Wage Policies and Incentives to Invest in Firm-Specific Human Capital

George Baker
Nancy Dean Beaulieu
Cristian Voicu

Harvard Business School

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Comments Welcome

Abstract

The accumulation of firm-specific knowledge improves firm productivity and employee retention, by creating a wedge between what the employee is worth inside and outside the firm. How does the firm create incentives for investment in firm-specific human capital when this investment requires costly employee effort and is not contractible? We characterize the optimal linear wage policies that provide such incentives. In a one-period model with stochastic outside offers, we obtain several results. (1) The firm-specific and general investments are substitutes in the employee's utility function, even if they are not in the cost function. (2) The firm-specific investment is zero below a threshold sharing rule, but it can jump discontinuously to a large level above the threshold. (3) For some parameters, the firm is fragile (the employees are indifferent between staying and leaving), but for others it is stable. We go on to derive the optimal wage policy in a dynamic framework when the firm can credibly commit not to change the policy over time. We show that this full-commitment wage policy is not dynamically consistent: the firm would like to renege on its commitment to pay high wages in later periods. This research sheds light on how the incentives for and the accumulation of firm-specific knowledge lead some firms to the brink of dissolution and others to a virtuous circle of stability and high profits.

Section 1: Introduction

Firm-specific human capital creates a gap between what an employee is worth to her current employer, and what she is worth in her next-best alternative job. This gap is a source of rents for both the employee and the firm, allowing employees to make more than they would make in another job, and allowing a firm to pay less than the worker is actually worth to the firm. How this rent is divided is an important part of a firm's wage policy.

Becker (1964) argued that employees should pay for all investments in general human capital (GHC), while investments in firm-specific human capital (FSHC) should be split between the firm and the worker. An implicit assumption of this argument is that investment in human capital is contractible, and that firms and workers can agree on the level of this investment. However, often investment in human capital involves subtle actions by employees that are difficult to contract on. In addition, they often involve private costs to the employee that are hard to measure and compensate for. Thus, investment in human capital has many of the characteristics of a moral hazard problem.

In this paper, we examine how a firm's wage policy should split the rents generated by the existence of FSHC. The trade off is simple: the more of the rents the firm's wage policy gives to the employee, the higher are the employee's incentives to invest in FSHC. However, the more of these rents the firm gives to employees, the less it takes for itself. Thus the firm has to balance the costs of a wage policy that grants much of the surplus to workers with the benefits that such policies have in inducing more investment in FSHC.

This trade-off is not the end of the story, however. Investment in FSHC, if it crowds out investment in general human capital, is risky to the employee since it lowers her outside wage (relative to what it could be) and raises the possibility that the firm will exploit the worker by lowering wages in the future. We examine this possibility, and show that firms would like (*ex ante*) to commit to not exploit workers in this way.

We develop several models which allow us to explore the implications of the fact that employees choose their levels of investment in firm-specific human capital, and that firms design wage policies to affect this choice. The models allow us to probe the trade-offs discussed above. In addition, the models allow us to understand what we believe to be a vicious/virtuous circle that affects human capital intensive firms, illustrated by the following two anecdotes.

Two Anecdotes

On January 14, 2005, the partners of the Boston law firm Testa Hurwitz & Thibeault decided to disband after 31 years in practice. The move came only a month after 10 partners decided to leave the firm to pursue "opportunities elsewhere that they considered more attractive." The firm of 600 lawyers and staff disappeared within 60 days. In making the partner vote announcement, Managing Partner George Davitt said, "A law firm such as ours, although prominent, profitable and filled with talented lawyers is – like any professional services organization – knit of a fabric that, if stretched too thin, can unravel." [1]

This paper attempts to understand the fragility of Testa Hurwitz and other human capital intensive firms. It often seems that some professional firms are perpetually on the brink of dissolution, with their most valuable partners constantly on the lookout for better opportunities, threatening to take their skills elsewhere, and devoting energy to increasing their value on the market (that is, investing in general human capital) rather than maximizing their value to their

[1] http://www.tht.com/News/news_tht_news_disbanded.htm

current firm (by investing in firm-specific human capital). These firms are threatened by a vicious circle in which professionals choose to maximize their value outside the firm, and so do not invest in the firm-specific human capital that would raise their value to their current employer above what they are worth on the market. As a result, in order to keep them from leaving, the firm is forced to reward them on the basis of their market value, which only encourages the professionals to invest in yet more general human capital.

At the same time, other firms seem immune to this problem. For the first 20 years of its existence as a firm, Kohlberg Kravis Roberts (KKR), a New York investment bank, never saw a professional quit to join another financial services firm.[2] When asked why they were not tempted by competitors, these professionals said simply that they were paid more at KKR than anyone else offered them. The firm's compensation system kept people's pay well above their outside value, which gave them no incentives to invest in their outside value, and so they focused their energies on investing in firm-specific human capital. This investment further increased the spread between their value to KKR and what they were worth to other firms, making it even less likely that they would leave.

Outline of the paper

In Section 2, we provide a review of the related literature. In Section 3, we first develop a one-period model with an outside labor market that makes stochastic offers to employees. With this model we prove four results. First, investment in firm-specific human capital and investment in general human capital are substitutes in the employee's utility function. Second, the employee's decision about how much to invest in firm-specific human capital is not convex in

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[2] George Baker and George Smith, *The New Financial Capitalist,* 1998.

the firm's wage policy: she is likely to jump from no investment in FSHC to a large positive investment for a small change in the firm's wage policy. Our third result is that there may be no profitable wage policy that induces non-zero investment in FSHC. This occurs either when the value of FSHC is too low, or the outside labor market generates wage offers that are too high. The fourth result is also an existence result: there are situations in which the firm does not pin the worker to her participation constraint. That is, the optimal wage policy makes the employee strictly better off than she would be in her next best alternative.

This model suggests some important questions that cannot be explored in a one-period context. In particular, the firm's optimal wage policy will induce a set of investments in firm-specific and general human capital that would seem to tempt the firm, in subsequent periods, to exploit the fact that the worker has made significant investments in FSHC by lowering her wages. In order to explore this dynamic incentive, in Section 4 we develop two multi-period models. We first simplify the outside labor market, and examine the evolution of the wage policy through time assuming that both firm and worker are myopic: that is they do not foresee the effects of current investments in human capital on future productivity, wages, etc. We then derive the optimal wage policy for the firm assuming that it is able to fully commit to this policy over multiple periods. This wage policy results in high and growing wages for the employee, and large investments in FSHC in all periods. However, it also generates significant temptation for the firm to alter its wage policy going forward: that is, the multi-period wage policy is not dynamically consistent. The firm would like to lower the wage in all periods after the first one, taking advantage of the fact that the worker has a large stock of firm-specific human capital. Section 5 concludes.

Section 2: Literature Review

(To be completed)

Section 3: Model Set-up and One Period Model

The presence of firm-specific human capital creates a gap between what an employee is worth to the firm and the value to the next best alternative job. The value to the firm M_t at time t is the marginal contribution generated by the employee. It is a function of firm-specific and general human capital.

$$M_t = G_t + S_t + \varepsilon_t \tag{1}$$

In the expression above, G_t is the stock of general human capital at time t, S_t is the stock of firm-specific human capital, and ε_t is a random variable representing unanticipated shocks.

The employee can invest in the stocks of general and firm-specific human capital over time. Let the productivity of investments in GHC be a, and the productivity of investments in FSHC be m. The stocks of human capital depreciate at rate δ. Let the investment in human capital at time t be e_{st} and e_{gt}. The letter e is used here to indicate effort: investments in human capital require the worker's personal effort in order to become productive.

$$G_t = (1-\delta)G_{t-1} + ae_{gt} \tag{2}$$

$$S_t = (1-\delta)S_{t-1} + me_{st} \tag{3}$$

Combining equations (1), (2) and (3), productivity is:

$$M_t = (1-\delta)G_{t-1} + (1-\delta)S_{t-1} + ae_{gt} + me_{st} + \varepsilon_t \tag{4}$$

The employee does not have to stay with the firm if the compensation is too low. There is always the possibility of seeking alternative employment. The outside offer A_t is a function of the amount of general human capital, but not of the amount of firm-specific human capital she possesses. By definition, the stock of firm-specific human capital can only be used at the present firm.

The employee's best outside offer A_t might be higher than G_t by ϕ_t. The variable ϕ_t could represent differences in worker productivity at a different firm or it could represent measurement error in G_t by other firms. We treat ϕ_t as a random variable, reflecting the fact that the best outside offer is not known ex-ante. The highest outside offer made to the employee is:

$$A_t = \left(1-\delta\right)G_{t-1} + ae_{gt} + \phi_t \tag{5}$$

It will generally be the case that $M_t > A_t$. This bargaining range between A_t and M_t leads to indeterminacy in the wage. We assume that the firm offers the worker a take-it-or-leave-it wage contract in order to reduce haggling (Williamson, Wachter, and Harris 1975). We will analyze a simple wage contract that is a linear function of the contractibles in the model.

In our model, the stocks of general and firm-specific human capital G_t and S_t are not contractible. They cannot be reliably inferred by the firm from the total output M_t because of the random error ε_t. The investments in human capital e_{gt} and e_{st} are also not contractible because they are not observed by the firm. Furthermore, we assume the wage can be contractually based on M_t but not A_t. We base this assumption on several arguments. First, firms invest in many performance measurement systems to allow them to determine an employee's total contribution to the firm; they invest in few such systems to determine an employee's value to other firms.[3] In addition, using A_t in the co ntract would invite a different kind of moral hazard problem, since the worker would have an incentive to generate outside offers.

We will focus on linear wage policies, which are simple and robust in a variety of settings. We further assume that the worker is wealth-constrained (any fixed component of the wage policy must be positive) to avoid the trivial solution of selling the firm to the worker.

$$W_t = \alpha_t M_t + \gamma_t \tag{6}$$

where $\gamma_t \geq 0$.

The investments in general and firm-specific human capital are costly for the employee. The cost of investment is in general a function $C\left(e_{st}, e_{gt}\right)$ where $e_{st} \geq 0$ and $e_{gt} \geq 0$. The cost function is assumed to be increasing and convex in each argument.

We abstract way from risk aversion considerations and assume that both the employee and the firm are risk neutral. Risk-neutrality, the linearity of the wage policy, and the fact that the mean-zero random shock ε_t is observed only after the employee decides whether to stay at the firm allows us to ignore the ε_t random variable.

The employee's and the firm's expected utility and the firm's profit in each period t are specified below:

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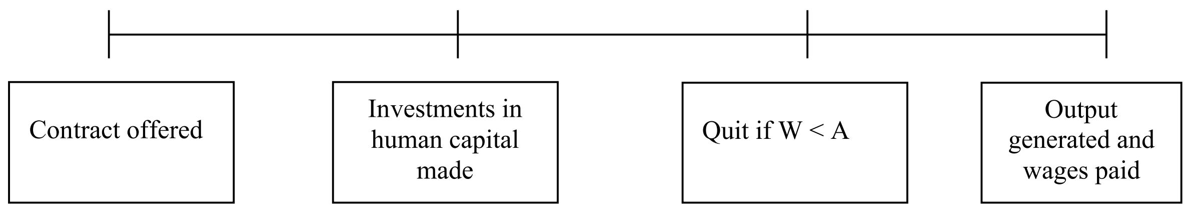
[3] Firms may engage in market surveys to determine the range of outside wages for certain types of jobs, but these are generally designed to evaluate a job, rather than an individual.

$$EU_t = E_t\left(\max\left\{W_t, A_t\right\}\right) - C\left(e_{st}, e_{gt}\right) \tag{7}$$

$$\pi_t = \left(M_t - W_t\right) \cdot \Pr\left\{W_t \geq A_t\right\} \tag{8}$$

The timing of the model in each period t is as follows:



The firm offers a wage contract, then the employee chooses e_{st} and e_{gt}. If the worker chooses $e_{st} = 0$, then she is choosing not to invest in FSHC. We will refer to this as a participation choice by the worker. In order for the firm to make profits, it must satisfy the worker's participation constraint and induce the employee to invest $e_{st} > 0$. After making her investment decisions, the employee then checks to see whether the realized outside alternative is better then the expected wage inside the firm. If so, the employee produces output and the firm pays the wage. If not, the employee leaves with her stocks of human capital, earns A_t and the firm gets no profit.

One Period Model

We begin by analyzing a one-period version of our model. The firm chooses a wage policy that maximizes total profitability. The wage policy aims to provide incentives for the worker to invest in FSHC. For the one-period model, we drop all time subscripts[4]. We also assume that there is no pre-existing stock of firm-specific human capital (i.e. $S = 0$).

We solve this moral hazard problem in two steps. First, we find the optimal investments in FSHC and GHC for a given wage policy. Second, we explore the wage policy that maximizes total firm profits.

$$\begin{aligned} &\max_{\alpha,\gamma} \pi\left(\alpha,\gamma,e_s^*,e_g^*\right) \\ &\text{s.t. } \left(e_s^*,e_g^*\right) = \arg\max_{e_s,e_g} EU\left(\alpha,\gamma,e_s,e_g\right) \end{aligned} \tag{9}$$

Recall that the worker's outside offer, received after investing in human capital, is

$$A = G + ae_g + \phi \tag{10}$$

The dollar amount A can be thought of as the highest offer made for the employee by n outside firms. The variance in these outside offers could have several sources. Perhaps other firms misestimate the worker's general human capital. Or perhaps there is some type of idiosyncratic match capital between the worker and other firms. Regardless of the source of this variance, ϕ will have an extreme value distribution, F(ϕ) (David and Nagaraja, 2003). As n gets larger the mean of ϕ increases, and F(0) and f(0) both approach 0. It is intuitive to think of the mean of ϕ ($\bar{\phi}$) as capturing how active the outside labor market is. As either the number of outside firms goes up, or the variance of their offers increases, $\bar{\phi}$ increases.

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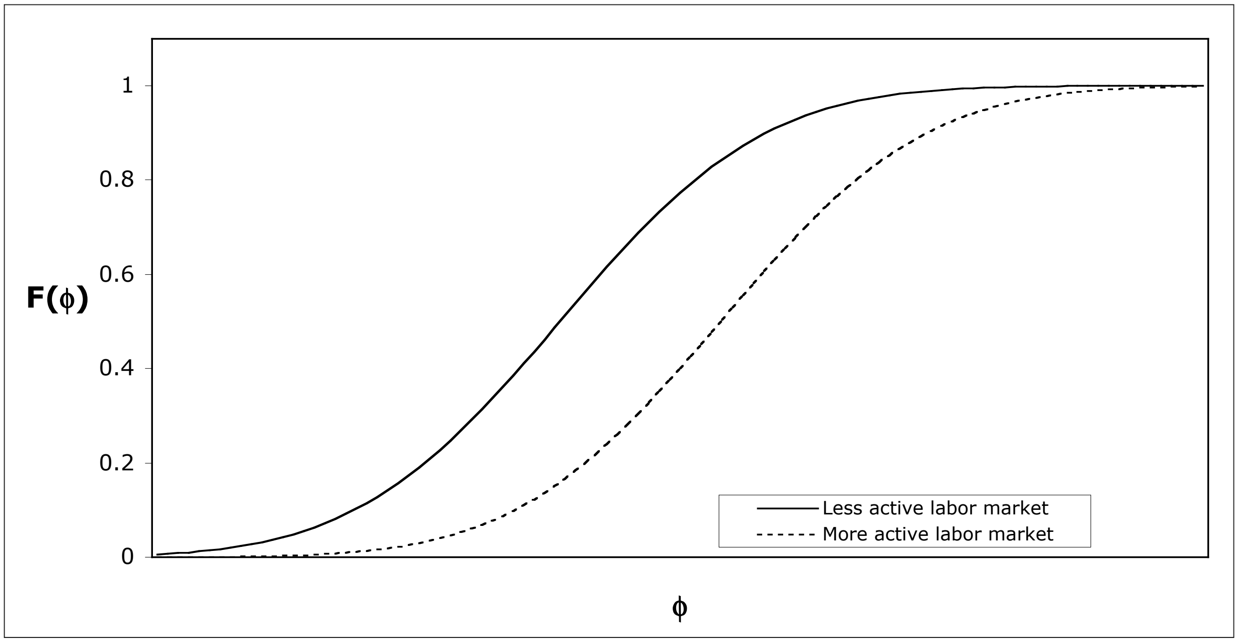
[4] For more compact notation, let the stock of general human capital be $G \equiv \left(1-\delta\right)G_{t-1}$.



Note that $F(0)$ approaches zero as n increases. This means that the worker can receive at least the value of her general human capital in alternative employment. Therefore, the firm needs to pay above the value of the worker's general human capital in order for there to be any possibility that the worker will stay and work at the firm.

The probability of staying can be expressed as follows

$$\Pr(stay) = \Pr(W > A) = \Pr\left(W > ae_g + G + \phi\right) = \Pr\left(\phi < W - ae_g - G\right) = F\left(W - ae_g - G\right) \tag{11}$$

It will be useful to define a variable x for the difference between the wage and the total amount of general human capital, such that $\Pr\left(W > A\right) = F\left(x\right)$

$$x = W - G - ae_g = \alpha\left(G + ae_g + me_s\right) + \gamma - \left(G + ae_g\right) \tag{12}$$

We can now write the overall problem in the model with stochastic outside offers

$$\max_{\alpha,\gamma} F\left(x\right)\left(M-W\right)$$
$$\text{s.t. } \left(e_s^*,e_g^*\right)=\underset{e_s\geq 0,e_g\geq 0}{\arg\max}\int_0^\infty \left[\max\left(W,G+ae_g+\phi\right)\right]f\left(\phi\right)d\phi-C\left(e_s,e_g\right) \tag{13}$$

The first step is to analyze the worker's optimal investments in human capital. The worker's expected utility maximization problem can also be formulated as

$$\max_{e_s\geq 0,e_g\geq 0} G+ae_g+xF\left(x\right)+\int_x^\infty \phi f\left(\phi\right)d\phi-C\left(e_s,e_g\right) \tag{14}$$

The intuition for this formula is as follows. The employee receives compensation of at least the stock of general human capital $G+ae_g$. On top of that, the worker receives either $x=W-G-ae_g$ or ϕ. x is received if she stays with the firm, with pro bability *F(x)*. If she leaves, she receives the mean of ϕ conditional on $\phi>x$.

The first order conditions at an interior solution are derived in the Appendix 1 and stated below

$$EU_1\left(e_s,e_g\right)=\alpha mF\left(x\right)-C_1\left(e_s,e_g\right) \tag{15}$$

$$EU_2\left(e_s,e_g\right)=\alpha aF(x)+a\left[1-F(x)\right]-C_2\left(e_s,e_g\right) \tag{16}$$

The marginal utility of investment in firm-specific human capital is αm when the worker stays, but it is 0 otherwise. The marginal benefit of investment in general human capital is αa when the worker stays and a otherwise.

Equations 15 and 16 can be used to show our first result: e_s and e_g are substitutes in the worker's expected utility function, even if they are not substitutes in her disutility of effort function.

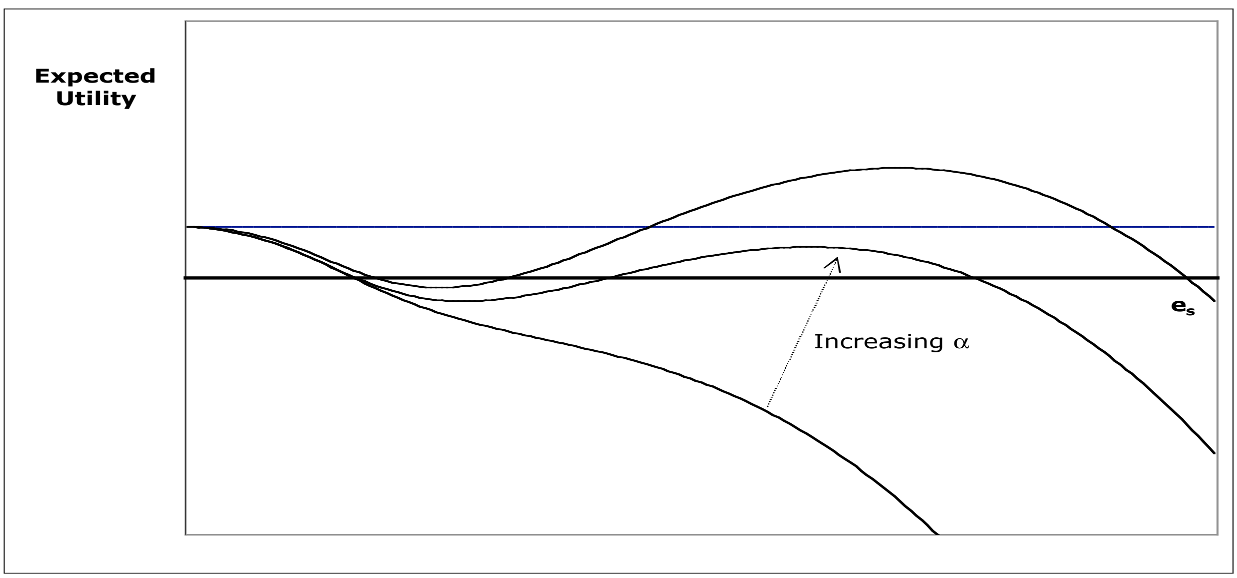
$$EU_{12}\left(e_s,e_g\right) = -am\alpha\left(1-\alpha\right)f\left(x\right) - C_{12}\left(e_s,e_g\right) < 0$$

Consider the case where $C_{12}\left(e_s,e_g\right) = 0$: i.e. e_s and e_g are not substitutes in the cost function. Then an increase in e_g will have no effect on the $C_1\left(e_s,e_g\right)$, but it will reduce *x,* and therefore *F*(*x*). Thus the marginal utility of e_s will decline as e_g increases, implying that they are substitutes in the worker's utility function. The intuition for this result is simple: when the worker invests in more GHC, she reduces the likelihood that she will stay with the firm. This reduces the marginal utility of investing in FSHC. If she invests more in FSHC, she lowers the probability of leaving, and decreases the marginal utility of investment in GHC.

Having solved for the employee's optimal investment decisions, we can now turn to exploring the firm's optimal wage policy. Even with explicit functional forms for *F(x)* and $C(e_s,e_g)$ it is not possible to find a closed-form solution to the firm's maximization problem. Even without a closed-form solution, however, we can still prove several results about the firm's optimal wage policy. We first note that γ^* is zero. (See Appendix 2.) The firm would like to set $\gamma < 0$, but the fact that the worker is wealth constrained forbids this. Thus the optimal wage policy is characterized by a single parameter, α^*.

We begin our analysis of the optimal α by focusing on the worker's marginal utility of investing in FSHC: $\alpha mF\left(x\right) - C_1(e_s,e_g)$. Start by assuming that $\alpha < 1$, so that the firm makes positive profits. Note first that for $e_s = 0$, $x < 0$ and (for large n) $F(x) \approx 0$. In this case, the worker leaves for sure, and expected utility is independent of α_t.

When α is very small, expected utility is decreasing for all $e_s > 0$. Therefore in that case the worker's optimal solution is $e_s^* = 0$. As α increases, the worker's utility from investing in firm-specific human capital increases unboundedly for all $e_s > 0$. (See figure below.) Thus for α large enough, the employee will always choose to invest in a positive level of firm-specific human capital.[5] However, this move from $e_s = 0$ for small α to $e_s > 0$ for larger α is not likely to be continuous. This is our second result: for many parameter values, there is a critical value of α at which the employee's investment in FSHC will jump from zero to a positive level.



[5] Note that when $\alpha > 1$, expected utility evaluated at e_s=0 is no longer independent of α. However, expected utility at e_s=0 rises more slowly than expected utility with e_s>0 so it is still true that there exists an α such that $EU|e_s=0 < EU|e_s>0$.

With this understanding of the non-convexity of the employee's investment decision problem, we can now reformulate the firm's profit maximization problem as one of choosing α subject to an incentive compatibility constraint (e_s^*, the interior solution, satisfies the first-order condition of Equation 14) and a participation constraint $\left(EU\big|_{e_s^*>0} \geq EU\big|_{e_s=0} \right)$. This reformulated program, while analytically identical to the one shown in equation 13, is convenient for understanding our next two results.

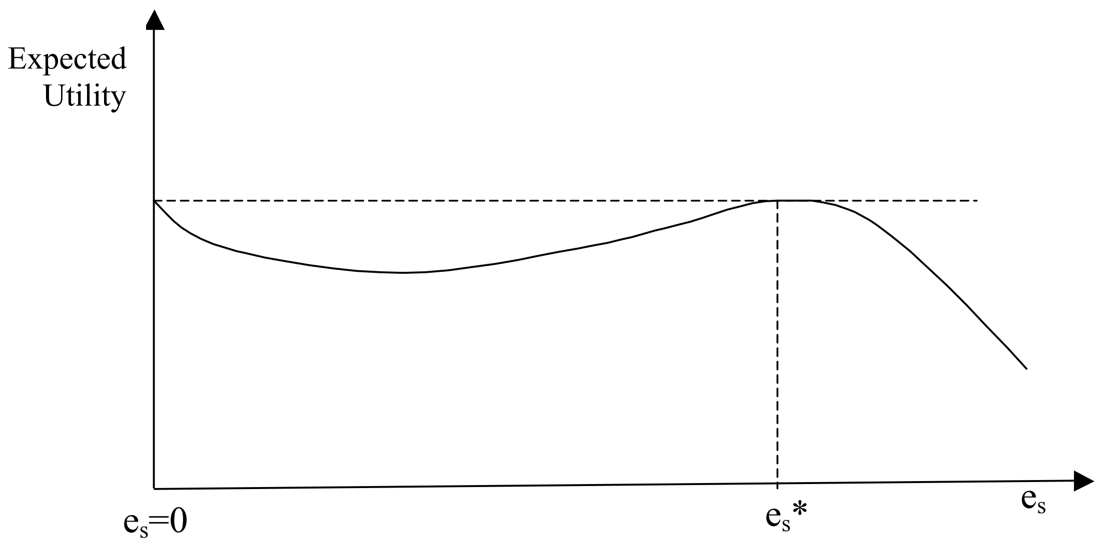
Our third result is that, for certain parameter values, no profitable wage policy exists. That is, there is no $\alpha < 1$ that satisfies the participation constaint. (See Appendix 3 for proof). This occurs when the productivity of investment in FCHS (m) is low and/or the outside labor market is very active ($\bar{\phi}$ is large). This result—that there is no profitable wage policy—occurs when the returns to the employee of investing in FSHC, in terms of both increased wages and the increased probability of staying, are not high enough to get the worker to forgo the possibility of an attractive outside offer.

Our fourth result is that there exist parameter values for which the firm sets α so that the participation constraint is slack. That is, at the optimal α the employee is strictly better off investing in positive levels of FSHC than she would be at $e_s = 0$. (Proof in Appendix 4.) This occurs when the value of FSHC investment is high (*m* is large), and the probability of large outside wage offers is low.

This result demonstrates that, when investment in human capital is important, firms may pay employees well above their opportunity wage. Note that in this situation, the firm could lower wages and still not risk losing the employee, but chooses not to so as to increase her incentive to invest in FSHC.

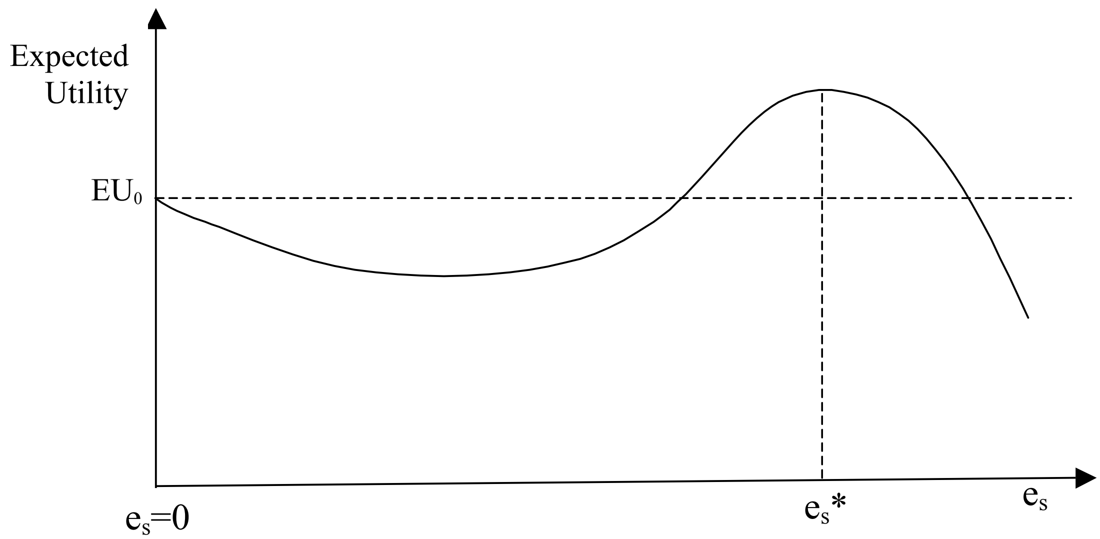
Discussion and Empirical Implications

This model makes predictions about the stability of human capital intensive firms. In particular, the non-convexity of the employee's investment problem, and the resultant lack of continuity in her investment choice and the firm's optimal wage policy, lead to the conclusion that some firms might be quite fragile. This fragility occurs when the participation constraint binds at the optimal wage policy. In certain environments, the firm's optimal wage policy makes the worker just indifferent between investing in positive levels of FSHC and investing in none. The figure below illustrates this indifference.

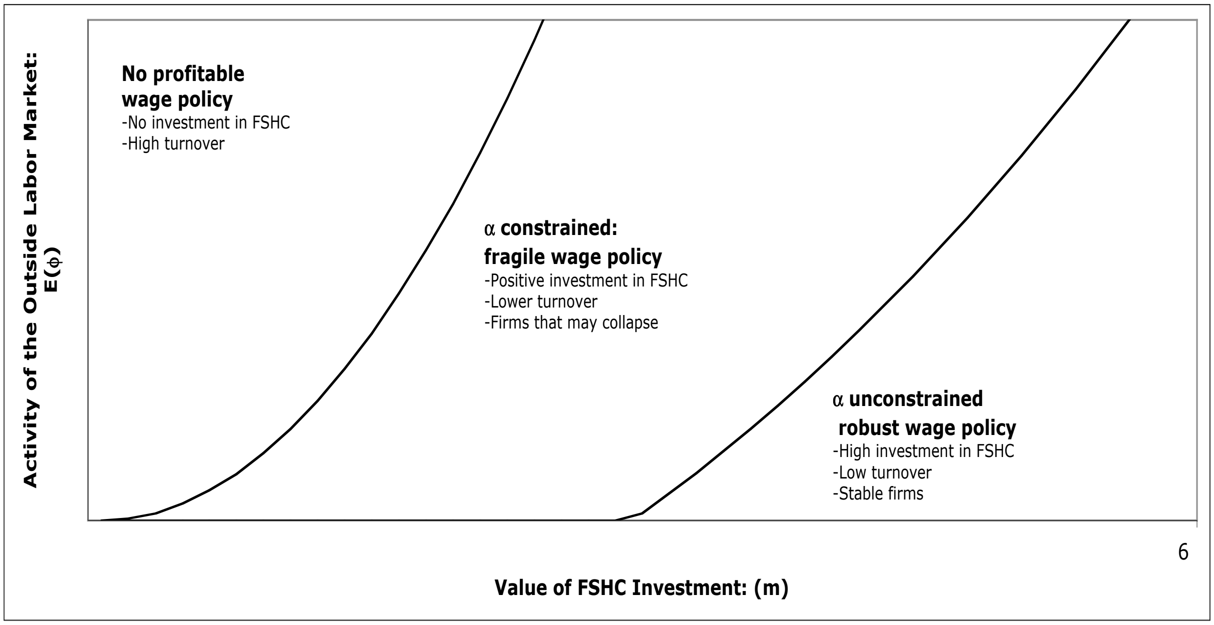


Consider the consequences of a small decrease in the marginal value of investment in FSHC (m) or a small increase in the competitiveness of the outside labor market ($\bar{\phi}$) for a firm with this optimal α. Either of these changes makes the employee's expected utility at e_s^* less than the expected utility at $e_s = 0$. In the event of such a change, the firm must react immediately with an increase in α, in order to keep the employee investing in FSHC. If it does not, the employee will no longer find investment in any positive level of FSHC optimal, and will reduce her investment to zero. This scenario is reminiscent of the story of the law firm in the introduction. In those firms where the optimal wage policy is bound by the participation constraint, employees are constantly on the brink of defecting, reducing their investments in firm-specific human capital, and focusing on their outside market value.

In other environments (when m is higher or $\bar{\phi}$ is lower) the firm will have a more robust wage policy. In these cases, the firm's optimal wage policy gives the employee strictly more utility than she would get from not investing in FSHC. This situation is depicted below: the employee invests e_s* in firm-specific human capital and earns a positive utility increment over EU_0. Note that even if the firm does not respond immediately to small changes in *m* or $\bar{\phi}$, the employee will still choose to invest in positive levels of FSHC, will continue to produce value for the firm, and will not leave. Thus small changes in the environment do not threaten such firms with mass defections.



More generally, the model predicts that there will be three different regimes, determined mainly by differences in m and $\bar{\phi}$, that result in three different outcomes for firms. In the first regime, characterized by low levels of m and/or high levels of $\bar{\phi}$, there will be no profitable wage policy. Employees will invest only in general human capital, and turnover will be high. In the second regime, characterized by intermediate levels of m and $\bar{\phi}$, firms will have fragile wage policies. Employees will invest in FSHC, but firms will have a tendency to be short-lived and fragile, since they must be constantly adjusting their wage policies to keep employees from defecting. In the third regime (high levels of m and/or low levels of $\bar{\phi}$) firms will have robust wage policies, employees will invest in FSHC, and firms will have a tendency to be long-lived and stable.



The model makes some new empirical predictions about the relationship between the value of human captial, labor market conditions, wages and profits. It also complicates the causal link between wage levels and turnover. In a traditional model, exogenously high levels of firm-specific human capital leads to low levels of turnover, since turnover is costly (and wasteful) in the presence of FCHC. In our model, this causal link also runs in reverse: when employees are less likely to turnover (for exogenous reasons) they will invest more in FSHC. In addition, exogenously high levels of turnover increase investment in general human capital. This leads to our first set of potentially testable emprical predictions: while the effect of low turnover on wages within the firm are ambiguous, low turnover should unambiguously lower workers' alternative wages by reducing their equilibrium level of investment in general human capital.

Thus, for instance, workers laid off from low turnover firms should get lower wages *ceteris paribus* than those laid off from high turnover firms.[6]

The model also makes predictions about wage differentials in different labor markets. Specifically, we predict that workers laid off in labor markets characterized by high turnover should get higher wages on re-employment than workers laid off in low turnover labor markets. As an example, consider the differences between the labor market for engineers in Silicon Valley in Californai and Rt. 128 in Massachusetts. AnnaLee Saxenian (1994) has argued that these two regions differed significantly in their degree of labor market turnover. Our model would predict that otherwise similar workers laid off in Silicon Valley (which has traditionally had high turnover levels) should get higher wages on re-employment than workers laid off from Route 128 firms.

Our model also makes sense of a seemingly puzzling practice in many firms. Human Resource Departments in firms often strive to maintain high levels of employee satisfaction by developing policies, sponsoring activities, and even providing facitlities that will make employees enjoy their workplace and their co-workers more. Such practices make sense in a world in which more satisfied employees are less likely to leave, and so are more likely to invest in (profitable) firm-specific human capital.

Suppose that the firm could induce "affiliation" among employees, that would make them like their work and/or their co-workers, but had no effect on productivity. Affiliation increases workers' utility from this job, but not any other: it is firm-specific, but not productive. Such

[6] Note that the result is not that they will suffer larger *wage declines* (because the effect of turnover on current wages is ambiguous) but rather than their wages will be lower.

affiliation is valuable to workers (because they get utility from it), and also to the firm, since it lowers turnover and increases investment in FSHC for any given level of α.

An additional empirical prediction is generated by a model that includes affiliation, to the extent that affiliation is measurable. Many firms collect data on employee satisfaction. To the extent that these measures of satisfaction are in fact capturing something about the level of affiliation felt by workers towards their firm and co-workers, then our model would predict that high levels of employee satisfaction will lead to higher levvels of investment in firm-specific human capital, lower levels of investment in general human capital, and lower alternative wages. Thus, employees laid off from high-satisfaction firms should have lower wages on re-employment than workers laid off from low-satisfaction firms.

Section 4: Multi-Period Models

The model above suggests questions that cannot be answered in the context of a single period model. For instance, as stocks of firm-specific capital accumulate and turnover probabilities go down, how does the optimal wage policy change? What is the risk to the employee of investing in firm-specific human capital, when this will make her vulnerable to the firm exploiting her in future periods? We now turn to analyzing a multi-period version of this model in order to understand the dynamics of these stocks, and the firm's dynamic wage policy.

In order to study the dynamics of this model, we make an important simplification. We assume that the employee faces a deterministic outside offer A_t, which we assume is equal to:

$$A_t = (1-\delta)G_{t-1} + ae_{gt} + \phi \tag{19}$$

where ϕ is a known number.[7] When the parameter ϕ is constant, the wage policy determines unambiguously the worker's quit/stay decision. Either the worker stays with probability one, or she leaves with probability one. As long as positive profits can be made, the firm will offer a wage policy that induces the worker to stay even after receiving the outside offer. As in the model with stochastic outside offers, there are parameters for which a robust wage policy is optimal, parameters for which a fragile wage policy is optimal, and parameters for which no profitable wage policy exists.

A Myopic Model

We first analyze a myopic multi-period model, in which neither the worker nor the firm anticipates the future effects of the investments made in firm-specific and general human capital. We develop this model not because we think it is realistic, but because it allows us to establish a baseline model against which to compare more sophisticated multi-period models.

The within-period timing is the same as above: the firm offers the worker a wage contract (γ_t and α_t) and the worker either accepts this wage offer knowing that she will invest in FSHC

[7] This would be the case, for instance, if the distribution of other firms' values had an upper support, and there were a large number of other firms.

or chooses to invest in no FSHC. Since ϕ is deterministic, whether the employee stays or quits is determined by the wage policy. We thus once again think of the worker's decision about whether or not to invest in FSHC as a participation constraint. In each period, the worker chooses to stay if:

$$\max_{e_{st},e_{gt}}\left\{W_t - C\left(e_{st},e_{gt}\right)\right\} \geq \max_{e_{st},e_{gt}}\left\{A_t - C\left(0,e_{gt}\right)\right\} \tag{20}$$

If the worker stays with the firm, her maximization problem is

$$\max_{e_{st},e_{gt}} \alpha_t\left[\left(1-\delta\right)\left(G_{t-1}+S_{t-1}\right)+ae_{gt}+me_{st}\right]-C\left(e_{st},e_{gt}\right) \tag{21}$$

The first order conditions of this problem can be easily determined. Let $C_1\left(\cdot\right)$ and $C_2\left(\cdot\right)$ be the derivatives of the cost function with respect to the first and second argument.

$$\begin{aligned} C_1\left(e_{st},e_{gt}\right) &= \alpha_t m \\ C_2\left(e_{st},e_{gt}\right) &= \alpha_t a \end{aligned} \tag{22}$$

This system of two equations and two unknowns determines the optimal solution $\left(e_{st}^*,e_{gt}^*\right)$. In order to generate closed-form solutions, we consider the particular case of the quadratic cost function

$$C\left(e_{st},e_{gt}\right) = \tfrac{e_{st}^2}{2} + \tfrac{e_{gt}^2}{2} \tag{23}$$

Optimal investment levels are:

$$\begin{aligned} e_{st}^* &= \alpha_t m \\ e_{gt}^* &= \alpha_t a \end{aligned} \tag{24}$$

If the worker intends to quit, the maximization problem is

$$\max_{e_{st}, e_{gt}} \left(1-\delta\right) G_{t-1} + a e_{gt} + \phi - \tfrac{e_{st}^2}{2} + \tfrac{e_{gt}^2}{2} \tag{25}$$

and the worker chooses the following investment levels:

$$\begin{aligned} e_{st} &= 0 \\ e_{gt} &= a \end{aligned} \tag{26}$$

We next analyze the firm's optimal wage policy in each period, taking into account the participation constraint. Again, it is easy to show that if the worker is wealth constrained, the optimal linear wage policy must have $\gamma_t = 0$. The firm chooses α_t to optimally balance three factors. It would like to keep as large a share of profits as possible by lowering α_t; it needs to provide incentives for investment in human capital; and the wage policy needs to be sufficiently attractive to meet the participation constraint. The participation constraint binds when the firm would like to set a low α_t. When the participation co nstraint is slack, the wage policy is geared towards providing optimal incentives to invest in human capital rather then towards matching the outside alternative.

With a quadratic cost function and a linear wage policy, the firm's problem is

$$\begin{aligned} &\max_{\alpha_t} \left(1-\alpha_t\right)\left[\left(1-\delta\right)\left(G_{t-1}+S_{t-1}\right)+\left(a^2+m^2\right)\alpha_t\right] \\ &\text{s.t. } \alpha_t\left(1-\delta\right)\left(G_{t-1}+S_{t-1}\right)+\tfrac{1}{2}\left(a^2+m^2\right)\alpha_t^2 \geq \left(1-\delta\right)G_{t-1}+\phi_t+\tfrac{1}{2}a^2 \end{aligned} \tag{27}$$

The optimal solution is either

$$\alpha_t = \tfrac{1}{2} - \tfrac{\left(1-\delta\right)\left(G_{t-1}+S_{t-1}\right)}{2\left(a^2+m^2\right)} \tag{28}$$

or

$$\alpha_t = \frac{-(1-\delta)\left(G_{t-1}+S_{t-1}\right)+\sqrt{(1-\delta)^2\left(G_{t-1}+S_{t-1}\right)^2+2\left(a^2+m^2\right)\left((1-\delta)G_{t-1}+\phi_t+a^2/2\right)}}{a^2+m^2} \tag{29}$$

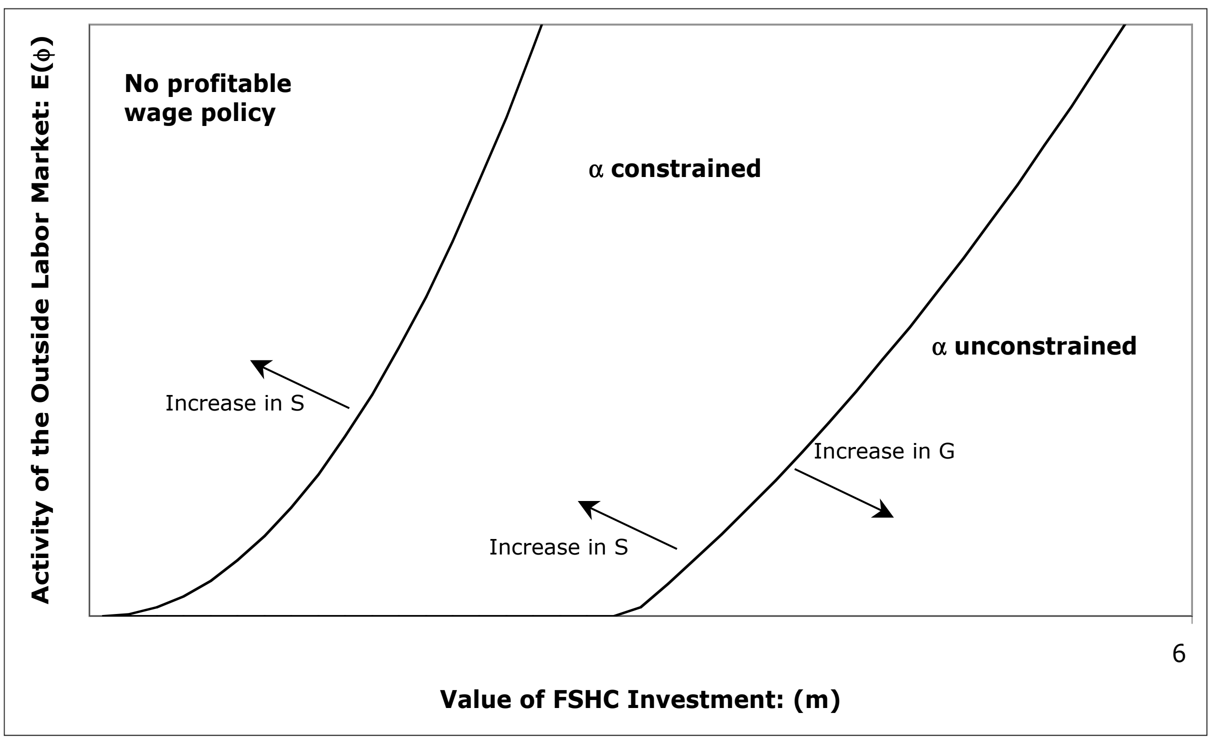
The first solution (equation 28) is optimal when the constraint is slack. This occurs when ϕ is small.

$$\phi < \tfrac{a^2+m^2}{8} - \tfrac{1}{2}a^2 - \tfrac{3}{4}\left(1-\delta\right)G_{t-1} + \tfrac{1}{4}\left(1-\delta\right)S_{t-1} - \tfrac{3}{8}\left(1-\delta\right)^2 \tfrac{\left(G_{t-1}+S_{t-1}\right)^2}{a^2+m^2}$$

The unconstrained α is decreasing in the stocks of human capital. Further analysis shows that the wage itself is actually decreasing in the stocks of human capital. This implies that as the worker's stock of general human capital increases, her wage declines. This seemingly counter-intuitive result highlights a feature of what we called, in the last section, the robust (unconstrained) wage policy. The firm's wage policy is disconnected from the labor market. Firm-specific and general human capital are identical, since there is no chance that the worker will leave the firm, and any increase in the stocks of human capital induce the firm to lower α_t because the firm wants to exploit these existing stocks. The level of sharing increases with the parameters a and m, since the importance of inducing investment in human capital is higher. The unconstrained solution does not depend on ϕ, of course, because the outside labor market plays no role in the robust wage policy.

When $\phi > \dfrac{m^2}{2}$, there is no profitable wage policy: no $\alpha_t < 1$ will satisfy the participation constraint and induce the worker to choose investing in FSHC over investing solely in general human capital. In the intermediate region the firm's wage policy is constrained by the outside labor market. In this region, α_t increases with G_{t-1} and decreases with S_{t-1}.

Having solved for the optimal wage policy in each period as a function of the constant parameters (a, m and ϕ) and the state variables (G_t and S_t), we now briefly explore the dynamic properties of this model. The main point to make is that as the stocks of FSHC grow, the firm is less likely to be in a region with no profitable wage policy, and more likely to find itself with an unconstrained wage policy. As the stocks of general human capital grow, the firm is more likely to be constrained by the outside labor market.



Given the unrealistic assumptions about the firm's and worker's horizons in this model, further analysis of the dynamics are not very insightful. We now proceed to analyze this model under more realistic assumptions.

Multiple Periods with Full Commitment

In this section we focus on the equilibrium with perfect foresight on the part of both the firm and the worker, and full commitment by the firm. That is, the firm is able to commit to the complete path of future α_t's. The case in which the firm cannot commit to a wage policy is studied in the next section. As will be seen, a long horizon and the ability to commit to a wage policy increase the incentives to invest in human capital substantially.

If the worker stays for the entire horizon, her expected utility is

$$EU_{Stay} \equiv \max_{\{e_{s\bullet}, e_{g\bullet}\}} \sum_{t=1}^{T} \beta^{t-1} \left\{ \alpha_t \left[G_0 (1-\delta)^t + m \sum_{j=1}^{t} e_{sj} (1-\delta)^{t-j} + a \sum_{j=1}^{t} e_{gj} (1-\delta)^{t-j} \right] - C(e_{st}, e_{gt}) \right\}$$

where β is the per-period discount factor.

With quadratic cost function, the optimal effort levels are

$$\begin{aligned} e_{st}^{*} &= m \sum_{i=t}^{T} \beta^{i-t} (1-\delta)^{i-t} \alpha_i \\ e_{gt}^{*} &= a \sum_{i=t}^{T} \beta^{i-t} (1-\delta)^{i-t} \alpha_i \end{aligned} \tag{30}$$

If the worker quits at the beginning of period Q, the expected utility is

$$\begin{aligned} EU_{Quit(Q)} \equiv \max_{\{e_{s\bullet}, e_{g\bullet}\}} & \sum_{t=1}^{Q-1} \beta^{t-1} \left[\alpha_t \left(G_0 + S_0 + m \sum_{j=1}^{t} e_{sj} + a \sum_{j=1}^{t} e_{gj} \right) - C(e_{st}, e_{gt}) \right] + \\ & + \sum_{t=Q}^{T} \beta^{t-1} \left[\left(G_0 + a \sum_{j=1}^{t} e_{gj} \right) + \phi_t - C(e_{st}, e_{gt}) \right] \text{ for } Q \in \{1, 2, .., T\} \end{aligned}$$

The optimal effort levels before quitting are

$$
\begin{aligned}
e_{si}^* &= m\sum_{j=i}^{Q-1}\beta^{j-i}\alpha_j \\
e_{gi}^* &= a\sum_{j=i}^{Q-1}\beta^{j-i}\alpha_j + a\sum_{j=Q}^{T}\beta^{j-i}
\end{aligned} \tag{31}
$$

and the optimal effort levels after quitting are

$$
\begin{aligned}
e_{si}^* &= 0 \\
e_{gi}^* &= a\sum_{j=i}^{T}\beta^{j-i}
\end{aligned} \tag{32}
$$

We prove in the appendix that the worker either stays for the entire horizon or quits in period 1: $EU_{Quit(Q)} \leq \max\left\{EU_{Stay}, EU_{Quit(1)}\right\}$ for any $Q \in \left\{2,3,..,T\right\}$

A sketch of this proof is as follows: the optimal effort levels insure that $EU_{Quit(2)} \leq \max\left\{EU_{Stay}, EU_{Quit(1)}\right\}$ and $EU_{Quit(3)} \leq \max\left\{EU_{Stay}, EU_{Quit(2)}\right\}$, which implies that $EU_{Quit(3)} \leq \max\left\{EU_{Stay}, EU_{Quit(1)}\right\}$. By induction, the result must be true for any period Q. Therefore, the worker will either leave in period 1 or stay until the end.

Because of this result, we can develop a two period model, which greatly simplifies the derivations at little cost to intuition or generality. The worker either quits in period one, or stays through period 2. The overall participation constraint for the worker is:

$$
\begin{aligned}
EU_{Stay} &= G_0\sum_{t=1}^{2}\beta^{t-1}\left(1-\delta\right)^t\alpha_t + \tfrac{1}{2}\left(a^2+m^2\right)\left[\sum_{t=1}^{2}\sum_{i=t}^{2}\sum_{j=t}^{2}\beta^{i+j-t-1}\left(1-\delta\right)^{i+j-2t}\alpha_i\alpha_j\right] > \\
EU_{Quit} &= \phi\sum_{t=1}^{2}\beta^{t-1} + G_0\sum_{t=1}^{2}\beta^{t-1}\left(1-\delta\right)^t + \tfrac{1}{2}a^2\left[\sum_{t=1}^{2}\sum_{i=t}^{2}\sum_{j=t}^{2}\beta^{i+j-t-1}\left(1-\delta\right)^{i+j-2t}\right]
\end{aligned} \tag{33}
$$

A profitable wage policy exists if $EU_{Quit} < EU_{Stay}$ for $\alpha_1 = \alpha_2 = 1$, in which case the worker will stay in both periods. This condition is equivalent to

$$\phi < \tfrac{1}{2} m^2 \left[\frac{1+\beta+2\beta(1-\delta)+\beta^2(1-\delta)^2}{1+\beta} \right] \tag{34}$$

Notice that this condition is less restrictive (and thus the firm is more likely to have a profitable wage policy) than in the myopic model whenever the future matters: that is whenever $\beta > 0$ and $\delta < 1$.

If Equation 34 is satisfied, then a profitable wage policy exists. However, as in the one-period model, the participation constraint may or may not bind: we will need to check whether or not it binds in both period 1 and period 2. We begin by solving the firm's profit maximization problem subject to the period 1 participation constraint, assuming that the period 2 participation constraint is slack.

$$\max_{\{\alpha.\}} \sum_{t=1}^{2} \beta^{t-1}\left(1-\alpha_t\right)\left[\left(G_0\right)\left(1-\delta\right)^t + m\sum_{j=1}^{t} e_{sj}\left(1-\delta\right)^{t-j} + a\sum_{j=1}^{t} e_{gj}\left(1-\delta\right)^{t-j}\right]$$
$$\text{s.t.}\quad EU_{Stay} \geq EU_{Quit}$$

The Lagrangian of this optimization problem is

$$\begin{aligned} L = {} & G_0 \sum_{t=1}^{2} \beta^{t-1}\left(1-\delta\right)^t\left(1-\alpha_t\right) + \left(a^2+m^2\right)\left[\sum_{t=1}^{2}\sum_{i=t}^{2}\sum_{j=t}^{2} \beta^{i+j-t-1}\left(1-\delta\right)^{i+j-2t}\left(1-\alpha_i\right)\alpha_j\right] \\ & + \lambda\left\{ G_0 \sum_{t=1}^{2} \beta^{t-1}\left(1-\delta\right)^t \alpha_t + \tfrac{1}{2}\left(a^2+m^2\right)\left[\sum_{t=1}^{2}\sum_{i=t}^{2}\sum_{j=t}^{2} \beta^{i+j-t-1}\left(1-\delta\right)^{i+j-2t} \alpha_i\alpha_j\right] - EU_{Quit} \right\} \end{aligned}$$

where $\lambda \geq 0$.

The first order conditions with respect to the wage policy in each period are

$$\sum_{t=1}^{k}\sum_{i=t}^{2}\beta^{i-t}\left(1-\delta\right)^{i-2t+1}\left(\tfrac{1}{2-\lambda}-\alpha_i\right)=\tfrac{1-\lambda}{2-\lambda}\tfrac{\left(1-\delta\right)G_0}{a^2+m^2}\text{ for } k=1,2$$

This is a linear system of 2 equations with 2 unknowns. It can be verified directly that the unique solution is

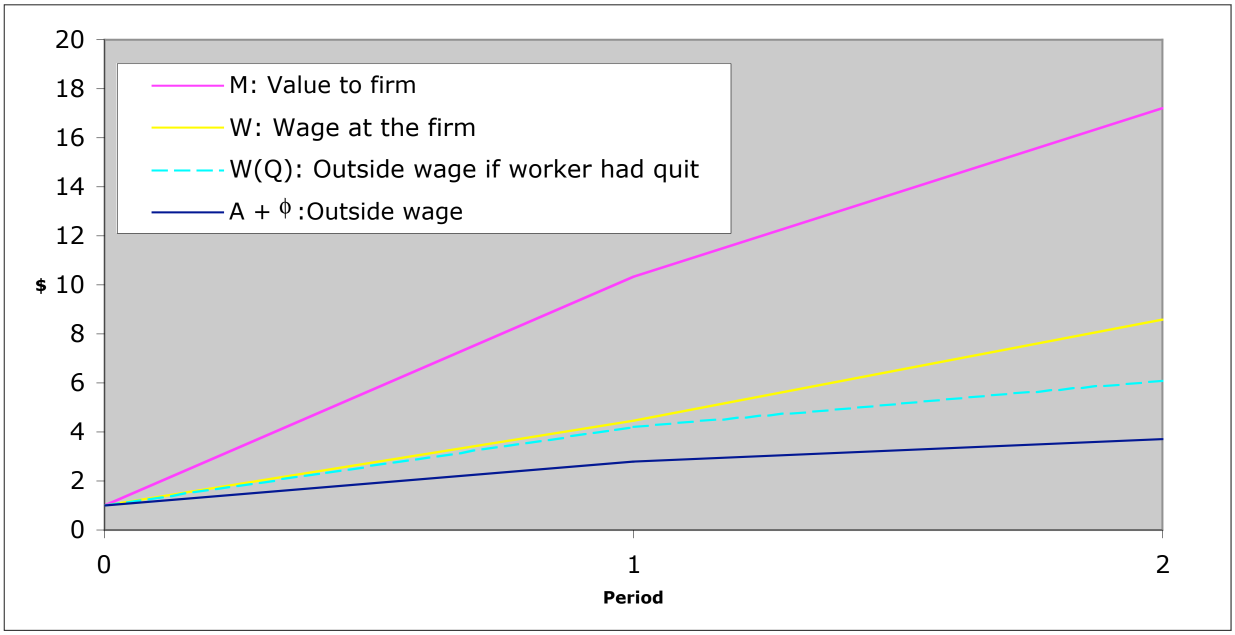
$$\alpha_1=\frac{1}{2-\lambda}-\frac{1-\lambda}{2-\lambda}\frac{\left(1-\delta\right)G_0}{a^2+m^2}$$

$$\alpha_2=\frac{1}{2-\lambda}$$

where $0 \le \lambda < 1$. (When $\lambda \ge 1$, there is no profitable wage policy.)

Notice that $\alpha_2 > \alpha_1$; the firm raises the employee's share of her output after the first period. This assures that the second period participation constraint does not bind, and demonstrates the dynamic inconsistency of the full commitment model. If the firm had the chance, it would lower α_2 and "take back" some of the human capital investment that the worker made in period 1.

This dynamic inconsistency is also demonstrated in the figure below. This figure shows four trajectories: the value of the worker to the firm over time, the worker's wage over time, her wage (equal to her value) if she had quit and invested only in general human capital, and her outside wage (that is, the value of her current level of general human capital). Comparison of the lowest line (the value of the worker's general human capital) with the next-to-lowest line (the value of her general human capital if she had invested only in GHC) demonstrates the risk to the employee of specific human capital investments. When an employee invests in firm-specific human at the expense of general human capital, she lowers her outside alternative and makes herself vulnerable to the firm.



The proof that the full commitment wage policy is not dynamically consistent is provided in the appendix.

Section 5: Conclusion

In this paper, we explore the implications of two facts: investments in firm-specific human capital are often non-contractible, and the firm uses its wage policy to induce employees to make such investments. We show that the firm faces a simple trade-off when designing a wage policy in such a world: it wants to motivate employees to invest in FSHC, but it also wants to capture a large fraction of the value produced by the workers' firm-specific human capital.

In a one-period model with stochastic outside offers, we show that employees often face a non-convex optimization problem: the optimal FSHC investment level is either zero or large.

This comes about because a small investment in FSHC only raises the probability that an employee will stay with the firm slightly, and so does not justify the costly investment in human capital that is only valuable if the employee stays.

There are three wage policy regimes in this one-period model. In the first, the firm has no way to make profits, because no wage policy will induce employees to invest in positive level of FSHC and allow the firm to make profits. In this regime, the firm would have to pay the employee more than the total value of her output to get her to be willing to invest in FSHC. In the second regime, the firm sets a wage policy that makes the employee just indifferent between investing in positive levels of FSHC, and investing in no FSHC. In this regime, the firm is fragile, since small changes in the environment can tip the employees towards defecting, and making investments only in general human capital. In the third regime, the firm adopts a wage policy that gives the employee a utility level that is above what she would achieve if she invested in no FSHC, and so employees will not defect in the face of small changes in the environment.

While the results from this one-period model are informative, the model is unable to explore any of the issues that arise from the fact that investments in human capital are long-lived, and the evolution of these stocks of human capital affect the optimal wage policy. We therefore model the wage policy of a firm with perfect foresight, which is able to make binding commitments to the future path of its wage policy. We show that this wage policy generates higher levels of investment in human capital than in the myopic model, but this full-commitment wage policy is not dynamically consistent. The firm would like to "take back" the early investments that the worker has made, by reducing the fraction of her output that the worker is able to keep.

These models are just a start, but they do highlight some of the issues prevalent in firms trying to build and profit from investments in firm-specific human capital.

Appendix 1: Derivation of the first order conditions with stochastic outside offer

$$\max_{e_s \geq 0, e_g \geq 0} EU = G + ae_g + E\left(\phi\right) + \int_0^x F\left(\phi\right) d\phi - C\left(e_s, e_g\right) \tag{A.1}$$

By Lagrange's formula for limits of integration

$$\frac{\partial \int_0^x F\left(\phi\right) d\phi}{\partial x} = F\left(x\right) \tag{A.2}$$

The first order condition with respect to investment in FSHC is

$$\begin{aligned} &EU_1\left(e_s, e_g\right) = 0 \\ &F\left(x\right)\tfrac{\partial x}{\partial e_s} - C_1\left(e_s, e_g\right) = 0 \\ &C_1\left(e_s, e_g\right) = F\left(x\right)\tfrac{\partial x}{\partial e_s} \\ &C_1\left(e_s, e_g\right) = \alpha m F\left(x\right) \end{aligned} \tag{A.3}$$

The first order condition with respect to investments in GHC is

$$\begin{aligned} &EU_2\left(e_s, e_g\right) = 0 \\ &a + F(x)\tfrac{\partial x}{\partial e_g} - C_2\left(e_s, e_g\right) = 0 \\ &C_2\left(e_s, e_g\right) = a + F\left(x\right)\tfrac{\partial x}{\partial e_s} \\ &C_2\left(e_s, e_g\right) = \alpha a F\left(x\right) + a\left[1 - F\left(x\right)\right] \end{aligned} \tag{A.4}$$

Appendix 2: Proof that γ = 0 in the stochastic model

First, we make the technical assumption that $aC_{12} < mC_{22}$. Otherwise, the substitution between investments is so strong that one never gets positive investments in both FSHC and GHC. Later, we discuss what happens when C_{12} is so large that it forces $e_g^* = 0$.

Proof by contradiction. Suppose that the wealth constraint $\gamma \geq 0$ is slack at the optimal solution. This implies we could drop the wealth constraint without affecting the solution.

$$\max_{\alpha,\gamma} \ \Pi = F(x)(M-W) = F(x)(me_s^* - x)$$

$$\text{s.t. } (e_s^*, e_g^*) = \underset{e_s \geq 0, e_g \geq 0}{\arg\max}\, ae_g + \int_0^x F(\phi)\, d\phi - C(e_s, e_g)$$

$$ae_g^* + \int_0^x F(\phi)\, d\phi - C(e_s^*, e_g^*) \geq ae_g^0 - C(0, e_g^0)$$

The participation constraint must be binding at the optimal solution. Otherwise we could consider a perturbation of this solution by changing α and γ such that x stays constant.

Using the FOC of the worker's problem,

$$C_{11}\tfrac{\partial e_s^*}{\partial \alpha} + C_{12}\tfrac{\partial e_g^*}{\partial \alpha} = mF(x)$$

$$C_{21}\tfrac{\partial e_s^*}{\partial \alpha} + C_{22}\tfrac{\partial e_g^*}{\partial \alpha} = aF(x)$$

which implies that $\frac{\partial e_s^*}{\partial \alpha} = \frac{mC_{22} - aC_{12}}{C_{22}C_{11} - C_{12}^2} F(x)$.

An optimal solution satisfies the firm's FOC:

$$\tfrac{\partial \Pi}{\partial \alpha} = F(x)m\tfrac{\partial e_s^*}{\partial \alpha} = 0 \rightarrow \tfrac{\partial e_s^*}{\partial \alpha} = 0 \rightarrow mC_{22} = aC_{12}$$

This never happens because $aC_{12} < mC_{22}$. So the participation constraint must be binding.

Consider a perturbation of α and γ such that the participation constraint is binding. Take the derivative of the participation constraint with respect to α:

$$a\frac{\partial e_g^*}{\partial \alpha} + F(x)\frac{\partial x}{\partial \alpha} - \left(C_1\frac{\partial e_s^*}{\partial \alpha} + C_2\frac{\partial e_g^*}{\partial \alpha}\right) = 0$$

Substitute the FOC of the worker's problem to derive

$$\frac{\partial x}{\partial \alpha} = \alpha m\frac{\partial e_s^*}{\partial \alpha} - a(1-\alpha)\frac{\partial e_g^*}{\partial \alpha}$$

By definition, $x = \alpha M + \gamma - \left(ae_g^* + G\right)$. Take the derivative w.r.t. α

$$\frac{\partial x}{\partial \alpha} = M + \alpha m\frac{\partial e_s^*}{\partial \alpha} - a(1-\alpha)\frac{\partial e_g^*}{\partial \alpha} + \frac{\partial \gamma}{\partial \alpha}$$

Combining the previous two equations, it follows that $\frac{\partial \gamma}{\partial \alpha} + M = 0$ when performing a perturbation such that the PC remains binding.

The firm's marginal change in profit is

$$\frac{\partial \Pi}{\partial \alpha} = \left[(1-\alpha)\frac{\partial M}{\partial \alpha} - M - \frac{\partial \gamma}{\partial \alpha}\right]F(x) + (M - W)\frac{\partial F(x)}{\partial a}$$

Using the result that $\frac{\partial \gamma}{\partial \alpha} + M = 0$,

$$\frac{\partial \Pi}{\partial \alpha} = (1-\alpha)\frac{\partial M}{\partial \alpha}F(x) + (M - W)\frac{\partial F(x)}{\partial \alpha}$$

Using the worker's FOC:

$$C_{11}\frac{\partial e_s^*}{\partial\alpha}+C_{12}\frac{\partial e_g^*}{\partial\alpha}=mF(x)+\alpha mf(x)\frac{\partial x}{\partial\alpha}$$
$$C_{21}\frac{\partial e_s^*}{\partial\alpha}+C_{22}\frac{\partial e_g^*}{\partial\alpha}=aF(x)-(1-\alpha)af(x)\frac{\partial x}{\partial\alpha}$$

$$\frac{\partial e_s^*}{\partial\alpha}=\frac{m[-EU_{22}]-a[-EU_{12}]}{EU_{11}EU_{22}-EU_{12}^2}F(x)=\frac{mC_{22}-aC_{12}-ma^2(1-\alpha)f(x)}{EU_{11}EU_{22}-EU_{12}^2}F(x)$$
$$\frac{\partial e_g^*}{\partial\alpha}=\frac{a[-EU_{11}]-m[-EU_{12}]}{EU_{11}EU_{22}-EU_{12}^2}F(x)=\frac{aC_{11}-mC_{12}-am^2\alpha f(x)}{EU_{11}EU_{22}-EU_{12}^2}F(x)$$

$$\frac{\partial M}{\partial\alpha}=m\frac{\partial e_s^*}{\partial\alpha}+a\frac{\partial e_g^*}{\partial\alpha}=\frac{\left(m\sqrt{-EU_{22}}+a\sqrt{-EU_{11}}\right)^2+2am\left(\sqrt{EU_{11}EU_{22}}+EU_{12}\right)}{EU_{11}EU_{22}-EU_{12}^2}F(x)>0$$

$$\frac{\partial x}{\partial\alpha}=\alpha m\frac{\partial e_s^*}{\partial\alpha}-a(1-\alpha)\frac{\partial e_g^*}{\partial\alpha}=\frac{\alpha\left(m^2C_{22}+a^2C_{11}-2amC_{12}\right)-\left(a^2C_{11}-amC_{12}\right)}{EU_{11}EU_{22}-EU_{12}^2}F(x)$$

The following results obtain:

- $\frac{\partial M}{\partial\alpha}>0$ for any alpha
- $\frac{\partial x}{\partial\alpha}<0$ on the interval $\alpha\in\left(0,\frac{a^2C_{11}-amC_{12}}{m^2C_{22}+a^2C_{11}-2amC_{12}}\right)$
- $\frac{\partial x}{\partial\alpha}>0$ for $\alpha\in\left(\frac{a^2C_{11}-amC_{12}}{m^2C_{22}+a^2C_{11}-2amC_{12}},\infty\right)$.

Furthermore, $aC_{12}<mC_{22}\leftrightarrow\frac{a^2C_{11}-amC_{12}}{m^2C_{22}+a^2C_{11}-2amC_{12}}<1$, so $\frac{\partial x}{\partial\alpha}$ is positive at $\alpha=1$: $\left.\frac{\partial x}{\partial\alpha}\right|_{\alpha=1}>0$.

The marginal change in profits at $\alpha=1$ is

$$\left.\frac{\partial\Pi}{\partial\alpha}\right|_{\alpha=1}=(M-W)f(x)\left.\frac{\partial x}{\partial\alpha}\right|_{\alpha=1}>0$$

The profits are maximized by setting $\alpha > 1$. Even if there existed an $\alpha < 1$ for which $\frac{\partial \Pi}{\partial \alpha} = 0$, this would be a local minimum. Since $\alpha^* > 1$, the firm must set $\gamma^* < 0$ to achieve positive profits.

How does this solution relate to "selling the firm to the worker"? If the probability of staying were insensitive to α at the optimal solution (i.e. $\frac{\partial F(x)}{\partial \alpha} = 0$), we would have

$$\frac{\partial \Pi}{\partial \alpha} = (1-\alpha) \frac{\partial M}{\partial \alpha} F(x)$$

This would immediately lead to $\alpha = 1$. So the result $\alpha^* > 1$ is a consequence of the sensitivity of $F(x)$ with respect to α.

Setting $\gamma^* < 0$ contradicts the wealth constraint $\gamma \geq 0$. Therefore, the wealth constraint forces the firm to set $\gamma = 0$.

If C_{12} is so large that $e_g^* = 0$, we still obtain the results that $\gamma = 0$.

The participation constraint must be binding. If not, consider a perturbation of the solution by changing α and γ such that x stays constant. Using the FOC of the worker's problem,

$$C_{11} \frac{\partial e_s^*}{\partial \alpha} = mF(x) \rightarrow \frac{\partial e_s^*}{\partial \alpha} = \frac{mF(x)}{C_{11}} > 0$$

This means we could improve the chosen solution by increasing α and lowering γ such that x stays constant. Thus, the participation constraint is binding.

Next, consider a perturbation of α and γ for which the PC is binding. Take the derivative of the PC with respect to α:

$$\frac{\partial x}{\partial \alpha} = \alpha m \frac{\partial e_s^*}{\partial \alpha}$$

Take the derivative of $x = \alpha M + \gamma - \left(a e_g^* + G\right)$ with respect to α:

$$\frac{\partial x}{\partial \alpha} = \alpha m \frac{\partial e_s^*}{\partial \alpha} + M + \frac{\partial \gamma}{\partial \alpha}$$

Combining these two equations, $M + \frac{\partial \gamma}{\partial \alpha} = 0$

The first order condition for the firm's profits is then

$$\frac{\partial \Pi}{\partial \alpha} = \left(1 - \alpha\right) \frac{\partial M}{\partial \alpha} F\left(x\right) + \left(M - W\right) f\left(x\right) \frac{\partial x}{\partial \alpha} = 0$$

Consider again the worker's first order condition, which implies:

$$\frac{\partial e_s^*}{\partial \alpha} = \frac{mF\left(x\right)}{C_{11} - \alpha^2 m^2 f\left(x\right)} = \frac{mF\left(x\right)}{-EU_{11}} > 0$$

Note that $\frac{\partial M}{\partial \alpha} = m \frac{\partial e_s^*}{\partial \alpha} > 0$, $\frac{\partial x}{\partial \alpha} = \alpha m \frac{\partial e_s^*}{\partial \alpha} > 0$, and $M - W > 0$ for positive profits. Therefore the firm must set $\alpha^* > 1$ at the optimum, which forces $\gamma^* < 0$ for positive profits. But this again violates the wealth constraint.

Appendix 3: Parameters for which no wage policy exists

A profitable wage policies exists if and only if the worker's participation constraint is satisfied for $\alpha = 1$. If $\alpha = 1$, then it also follows that $x = me_s$.

$$EU\left(e_s^* > 0, e_g^*\right) = G + ae_g^* + E\left(\phi\right) + \int_0^{me_s^*} F\left(\phi\right) d\phi - C\left(e_s^*, e_g^*\right)$$

$$EU\left(0, e_g^0\right) = G + ae_g^0 + E\left(\phi\right) - C\left(0, e_g^0\right)$$

If $m = 0$, no profitable wage policy exists because

$$EU\left(e_s^* > 0, e_g^*\right) = G + ae_g^* + E\left(\phi\right) - C\left(e_s^*, e_g^*\right) < EU\left(0, e_g^*\right) \leq EU\left(0, e_g^0\right)$$

Set $e_s = \overline{e}_s > 0$ and $e_g = \overline{e}_g$ to some fixed levels, such that $EU\left(e_s^* > 0, e_g^*\right) \geq EU\left(\overline{e}_s, \overline{e}_g\right)$.

Take the limit for $m \to \infty$ to obtain $\lim_{m \to \infty} EU\left(e_s^* > 0, e_g^*\right) \geq \lim_{m \to \infty} EU\left(\overline{e}_s, \overline{e}_g\right) = \infty$.

Therefore, a profitable wage policy does exist if *m* is large enough.

Furthermore, when an interior solution exists, the worker's optimal expected utility is increasing in *m* by the Envelope Theorem:

$$\frac{dEU\left(e_s^*, e_g^*; m\right)}{dm} = \frac{\partial EU\left(e_s^*, e_g^*; m\right)}{\partial m} = e_s^* F\left(me_s^*\right) > 0$$

To summarize, a profitable wage policy exists if and only if m is above a certain threshold.

Next, we discuss how the distribution of outside offers affects the existence of a profitable wage policy. Suppose the cost function is separable $C(e_s, e_g) = c(e_s) + c(e_g)$. For $\alpha = 1$, the separability of the cost function ensures that $e_g^* = e_g^0$. The participation constraint is then satisfied only if

$$\int_0^{me_s^*} F(\phi)\, d\phi \geq c(e_s^*)$$

Result. Consider the distribution $F_{indifference}(x) = \frac{1}{m} c'\left(\frac{x}{m}\right)$ for $x \in \left(0, mc'^{-1}(m)\right)$. If $F(x) < F_{indifference}(x)$ on the range $x \in \left(0, mc'^{-1}(m)\right)$ (first-order stochastic dominance), no profitable wage policy exists. If $F(x) > F_{indifference}(x)$ on the range $x \in (0, \varepsilon)$ for any $\varepsilon > 0$ then a profitable wage policy exists.

Proof: If $F(x) < F_{indifference}(x)$ on the range $x \in \left(0, mc'^{-1}(m)\right)$, then

$$EU_1(e_s, e_g) = mF(x) - c'\left(\tfrac{x}{m}\right) < mF_{indifference}(x) - c'\left(\tfrac{x}{m}\right) = 0$$

The worker's utility is monotonically decreasing in FSHC investments for any $x < mc'^{-1}(m) \leftrightarrow c'(e_s) < m$. If an interior solution existed, the first order condition $c'(e_s) = mF(x)$ should be satisfied, but the expected utility is decreasing in e_s on this range. No profitable wage policy exists.

If $F(x) < F_{indifference}(x)$ on the range $x \in (0, \varepsilon)$, then

$$EU_1(e_s, e_g) = mF(x) - c'\left(\tfrac{x}{m}\right) > mF_{indifference}(x) - c'\left(\tfrac{x}{m}\right) = 0$$

Take the limit for $e_s \to 0$: $\lim_{e_s \to 0} EU_1\left(e_s, e_g\right) > 0$. Therefore, $e_s = 0$ cannot be the optimal solution. Since the worker stays when $\alpha = 1$, a profitable wage policy exists.

To summarize, no profitable wage policy exists when $\Pr\left(\phi \le x\right) = F\left(x\right) < F_{indifference}\left(x\right)$ (i.e. the outside offers are large), but a profitable wage policy exists when $\Pr\left(\phi \le x\right) = F\left(x\right) > F_{indifference}\left(x\right)$ for small x (i.e. there is a sufficient likelihood that the outside offers will be very small.)

There is another proof that no wage policy exists when *m* is low and the outside offers are large. Suppose the distribution of outside offers is concentrated at a single point $\bar{\phi}$: $F\left(x\right) = 1$ for $x \ge \bar{\phi}$ and $F\left(x\right) = 0$ for $x < \bar{\phi}$ and that, furthermore, the cost function is separable. The first order condition of the worker's maximization problem is $c'\left(e_s^*\right) = m$, which implies that $e_s^* = c'^{-1}\left(m\right)$. A profitable wage policy exists if and only if

$$mc'^{-1}\left(m\right) - c\left(c'^{-1}\left(m\right)\right) \ge \bar{\phi}$$

In particular, if $c\left(e_s\right) = \frac{1}{2} e_s^2$, a profitable wage policy exists if and only if $\frac{1}{2} m^2 \ge \bar{\phi}$. This occurs when m is low and the outside offers $\bar{\phi}$ are high.

Appendix 4: The participation constraint may be slack

Suppose the distribution of outside offers is concentrated at a single point $\bar{\phi}$: $F(x)=1$ for $x \geq \bar{\phi}$ and $F(x)=0$ for $x < \bar{\phi}$ and that, furthermore, the cost function is of the form $C(e_s, e_g) = \frac{1}{2}e_s^2 + \frac{1}{2}e_g^2$.

Using the first order conditions at an interior solution are $e_s^* = \alpha m$ and $e_g^* = \alpha a$,

$$\max_{\alpha} \Pi = (1-\alpha)\left[\left(a^2+m^2\right)\alpha + G\right]$$
$$\text{s.t. } \tfrac{1}{2}\left(a^2+m^2\right)\alpha^2 + \alpha G \geq \tfrac{1}{2}a^2 + G + \bar{\phi}$$

The participation constraint is equivalent to $\alpha \geq \frac{\sqrt{G^2+2\left(a^2+m^2\right)\left(\frac{1}{2}a^2+G+\bar{\phi}\right)}-G}{a^2+m^2}$

Ignoring the participation constraint, the firm would like to set the wage policy:

$$\alpha^* = \tfrac{1}{2} - \tfrac{1}{2}\tfrac{G}{a^2+m^2}$$

The participation constraint is slack at the optimal solution if

$$\frac{\sqrt{G^2+2\left(a^2+m^2\right)\left(\frac{1}{2}a^2+G+\bar{\phi}\right)}-G}{a^2+m^2} < \tfrac{1}{2} - \tfrac{1}{2}\tfrac{G}{a^2+m^2} \leftrightarrow$$
$$\bar{\phi} < \tfrac{a^2+m^2}{8} - \tfrac{1}{2}a^2 - \tfrac{3}{4}G - \tfrac{3}{8}\tfrac{G^2}{a^2+m^2}$$

The participation constraint must be binding at the optimal solution if

$$\bar{\phi} \in \left[\tfrac{a^2+m^2}{8} - \tfrac{1}{2}a^2 - \tfrac{3}{4}G - \tfrac{3}{8}\tfrac{G^2}{a^2+m^2}, \tfrac{1}{2}m^2\right]$$

Notice that $\lim_{m\to\infty}\left(\frac{a^2+m^2}{8}-\frac{1}{2}a^2-\frac{3}{4}G-\frac{3}{8}\frac{G^2}{a^2+m^2}\right)=\infty$. Therefore the participation constraint is slack when $\bar{\phi}$ is small or when m is large.

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