^+)^2} = -\mu \frac{f(x_i^+)/g(x_i^+)}{\big(f(x_i^+)/g(x_i^+)\big)'} \left[\frac{g(x_j)}{\lambda+\mu(1-G(x_j))} + \frac{f(x_j)}{\lambda+\mu\, F(x_j)}\right] > 0$$

Because reaction functions are positive and identical for both individuals, equilibria can only be symmetric. Let $R_i^+(x_j)$ be i's reaction function. Sufficient conditions for a unique, stable interior solution are: $R_i^+(0) > 0$, $R_i^+(1) < 1$ and $\left.\frac{\partial x_i^+}{\partial x_j}\right|_{x_j=x_i^+} < 1$. From (24), the first and second inequalities can be written as $\frac{f(0)}{g(0)} > \frac{p}{1-p}(1+\kappa)$ and $\frac{f(1)}{g(1)} < \frac{p}{1-p}\frac{1}{1+\kappa}$, which are strengthened versions of the conditions in Proposition 1. The third inequality boils down to **(A3.1)**. From (5) and (24), we finally get:

$$\frac{f(x^+)}{g(x^+)}\,\frac{\lambda+\mu\, F(x^+)}{\lambda+\mu(1-G(x^+))} = \frac{f(x^*)}{g(x^*)} = \frac{p}{1-p} \tag{27}$$

Competition. Taking the FOC in (23) and then differentiating it with respect to the other agent's threshold, we get:

$$\frac{\partial x_i^-}{\partial x_j} = -\frac{\partial^2 V_i^-/\partial x_i^- \partial x_j}{\partial^2 V_i^-/\partial (x_i^-)^2} = \mu \frac{f(x_i^-)/g(x_i^-)}{\big(f(x_i^-)/g(x_i^-)\big)'} \left[\frac{g(x_j)}{\lambda+\mu\, G(x_j)} + \frac{f(x_j)}{\lambda+\mu(1-F(x_j))}\right] < 0$$

Because reaction functions are negative there can only exist one symmetric equilibrium. Let $R_i^-(x_j)$ be i's reaction function. Sufficient conditions for the symmetric equilibrium to be stable and interior are: $R_i^-(0) > 0$, $R_i^-(1) < 1$ and $\left.\frac{\partial x_i^-}{\partial x_j}\right|_{x_j=x_i^-} > -1$. From (25), the first and second inequalities can be written as $\frac{f(0)}{g(0)} > \frac{p}{1-p}\frac{1}{1+\kappa}$ and $\frac{f(1)}{g(1)} < \frac{p}{1-p}(1+\kappa)$. The third inequality boils down to **(A3.2)**. From (5) and (25), we finally get:

$$\frac{f(x^-)}{g(x^-)}\,\frac{\lambda+\mu(1-F(x^-))}{\lambda+\mu\, G(x^-)} = \frac{f(x^*)}{g(x^*)} = \frac{p}{1-p} \tag{28}$$

Let $\tilde{x}$ be the value that solves $F(\tilde{x}) + G(\tilde{x}) = 1$. Combining (27) and (28), we have:

$$\frac{f(z)}{g(z)}\,\frac{\lambda+\mu\, F(z)}{\lambda+\mu(1-G(z))} \gtrless \frac{f(z)}{g(z)} \gtrless \frac{f(z)}{g(z)}\,\frac{\lambda+\mu(1-F(z))}{\lambda+\mu\, G(z)} \;\Leftrightarrow\; F(z)+G(z) \gtrless 1 \;\Leftrightarrow z \gtrless \tilde{x}$$

There exists $\tilde{p}$ such that $\frac{f(x^+(\tilde{p}))}{g(x^+(\tilde{p}))}\frac{\lambda+\mu\, F(x^+(\tilde{p}))}{\lambda+\mu(1-G(x^+(\tilde{p})))} = \frac{f(x^-(\tilde{p}))}{g(x^-(\tilde{p}))}\frac{\lambda+\mu(1-F(x^-(\tilde{p})))}{\lambda+\mu\, G(x^-(\tilde{p}))} = \frac{f(x^*(\tilde{p}))}{g(x^*(\tilde{p}))} = \frac{\tilde{p}}{1-\tilde{p}}$. For all $p \gtrless \tilde{p}$, then $x^+(p) \lessgtr x^*(p) \lessgtr x^-(p) \lessgtr \tilde{x}$.

A9. Proof of Proposition 11

Let $\alpha \equiv P(A)P(1|A) = p(1-G(x))$, $\beta \equiv P(B)P(1|B) = (1-p)(1-F(x))$, $\gamma \equiv P(A)P(0|A) = pG(x)$, and $\delta \equiv P(B)P(0|B) = (1-p)F(x)$. Also, cell firing being informative, we need $\overline{p}(x) = \Pr(A|c > x) > 1/2 \Rightarrow \frac{\alpha}{\alpha+\beta} > 1/2 \Rightarrow \alpha > \beta$. Similarly, $\underline{p}(x) = \Pr(A|c < x) < 1/2 \Rightarrow \frac{\gamma}{\gamma+\delta} < 1/2 \Rightarrow \delta > \gamma$. We have:

$$\overline{p}(x, l_H) - \overline{p}(x) \equiv \overline{C}^+ = \frac{\alpha\beta(2\theta-1)}{[\alpha\theta + \beta(1-\theta)](\alpha+\beta)},$$

$$\overline{p}(x) - \overline{p}(x, l_L) \equiv \overline{C}^- = \frac{\alpha\beta(2\theta-1)}{[\alpha(1-\theta) + \beta\theta](\alpha+\beta)}$$

It is immediate to see that $\overline{C}^- - \overline{C}^+ \propto \alpha - \beta \geqslant 0$. Furthermore, $\frac{\partial \overline{C}^+}{\partial \theta} \propto \alpha\beta \geqslant 0$ and $\frac{\partial \overline{C}^-}{\partial \theta} \propto \alpha\beta \geqslant 0$. Also,

$$\frac{\partial \overline{C}^+}{\partial x} \propto \Big((1-\theta)\beta^2 - \alpha^2\theta\Big)\Big(\beta\frac{\partial \alpha}{\partial x} - \alpha\frac{\partial \beta}{\partial x}\Big).$$

Given $\alpha > \beta$ and $\theta > 1-\theta$, we have $(1-\theta)\beta^2 - \alpha^2\theta < 0$. Also, $\beta\frac{\partial \alpha}{\partial x} - \alpha\frac{\partial \beta}{\partial x} = (1-p)p[(1-G(x))f(x) - (1-F(x))g(x)] \geqslant 0$ for all x given MLRP. Therefore $\partial\overline{C}^+/\partial x \leqslant 0$. Last,

$$\frac{\partial \overline{C}^-}{\partial x} \propto \Big((1-\theta)\alpha^2 - \beta^2\theta\Big)\Big(\alpha\frac{\partial \beta}{\partial x} - \beta\frac{\partial \alpha}{\partial x}\Big)$$

so for each x there exists $\hat{\theta}$ such that $\frac{\partial \overline{C}^-}{\partial x} < 0$ for all $\theta < \hat{\theta}$ and $\frac{\partial \overline{C}^-}{\partial x} > 0$ for all $\theta > \hat{\theta}$. Similarly,

$$\underline{p}(x) - \underline{p}(x, l_H) \equiv \underline{C}^+ = \frac{\delta\gamma(2\theta-1)}{[\delta\theta + \gamma(1-\theta)](\delta+\gamma)},$$

$$\underline{p}(x, l_L) - \underline{p}(x) \equiv \underline{C}^- = \frac{\delta\gamma(2\theta-1)}{[\delta(1-\theta) + \gamma\theta](\delta+\gamma)}$$

It is again immediate to see that $\underline{C}^- - \underline{C}^+ \propto \delta - \gamma \geqslant 0$. Furthermore, $\frac{\partial \underline{C}^+}{\partial \theta} \propto \delta\gamma \geqslant 0$ and $\frac{\partial \underline{C}^-}{\partial \theta} \propto \delta\gamma \geqslant 0$. Also,

$$\frac{\partial \underline{C}^+}{\partial x} \propto \Big((1-\theta)\gamma^2 - \delta^2\theta\Big)\Big(\gamma\frac{\partial \delta}{\partial x} - \delta\frac{\partial \gamma}{\partial x}\Big).$$

Given $\delta > \gamma$ and $\theta > 1-\theta$, we have $(1-\theta)\gamma^2 - \delta^2\theta < 0$. Also, $\gamma\frac{\partial \delta}{\partial x} - \delta\frac{\partial \gamma}{\partial x} = (1-p)p[G(x)f(x) - F(x)g(x)] \leqslant 0$ for all x given MLRP. Therefore $\partial\underline{C}^+/\partial x \geqslant 0$. Last,

$$\frac{\partial \underline{C}^-}{\partial x} \propto \Big((1-\theta)\delta^2 - \gamma^2\theta\Big)\Big(\delta\frac{\partial \gamma}{\partial x} - \gamma\frac{\partial \delta}{\partial x}\Big)$$

so for each x there exists $\tilde{\theta}$ such that $\frac{\partial \underline{C}^-}{\partial x} > 0$ for all $\theta < \tilde{\theta}$ and $\frac{\partial \underline{C}^-}{\partial x} < 0$ for all $\theta > \tilde{\theta}$.

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