Time to Decide: Information Search and Revelation in Groups[*]

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Abstract

We analyze costly information acquisition and information revelation in groups in a dynamic setting. Even when group members have perfectly aligned interests the group may inefficiently delay decisions. When deadlines are far away, uninformed group members freeride on each others' efforts to acquire information. When deadlines draw close, informed group members stop revealing their information in an attempt to incentivize other group members to continue searching for information. Surprisingly, setting a tighter deadline may increase the expected decision time and increase the expected accuracy of the decision in the unique equilibrium. As long as the deadline is set optimally, welfare is higher when information is only privately observable to the agent who obtained information rather than to the entire group.

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1 Introduction

This paper studies decision making in groups in which the members must be motivated to acquire *and* share information in a timely manner prior to making a decision. These situations are very common. For example, in order to evaluate which corporate strategy to pursue individual members of an executive board must gather and share information about a project's profitability and likelihood of success with their fellow board members. Failure to reach an early decision may involve significant delay costs such as the loss of profit from delayed product introductions. In trial juries it is important that jurors pay attention to the evidence and bring this information to the attention of the other jurors in order to allow the jury to make an informed and timely judgement. Parents must investigate the quality of different schools and inform their partner of their findings before deciding where to send their offspring.

Teams thus face two important challenges in their decision making process, even in the absence of conflicting preferences between team members. First, team members must be willing to invest time and costly effort to search for information, such as analyzing market forecasts, evaluating judicial evidence and reading school quality assessment reports. Since these cost are borne privately, but the acquired information is a public good, this leads to a standard free-riding problem. Second, team members should share the acquired information efficiently and in a timely manner. However, the desire to maintain the motivation of other committee members to search for information can lead some members to stay silent about their own discoveries and to delay decisions. When members who previously succeeded in finding information about possible courses of action fail to communicate their information, the committee may unduly delay its decision or even make the wrong choices due to missing information. Interestingly, the reluctance to share information in a timely manner may then in turn undermine the group members' incentives to search for information.

While an extensive literature on group decision making has studied in isolation either the distorted incentives to gather or the distorted incentives to reveal private information, this paper jointly analyzes these two decisions in a dynamic framework. Team members can decide whether and when to exert effort to gain information *and* whether and when to communicate their information to the rest of the group. We show that these two considerations are in conflict and lead to a trade-off between incentives for private information gathering and intra-committee communication. This trade-off is critical to the understanding of why groups often fail to make decisions in a timely manner and of whether common practices like deadlines and public disclosure actually help groups achieving their goals. First, in their seminal research on group decision making Stasser and Titus (1985) and Stasser (1999) show that groups do not share information effectively when members possess private information. When members of a group have different pieces of information, people tend to discuss the information that they all posses in common, and they do not always share or emphasize the information privately held by each group member. The lack of proper information sharing and integration inhibits group problem-solving effectiveness. In our model, the reluctance to share information is a result of each agent's desire to maintain the motivation of other team members. Second, management scholars have long stressed that while group decision making tends

to lead to more information and knowledge being available when decisions are made, the decision making process often takes longer and is costlier than individual decision making. Delay and indecision are common in top management teams and can have serious consequences including missed market opportunities (Eisenhardt 1989). In his popular textbook on management practices Griffin (2006, page 250) notes that "perhaps the biggest drawback from group and decision making is the additional time and hence the greater expense entailed. [...] Assuming the group or team decision is better, the additional expense may be justified." In our model group decision making also results in excess delay which is a result of both a lack of information search and a lack of information disclosure, but groups also acquire more information and, perhaps paradoxically, may even overacquire information in expectation despite the presence of a free-riding problem. Third, to help promote the effectiveness of group and team decision making Griffin (2006) advocates the careful use of deadlines: "Time and cost can be managed by setting a deadline by which the decision must be made final." Our formal analysis of the interaction between incentives for information acquisition and information sharing shows how standard team practices to incentivize group members, like the imposition of deadlines and disclosure rules, though often beneficial, can also backfire when used incorrectly. While deadlines are expected to increase the cost of freeriding, the resulting increase in search efforts reduces the incentives to reveal information when this information discourages group members from searching intensively. This view is in line with Carrison's (2003, page 122) case study analysis of how organizations manage to meet critical time challenges. He argues that "whenever the workplace is charged with the electricity of a race against time, clear communication can suffer." However, we show that disclosure rules aimed at improving communication may reduce the incentives to become informed.

We consider a continuous-time model where each team member of a group of two can choose to gather information and call a decision before a finite time horizon T. Since the group members equally share the benefits of a more accurate decision, the costly acquisition of information becomes a public good and members of the group will attempt to free-ride on the information acquisition efforts of other members. Team members have also strong incentives not to communicate what they know before a decision is called. With decreasing returns to information revealing one's own information reduces the incentives for the other team member to search for more information. Hence, an informed team member would like to pretend she is not informed, in particular, when she expects the other team member to search intensively for information. Of course, in equilibrium, team members will anticipate such behavior and accordingly update their belief that their fellow group member is informed. A higher belief that another team member is informed reduces the incentive to search for additional information, since this additional piece will contribute less to the decision. A higher belief also reduces the incentive to delay a decision, as more informed team members search less intensively for information.

We provide a characterization of the symmetric equilibrium strategies employed by the members of the team. Most notably, the behavior of the players is significantly affected by the length of the time that is still available until the final deadline at time T. When T is relatively small, that is to

3

say there is little time between the start of the game and the deadline to become informed, players exert maximum effort to become informed and reveal no information when successful. Players have little time to become informed and thus have strong incentives from the start of the game to exert effort themselves rather than to count on their partners' effort. In response to this high effort choice any informed agent prefers not to disclose her information and to delay a decision since she benefits from the potential acquisition of an additional signal by a hard-working uninformed agent. However, when is T is large, this equilibrium is no longer sustainable. The belief of uninformed agents that the other team member is informed would be too high and an uninformed agent would no longer be willing to exert such a high level of effort. As a result, uninformed agents choose lower effort levels to compensate for the fact that the other team member is likely to be informed and just delaying the decision. For even larger T, informed agents may no longer prefer to delay their decision since the cost of delaying a decision until the deadline increases. As a result, an informed agent will initially prefer to forego any delay costs and instead choose to immediately call a decision upon acquiring a signal. Uninformed agents initially free-ride on each others' efforts to acquire this signal. The unique equilibrium for long deadlines thus has two phases: a first phase of low effort intensity and full information disclosure, a second phase of high effort intensity and no information disclosure.

The unique symmetric equilibrium of the game suggests that inefficient delay is due to the lack of information search far from the deadline and the lack of information revelation close to the deadline. As no information is revealed close to the deadline, committees are expected to take decisions early on or to wait until the deadline. Relatedly, our model suggests an explanation for why committees may delay decisions without its members actually looking for more information. That is to say, when deadlines are very strict our model provides a formal characterization of "Parkinson's Law" (Parkinson 1955, 1958) which posits that "work fills the time available" as all decisions will be delayed until the final deadline. Tight deadlines are expected to reduce the decision time, but also to reduce the expected decision precision. However, the opposite may occur in this setting. The reason is that a longer deadline increases the probability that information is still acquired before team members stop revealing information and delay decisions until the deadline. Hence, this increases the probability of an early decision being taken with potentially less information in expectation. We show that there is a unique finite deadline that maximizes agents' ex-ante welfare by maximizing beneficial search efforts while avoiding unnecessary decision delays.

Our model also highlights the importance of observability of successful information acquisition. When information is immediately observable to all team members, free-riding on information acquisition will lead to a severe underprovision of search effort, but at least decisions will always be taken immediately once information has been obtained. In contrast, when information is only privately observable, team members have stronger incentives to search as they can reap informational rents from successful information acquisition. However, successful agents' attempts to benefit from another team member's information acquisition causes decisions to be taken unnecessarily late. We provide a clean comparison of the polar cases of private and public information. Surprisingly, as

long as the agents can optimally set the deadline ex ante, welfare is always higher when information is privately observable. We show that the strong search incentives and the lack of information disclosure may induce teams to overacquire information before taking a decision. Relatedly, when the returns to additional information are decreasing rapidly and the deadline is set inefficiently short, the team members may benefit from making information publicly observable.

Our model captures settings in which there are typically severe restrictions on the types of mechanisms and contracts that are available. We discuss the use of simple instruments and common features which are characteristics of some group-decision settings, like bonus payments, communication frictions and third-party information intermediaries such as a committee chairperson. We analyze how these differentially affect the incentives to search and reveal information. We also extend our baseline model to discuss some common features of group decision making processes, like communication frictions and committee chairpersons who act as information intermediaries. Finally, we show that our qualitative findings are robust to modifications in the information structure and how adding team members influences information acquisition and delay. It is important to note that while our analysis focuses on applications of information search and disclosure in groups, our findings naturally generalize to the analysis of the private provision of public goods in dynamic settings. Crucial ingredients are stochastic production, decreasing marginal returns to production and private information about agents' own level of production. For example, entrepreneurs engaged in a common venture exert effort to raise funds to implement their joint project. A partner may be unwilling to reveal that she already raised a sizable amount so as not to discourage her partners from raising even more funds. Our model relates the delayed implementation of projects to the lack of effort provision and the lack of information sharing.[1]

1.1 Related Literature

Our model is related to the large and growing literature on decision making in groups. Several previous contributions have focused on the distorted incentives to reveal private information in the presence of conflicting preferences (Li, Rosen and Suen 2001; Dessein 2007, Gerardi and Yariv 2007)[2], of reputation or career concerns (Ottaviani and Sorensen 2001; Levy 2007; Visser and Swank 2007) and of different voting rules (Feddersen and Pesendorfer 1998). In our model, preferences are perfectly aligned, conditional on the available information, and individuals strictly prefer to reveal their private information when a decision is taken. Another strand of the literature analyzes how incentives for individual information acquisition in committees can be optimally provided

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[1]Another example is that of an ongoing funding drive, where contributors to charitable causes may be unwilling to reveal that they have already made a sizeable gift so as not to discourage other potential donors, whose marginal value of contribution is now lower, from donating some of their own funds. Our model suggests that charitable giving can be increased when donors do not disclose their acts of charity. Also according to many religious and ethical traditions charity should not draw attention to the giver: "If ye disclose (acts of) charity even so it is well; but if ye conceal them and make them reach those (really) in need, that is best for you." (Holy Qur'an, Surah 11-271, Translation by Yusuf Ali).

[2]Another strand of literature studies how incentives for information acquisition arise if decision-makers have different preferences (Aghion and Tirole 1997) or beliefs (Che and Kartik 2009) and thus take different decisions conditional on holding the same information.

by structuring the decision procedure (Persico 2004) , the size of the committee (Mukhopadhaya 2003, Cai 2009), the voting rules (Li 2001, Gerardi and Yariv 2008, Lizzeri and Yariv 2011), or restricting the action space (Szalay 2005). Gershkov and Szentes (2009) are one notable exception in this literature as they also focus on influencing the prior information of the members acquiring information. They find that a social planner who accounts for members' efforts would leave any member as much in the dark as possible. Blanes-I-Vidal and Moeller (2011) also study the impact on team members' incentives from communicating private information, but their focus is on incentives to implement a common decision rather than on incentives to acquire information.

To our knowledge, the present paper is the first to study the interplay of incentives to acquire and to reveal information in a dynamic setting, thus closing the gap between the two above-mentioned strands of the literature on group decision making. Furthermore, in addition to having the dual instruments of information acquisition and disclosure available to them, in our model agents can also choose *when* to search for and to disclose information. In that respect, our modeling approach of the dynamic interaction is most closely related to Bonatti and Hörner (2011) who study effort incentives in teams in a continuous-time framework, but they abstract away from the incentive problems related to information sharing that are central to our analysis.

Finally, delay and optimal deadlines in group decision making are also the focus of Damiano, Li and Suen (2009, 2010) who study repeated voting games between team members with differing interests. Our model also sheds light on the literature on sequential public good provision (Admati and Perry 1991, Varian 1994, Teoh 1997) which studies voluntary contributions to a joint project over time, but does not address the timing of voluntary information disclosure of contributors regarding their own level of contributions.

The remainder of the paper is organized as follows. In Section 2 we introduce the model. In order to build intuition for our continuous-time results, we first focus on a simple two-period case in Section 3 which highlights the main driving forces of our analysis. We then present the continuous-time model and characterize and discuss the equilibrium strategies in Section 4. In Section 5, we derive several comparative statics results regarding welfare and expected decision time and decision precision as a function of the length of the game. In Section 6, we compare the equilibrium strategies and welfare when the acquired information is private and public. Finally, in Section 7 we discuss robustness and extensions. Section 8 concludes. All the proofs are in the Appendix.

2 Setup

A team of two agents is engaged in joint stochastic public good production. Both agents have identical preferences and can each exert costly private effort e to produce units of output of the public good that are to be consumed at the end of the game after production has ended. Each agent's effort increases the probability of producing an additional unit of output, but is unobservable to the other agent. This results in a standard moral hazard problem within the team; both agents would

like to freeride on each others' effort to produce units of output. We assume that while there are constant costs of effort, the returns to output production are decreasing. Denoting the *additional* value of the n-th unit of output when consumed by each agent at the end of the game by α_n, this implies $\alpha_n \geq \alpha_{n+1}$ for any n. Information about how many units of output have been produced by a single agent is only available to the agent who produced them. Each agent decides whether to conceal or disclose their information. We assume they cannot credibly reveal that they produced no output previously. Finally, agents may choose at any point in time whether or not to call an end to the production phase. Once an end to the production phase is called by either agent all the units of the public good produced by both agents are consumed and the game ends. As long as no end has been called, both agents incur an additive delay cost δ. This setting is sufficiently general to capture many forms of public good production such as the information acquisition in group decision making, initial funding of entrepreneurial ventures, contributions to charity, co-authorship or even more mundane tasks such as firewood collection.[3]

Our primary focus in this paper is on information acquisition and revelation in groups. We consider an application that generates the above mentioned reduced-form ingredients in which a team of two agents choosing a decision a to match a state of the world θ. Both agents have identical preferences regarding the decision to be taken for a given state of the world, but the state of the world is unknown. The decision utility is given by a quadratic loss function $-(\theta - a)^2$. By exerting costly private effort e an agent increases the probability of acquiring an additional private signal about the state of the world. We call an agent *informed* if she acquired a signal and *uninformed* if she did not obtain a signal before. A better informed decision reduces the expected quadratic utility loss. In particular, we assume that each agent starts with an identical normal prior $\theta \sim N\left(0, \frac{1}{\varepsilon}\right)$ with the precision denoted by ε. Agents can acquire additional signals s. Each signal is independent and normally distributed with precision τ, $s \sim N\left(\theta, \frac{1}{\tau}\right)$. The expected loss when taking a decision with n signals simplifies to $\frac{1}{\varepsilon + \tau n}$. Hence, the marginal return to the nth signal is $\alpha_n = \frac{\tau}{(\varepsilon + \tau n)(\varepsilon + \tau(n+1))}$. It is decreasing in n and is decreasing at a faster rate when each signal is more precise. By revealing information an agent reduces her partner's incentives to acquire more information as the expected value of the additional information is lower. We assume that agents cannot credibly reveal that they have no information. We also assume that agents may choose whether or not to call a decision. Calling a decision is irreversible so once it is called by either agent a decision must be made. Since the decision preferences are aligned, agents reveal any information they have when a decision is called before deciding on an action. The game ends after an action a is taken. As long as a decision is not called, agents incur an additive delay cost δ.

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[3]To fix ideas, consider the case of two entrepreneurs engaged in a common venture who are each trying to raise funding to implement their joint project. Investing effort in raising funds is privately costly to each entrepreneur as it requires convincing investors of the merits of the project and it may not necessarily result in fundraising success. Any funds raised contribute to the final value of the project when implemented, but they do so at a decreasing rate, and are only observable to the team member who obtained them. At any point in time each entrepreneur is free to communicate how much funding she has already raised and may decide to call an end to the fundraising process and force implementation of the project.

3 A Simple Model

We first consider a simple model to highlight some of the forces that govern the agents' decisions to search for information and to disclose information. In this simple two-period model, we assume that each agent starts the game with a signal with probability $1 - \phi$ and is uninformed with probability ϕ. The first agent moves in the first period, the second agent moves in the second period.

The first agent chooses the probability that a decision is called in the first period. We denote this decision probability by $d(\phi, n) : [0, 1] \times \{0, 1\} \to [0, 1]$ which depends on the probability ϕ that the second agent is uninformed and the number of signals n she has obtained. If a decision is called, both agents reveal their information and take the decision $a = E[\theta | \Omega_1]$ given all the information Ω_1 known in the first period. If no decision is called in the first period, both agents bear a cost δ from delaying the decision until the second period. However, delaying the decision to the second period allows the second agent to acquire more information.

The agent who moves in the second period, chooses effort according to the effort function $e(\hat{\phi}, n) : [0, 1] \times \{0, 1\} \to [0, e_{\max}]$ given her updated belief $\hat{\phi}$ that the other agent is uninformed. When exerting effort at cost ce, this agent obtains a signal with probability λe, where c and λ measure the marginal cost and return to effort. If the first agent has not called a decision in the first period, a decision is called in the second period after the effort choice of the second agent. Both agents reveal their information and take the decision $a = E[\theta | \Omega_2]$ given all the information Ω_2 available in the second period.

Incentives to exert effort When the second agent is uninformed, her marginal gain from exerting additional effort equals

$$\lambda \left[\hat{\phi} \alpha_1 + (1 - \hat{\phi}) \alpha_2 \right] - c,$$

where $\hat{\phi}$ is the second agent's belief that the first agent is uninformed given that she has not called a decision in the first period. The marginal return to effort depends on the increase in probability of obtaining an additional signal and the expected value of that signal. The marginal return is thus higher the more likely it is that the first agent is uninformed as this increases the expected value of an additional signal. Note also that the expected return to searching for information is at least as high for an uninformed agent as for an informed agent. We assume $\lambda \alpha_1 > c > \lambda \alpha_2$. Hence, an agent who is informed or knows that the first agent is informed will exert no effort.

Incentives to delay a decision When the first agent is informed, her gain from delaying a decision until the second period equals

$$\lambda e(\hat{\phi}, 0) \phi \alpha_2 - \delta.$$

By incurring a delay cost δ, the first agent allows the second agent to acquire an additional signal. The first agent anticipates that the second agent will exert effort only if she is uninformed and does not know that the other agent is informed. The gain from delaying a decision is higher the

higher the expected effort level exerted by the second agent. This depends on the probability ϕ that the other agent is uninformed and the effort level $e(\hat{\phi}, 0)$ exerted by the agent when uninformed. Clearly, an informed agent will never want to disclose her information when delaying a decision as this reduces the second agent's incentives to search for additional information. Note that the expected return to delaying a decision in the first period is at least as high for an uninformed agent as it is for an informed agent, since $\alpha_1 > \alpha_2$. We assume that δ is sufficiently small for an uniformed agent not to call a decision.

Equilibrium This simple model identifies a clear tension between the incentives to search for information and the incentives to disclose information. The more search effort an agent expects her partner to exert, the more willing she is to delay a decision, even when she is already informed. However, the more likely an agent expects her partner to be informed, the less willing she is to exert more search effort. For the relevant range of parameter values, we find that the unique equilibrium either involves an uninformed second player exerting maximum effort and an informed first player delaying decisions or an uninformed second player exerting less than maximum effort and an informed first player partly calling decisions. We formally characterize the equilibrium strategies in the appendix.

The private nature of the acquired information is essential. First, informed players use the option to conceal their information and delay decisions in equilibrium. When the cost of delay is relatively small, no equilibrium exists in which an informed player always calls a decision. If she did, the second player would believe that the first player is uninformed when given the chance to obtain a signal and thus would be willing to exert maximum effort. As a result, even when informed, the first player would delay the decision. Second, the option to conceal information affects the efficiency of the equilibrium. The private nature of information leads to inefficient delay by the first player when the second player is already informed and thus does not search for another signal. However, the fact that information is private also increases the value of being informed for the first player. An informed first player can induce the second player to acquire information because she is not required to disclose her own information. Hence, the value of being informed for the first player is higher when this information is private than when it is public. It may, however, be socially inefficient to delay the decision for an additional signal. In this case, the private nature of information leads to overacquisition of information.

4 A Continuous-Time Model

We now consider a continuous-time setup where t denotes the time of the game. This setup affords substantial tractability and allows a clean characterization of equilibrium strategies as a function of the deadline. The game ends at a (possibly arbitrarily large) finite horizon at time T or before if a decision has been called by an agent. Both agents start the game uninformed.

As long as a decision has not been called, each agent chooses how much effort to exert. Effort is

denoted by the function $e(t,n) : [0,T] \times N \rightarrow [0, e_{\max}]$ where n is the number of signals the agent has acquired up to time t. The effort function is piecewise continuous over time. An agent's effort level e determines the exponential rate λe at which an additional signal is acquired. The exponential arrival of signals is independent for both players, conditional on their respective efforts. The agent incurs a linear effort cost ce.

At each point in time, an agent chooses to call a decision or not, denoted by $\hat{d}(t,n) : [0,T] \times N \rightarrow \{not\ call, call\}$. Each agent incurs an additive delay cost δ as long as no decision has been called. When one agent calls a decision at time t, the two agents agree to take the decision $a = E[\theta|\Omega_t]$ given all the information Ω_t known at time t. At that point the quadratic loss $-(\theta - a)^2$ is realized and the game ends. When no decision has been called before the deadline is reached at time T, a decision is called with certainty.

The analysis in this paper focuses on equilibria in which uninformed agents search for information and informed agents call decisions. As in our analysis of the simple model, we assume that $c > \lambda\alpha_2$. Hence, the number of signals acquired by one agent will only ever be 0 or 1 in equilibrium.[4] We therefore drop the second argument of the effort function and write $e(t)$ for the effort strategy of the uninformed agent. We also assume that $[\lambda\alpha_1 - c]e_{\max} > \delta$ such that an uninformed agent acting alone would be prepared to search for information in order to make a decision. In Section 4.2 we also show that no symmetric equilibria involve uninformed agents calling a decision in equilibrium unless the equilibrium is one in which both types of agents call a decision with certainty at a point in time, which is in effect a deadline supported by off-equilibrium beliefs. To provide a concise description of the important elements of our model we will proceed by imposing $\hat{d}(t,0) = not\ call$ for all t. We therefore drop the second argument of the decision function and write $d(t)$ for the decision strategy of the informed agent.[5]

Common knowledge that a signal was received by either player would induce both players to call a decision. Although we make specific assumptions about the decision making process, analogous equilibria exist with voluntary verifiable disclosure of signals, cheap talk communication between agents as well as a decision protocol where both players have to agree on calling a decision. It is only essential that players cannot verifiably disclose that they do not hold any signal.

The probability that an agent does not acquire a signal by time t provided a decision has not yet been called by the other agent, equals

$$\sigma(t) = \exp\left(-\int_0^t \lambda e(s)\, ds\right)$$

We allow for mixing strategies regarding the decision to call at any given instance. From an agent's

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[4]In a more general setup one could assume that $\lambda\alpha_n > c > \lambda\alpha_{n+1}$ such that it is optimal to search for n signals if one's partner is uninformed. Here, for the sake of simplicity and tractability, we have chosen $n = 1$.

[5]Our assumptions imply that in the one-player version of the model, a player exerts maximum effort until she finds a signal. Once she has acquired a signal, she immediately takes a decision. In the two-player version, the efficient outcome is for both players to exert maximum effort until one signal has been found if $(2\lambda\alpha_2 - c)e_{\max} \leq \delta$. Otherwise, both players continue searching until a second signal has been found. A decision is called after respectively one or two signals are acquired.

perspective the probability that the other agent will call a decision by time t may be written as a weakly increasing function of time $\tilde{\rho}(t)$. Since agents may decide not to call a decision after acquiring a signal, an agent updates her belief that her partner is still uninformed when no decision has been called. We denote this belief by

$$\phi(t) = \frac{\tilde{\sigma}(t)}{1 - \tilde{\rho}(t)}.$$

In Section 4.2 we show that in all symmetric perfect Bayesian equilibria, subject to the earlier caveat, the equilibrium decision strategy is described by a continuous $\tilde{\rho}(t)$. In the interest of clarity we will restrict our attention to mixing strategies which result in a continuous $\tilde{\rho}(t)$ in the main body of the paper and refer the reader to the appendix for the general specification. We describe an agent's mixed strategy at different points in time by $d(t) : [0, T] \rightarrow \{call\} \times [0, \infty)$. If $d(t) = call$ and $\phi(t) = 1$, the hazard rate at which decisions are being made is the rate at which uniformed agents are becoming informed. Hence,

$$\frac{\frac{d\tilde{\rho}(t)}{dt}}{1 - \tilde{\rho}(t)} = \lambda e(t) \text{ if } d(t) = call \text{ and } \phi(t) = 1.$$

Otherwise, for $\phi(t) < 1$, the hazard rate at which decisions are being made is described by $d(t) \in [0, \infty)$ and $\phi(t)$ in the following way,

$$\frac{\frac{d\tilde{\rho}(t)}{dt}}{1 - \tilde{\rho}(t)} = d(t)(1 - \phi(t)).$$

Bayesian updating implies that an agent's belief evolves in the following way,

$$\frac{d\phi(t)}{dt} = \begin{cases} 0 \text{ if } d(t) = call \text{ and } \phi(t) = 1, \\ [d(t)(1 - \phi(t)) - \lambda e(t)]\phi(t) \text{ otherwise.} \end{cases} \tag{1}$$

Hence, if $d(t)(1 - \phi(t)) = \lambda e(t)$ and $\phi(t) < 1$ or $d(t) = call$ and $\phi(t) = 1$ the belief $\phi(t)$ remains constant over time.

Perfect Bayesian Equilibrium We consider symmetric perfect Bayesian equilibria of the continuous game with deadline T. The equilibrium strategy is the same for any continuation game starting at t and denoted by $\{e^*(t), d^*(t) | t \in [0, T]\}$. Any off-equilibrium strategy either ends the game or is without consequence for the optimal strategy. The posterior belief $\phi^*(t)$ is formed according to Bayesian updating for a given strategy profile as in (1).

To characterize the equilibrium strategies, we use the continuation value of the game at time t for the informed and uninformed player, denoted by $V^I(t)$ and $V^U(t)$ respectively. The sufficient conditions for $e^*(t), d^*(t)$ and $\phi^*(t)$ to constitute a perfect Bayesian equilibrium are as follows.

For any t, the continuation value for the informed player of the perfect Bayesian equilibrium equals

$$V^I(t) = \max_{\hat{t}\in[t,T]} V_0 + \alpha_1 + \int_t^{\hat{t}} \left[\alpha_2 - \delta(s-t)\right] \frac{\frac{d\tilde{\rho}^*(s)}{ds}}{1-\tilde{\rho}^*(t)} ds + \frac{1-\tilde{\rho}^*(\hat{t})}{1-\tilde{\rho}^*(t)} \left\{\left(1-\phi^*(\hat{t})\right)\alpha_2 - \delta\left[\hat{t}-t\right]\right\},$$

(2)

where $V_0 \equiv -\frac{1}{\varepsilon}$ denotes the value of taking an uninformed decision and $\tilde{\rho}^*(t), \sigma^*(t)$ are consistent with $d^*(t), e^*(t)$ as defined earlier. Defining $\hat{t}(t)$ as the set of maximizers $\hat{t} \subset [t,T]$ of the maximization in (2), the calling decisions satisfy:

$$
\begin{aligned}
d^*(t) &= call & &\text{if } \hat{t}(t) = \{t\}, \\
d^*(t) &= 0 & &\text{if } \min \hat{t}(t) > t, \\
d^*(t) &\in \{call\} \times [0,\infty) & &\text{otherwise.}
\end{aligned}
$$

(3)

For any t, the continuation value for the uninformed player of the perfect Bayesian equilibrium equals

$$
\begin{aligned}
V^U(t) = V_0 &+ \int_t^T \left[\alpha_1 - \delta(s-t) - c\int_t^s e^*(r)\,dr\right] \frac{\sigma^*(s)\frac{d\tilde{\rho}^*(s)}{ds}}{\sigma^*(t)(1-\tilde{\rho}^*(t))} ds \\
&+ \int_t^T \left[\left[V^I(s)-V_0\right] - \delta(s-t) - c\int_t^s e^*(r)\,dr\right] \frac{\frac{d\sigma^*(s)}{ds}(1-\tilde{\rho}^*(s))}{\sigma^*(t)(1-\tilde{\rho}^*(t))} ds \\
&+ \frac{\sigma^*(T)(1-\tilde{\rho}^*(T))}{\sigma^*(t)(1-\tilde{\rho}^*(t))} \left\{(1-\phi^*(T))\alpha_1 - \delta(T-t) - c\int_t^T e^*(s)\,ds\right\}.
\end{aligned}
$$

The effort decisions satisfy

$$e^*(t) \in \arg\max_{e\in[0,e_{\max}]} \lambda e\left(V^I(t) - V^U(t)\right) - ce.$$

(4)

We have so far written out the payoffs conditional on uninformed agents not calling decisions. To ensure that uninformed agents do not have a strict incentive to call a decision in the perfect Bayesian equilibrium, we require that

$$V^U(t) \geq V_0 + (1-\phi^*(t))\alpha_1.$$

Incentives to exert effort   An uninformed agent is more willing to exert effort the more valuable it is to become informed. The return to effort depends on the difference in the continuation values when informed and uninformed as in (4). When an agent is uninformed at time t, her marginal gain from exerting additional effort equals

$$\lambda\left[V^I(t) - V^U(t)\right] - c.$$

Before the deadline, the value of becoming informed also depends on the foregone expected cost of effort and delay when the agent were still uninformed. At the deadline, the value of becoming

12

informed only depends on the gained accuracy of the decision. Clearly, an informed agent can make a more accurate decision, but the value of having acquired a signal is lower when the partner has acquired a signal as well. Evaluated at the deadline T, the marginal gain of effort equals

$$\lambda \left[\phi \left(T \right) \alpha_1 + \left(1 - \phi \left(T \right) \right) \alpha_2 \right] - c.$$

This mirrors the expression in the simple model of the previous section. At the deadline, an uninformed player is unwilling to exert any effort if the probability that her partner is uninformed $\phi \left(T \right) < \bar{\phi}$, where the threshold $\bar{\phi}$ is defined by the equation

$$\lambda \left[\bar{\phi} \alpha_1 + \left(1 - \bar{\phi} \right) \alpha_2 \right] = c.$$

Incentives to delay a decision An informed agent is willing to delay a decision to give the opportunity to her partner to become informed as well. When deciding how long to delay a decision, the agent trades off the potential increase in the accuracy of the decision if an uninformed partner becomes informed with the expected cost from delaying the decision. The agent takes into account that her partner may or may not call a decision when becoming informed or may be informed already as is clear from the maximization in (2). However, just before the deadline, the incentive to delay a decision is approximately equal to

$$\lambda e \left(T \right) \phi \left(T \right) \alpha_2 - \delta.$$

The return to delaying only depends on the expected increase in accuracy, which happens with the probability that a still uninformed partner acquires a signal. This mirrors the expression in the simple model. We define $\bar{\phi}_d$ as the belief for which an informed agent is unwilling to delay a decision at the deadline when the other agent exerts maximum effort, i.e.,

$$\lambda e_{\max} \bar{\phi}_d \alpha_2 = \delta.$$

4.1 Equilibrium

We now characterize the symmetric equilibrium strategies of the continuous-time game with deadline T. Equilibrium strategies change as the agents approach the deadline at T, but they also depend on how large T is, that is to say how tight the deadline is set at the start of the game. When the deadline is sufficiently tight (T is sufficiently small), the unique equilibrium involves delay coupled with maximum effort throughout the game. When the deadline is sufficiently loose (T is sufficiently large), any equilibrium involves no delay coupled with low effort at the start of the game and full delay coupled with high effort close to the deadline.

4.1.1 Baseline case ($\bar{\phi} > \bar{\phi}_d$)

In the baseline case, we assume that at the deadline the incentives to search are small relative to the incentives to delay, i.e. $\bar{\phi} > \bar{\phi}_d$. This assumption implies that uninformed agents would stop exerting effort before informed agents stop delaying as the equilibrium belief $\phi^*(T)$ decreases. In this case, there will be at most two different equilibrium regions: no delay coupled with low effort, followed by delay and high effort.

The equilibrium strategies are characterized in Proposition 1. There are three distinct phases to consider depending on the length of the game denoted by T. We define the thresholds X and Y. The first threshold X equals the amount of time after which the belief $\phi^*(t)$ reaches $\bar{\phi}$ when an uninformed partner exerts maximum effort, but discloses no information upon becoming informed,

$$\exp\left(-\lambda e_{\max} X\right) = \bar{\phi}.$$

This threshold thus determines the maximum amount of time that uninformed players can be induced to exert maximum effort when no informed player is disclosing information. The second threshold Y equals the amount of time for which the total delay cost equals the expected value for an informed player of having a partner who is informed with probability $1 - \bar{\phi}$,

$$\delta Y = \left(1 - \bar{\phi}\right)\alpha_2.$$

This threshold thus determines the maximum amount of time for which an informed player is willing to delay a decision if the probability ϕ that her partner is uninformed falls from 1 to $\bar{\phi}$. Note that $X < Y$ as $\bar{\phi} > \bar{\phi}_d$.[6]

Proposition 1 *If $\bar{\phi} > \bar{\phi}_d$, then the equilibrium strategies and beliefs are as follows:*
i) If $T \leq X$, any informed player chooses not to call a decision, $d^*(t) = 0$, for all t, while any uninformed player chooses to exert maximum effort, $e^*(t) = e_{\max}$. The players' beliefs evolve according to $\phi^*(t) = \exp\left(-\lambda e_{\max} t\right)$.
ii) If $X < T \leq Y$, any informed player chooses not to call a decision, $d^*(t) = 0$, for all t, while any uninformed player chooses $e^*(t)$ which is not uniquely determined, but the effort choice must satisfy the conditions

$$\exp\left(-\lambda \int_{t_0}^{t} e^*(s)\, ds\right) \geq 1 - [t - t_0]\frac{\delta}{\alpha_2} \text{ for all } t \in [t_0, T], \tag{5}$$

$$\exp\left(-\lambda \int_{t_0}^{T} e^*(s)\, ds\right) = \bar{\phi}, \tag{6}$$

---

[6] Since $\bar{\phi} > \bar{\phi}_d$, an informed partner is willing to delay locally when $\phi^*(t) \geq \bar{\phi}$ and an uninformed partner exerts $e = e_{\max}$. That is, for $T = X$,

$$\delta < \lambda e_{\max}\phi^*(t)\alpha_2 \text{ for all } t \leq T.$$

Hence,

$$\delta X < \int_0^X \lambda e_{\max}\phi^*(t)\, dt\, \alpha_2 = \left(1 - \bar{\phi}\right)\alpha_2.$$

for $t_0 = 0$. The players' beliefs evolve according to $\phi^*(t) = \exp\left(-\lambda \int_0^t e^*(s)\, ds\right)$ for all t.

iii) If $T > Y$, any informed player calls an immediate decision for $t < t_e \equiv T - Y$ and no decision $d^*(t) = 0$ for $t \geq t_e$ while any uninformed player chooses $e^*(t) = \frac{\delta}{c}$ for $t < t_e$ and chooses $e^*(t)$ for $t \geq t_e$ which is not uniquely determined but the effort choice must satisfy the conditions (5) and (6) for $t_0 = t_e$. The players' beliefs evolve according to $\phi^*(t) = 1$ for $t < t_e$, $\phi^*(t) = \exp\left(-\lambda \int_{t_e}^t e^*(s)\, ds\right)$ for $t \geq t_e$ and $\phi^*(T) = \bar{\phi}$.

The theoretical results in the preceding proposition have an intuitive interpretation which builds on the features of our simple model of the previous section. The results illustrate the dynamics that emerge from the trade-off between the conflicting objectives of information acquisition and information sharing. Consider the first case where T is relatively small, that is to say there is little time between the start of the game and the deadline to become informed. In particular, the incentives to provide effort originate from the marginal value of information when a decision is taken. Since informed individuals always delay until the deadline, this is entirely determined by the marginal value of an extra signal at the deadline, that is

$$\phi^*(T)\alpha_1 + (1 - \phi^*(T))\alpha_2.$$

As long as the pursued signal is likely to be the first signal, the incentives for effort are sufficiently high to support maximal effort. Since T is smaller than X, the belief ϕ^* cannot fall below the threshold $\bar{\phi}$ and the agent thus chooses to exert maximum effort $e_{\max}$.[7] In response to this high effort choice any informed agent prefers not to call a decision and to delay since she benefits from the potential acquisition of an additional signal by a hard-working uninformed agent. Note that since $e^*(t) = e_{\max}$ and $d^*(t) = 0$, any agent correctly believes that as time passes it is more and more likely that the other agent is informed.

For T larger than X, the equilibrium outlined in the previous case is no longer sustainable. If the belief ϕ^* fell below the threshold $\bar{\phi}$, uninformed agents would no longer be willing to exert such a high level of effort. In equilibrium, uninformed agents now choose lower effort levels in a way that ensures that at time T the belief ϕ^* is exactly at the threshold $\bar{\phi}$, i.e., $\exp\left(-\lambda \int_0^T e^*(s)\, ds\right) = \bar{\phi}$. This belief at the deadline makes uninformed agents indifferent with respect to the level of effort they choose, both at and before the deadline. Note that

$$
\begin{aligned}
V^I(t) - V^U(t) &= \frac{c}{\lambda} + \exp\left[-\lambda \int_t^T e^*(s)\, ds\right]\left(V^I(T) - V^U(T) - \frac{c}{\lambda}\right) \qquad (7)\\
&= \frac{c}{\lambda} + \exp\left[-\lambda \int_t^T e^*(s)\, ds\right]\left[\phi^*(T)\alpha_1 + (1 - \phi^*(T))\alpha_2 - \frac{c}{\lambda}\right].
\end{aligned}
$$

Incentives to exert effort exist at t, that is $V^I(t) - V^U(t) \geq \frac{c}{\lambda}$, provided that they exist at time T, that is $V^I(T) - V^U(T) \geq \frac{c}{\lambda}$. As a result, when an uninformed agent is indifferent with regards to

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[7]It is important to emphasize that the maximum effort exertion of agents close to the deadline is the result of the increasing importance over time to become informed. It is not caused by any discounting motive since such an incentive is explicitly ruled out in our discounting-free setup.

her effort choice at the deadline $\phi^*(T) = \bar{\phi}$, she is also indifferent at any time t before.

The equilibrium path of effort is not unique, but to ensure that informed agents are willing to defer a decision until the deadline at any point during the game, uninformed agents need to backload their effort sufficiently.[8] One possible equilibrium is that uninformed agents choose effort levels $e^*(t) = 0$ for $t < T - X$ and $e^*(t) = e_{\max}$ for $t \geq T - X$. Notice also that for larger T, the aggregate effort exerted by uninformed agents remains the same, but their average effort intensity is lower.

For T larger than Y, informed agents no longer prefer to delay their decision from the start. While the aggregate benefit of delaying a decision through the potential information acquisition by the uninformed partner remains constant at $(1 - \bar{\phi})\alpha_2$, the aggregate cost of delay δT increases as T increases. As a result, when T exceeds Y, an informed agent will initially, that is as long as $t < t_e$, prefer to forego any delay costs and instead choose to immediately call a decision upon acquiring a signal. Hence, in contrast to the two previous cases the incentives for effort are now composed of the incentive to bring forward the time at which a decision is taken, thereby avoiding delay costs, and of the incentive to free ride on the effort of the other agent, thereby avoiding effort costs. In equilibrium, these two effects exactly balance each other when the other agent exerts effort $e_{-i} = \frac{\delta}{c}$. To see this, note that if an agent i shifts effort by Δe_i to the next instant, this allows her to avoid the expected effort costs $\lambda e_{-i} c \Delta e_i$, since the rate at which the other agent acquires information is λe_{-i}. On the other hand, the shift of effort in time increases delay costs δ at the rate $\lambda \Delta e_i$, hence the additional delay cost is $\lambda \delta \Delta e_i$. These two effects exactly offset one another when

$$\lambda e_{-i} c \Delta e_i = \lambda \delta \Delta e_i \Leftrightarrow e_{-i} = \frac{\delta}{c}.$$

Hence, an uninformed agent is indifferent with regards to her effort choice. The effort level exerted during this phase of full disclosure is lower than the average effort level in the phase of no disclosure, $\frac{\delta}{c} < \frac{X}{Y} e_{\max}$, reflecting the fact that a close deadline overcomes the temptation to free-ride.[9] Note also that when effort is low, $e_{-i} = \frac{\delta}{c}$, an informed agent is not willing to delay a decision since $\lambda \frac{\delta}{c} \alpha_2 < \delta$. If no decision has been called up to $t_e = T - Y$, the equilibrium is identical to case ii) from t_e onwards.

The three panels of Figure 1 illustrate the evolution over time t of the belief ϕ^*, the equilibrium effort e^* and the decision choice d^* for different lengths of the deadline T. In Panel 1.A, the deadline T is relatively tight, that is to say, $T \leq X$. Uninformed agents exert maximum effort $e_{\max}$ over the entire course of the game and informed agents delay making a decision. The belief ϕ^* declines from the complete certainty that the other agent is uninformed at $t = 0$ to $\phi^* \geq \bar{\phi}$ by the end of the game at T and with equality $\phi^* = \bar{\phi}$ at T if $T = X$.

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[8]This indeterminacy is due to our assumption that the cost of effort is linear which affords substantial tractability. In a related context Bonatti and Hörner (2011) also assume linear effort costs. When considering convex costs they are no longer able to obtain closed-form solutions, but their numerical illustrations for cost power functions suggest that the qualitative features of our analysis would remain intact.

[9]This follows as $\lambda \frac{\delta}{c} \alpha_2 Y < \delta Y = (1 - \bar{\phi})\alpha_2 = \int_0^X \lambda e_{\max} \phi^*(t)\, dt\, \alpha_2 < \int_0^X \lambda e_{\max} dt\, \alpha_2 = \lambda e_{\max} X \alpha_2$.
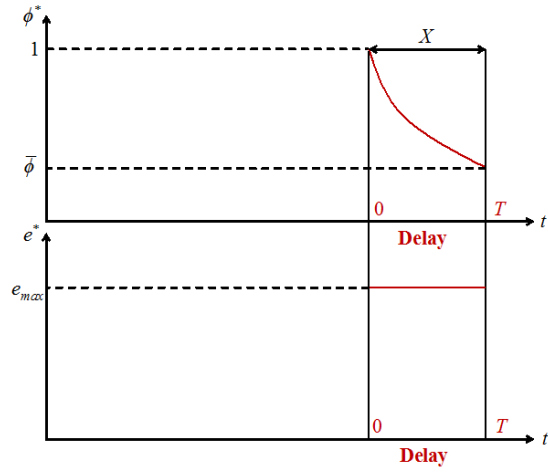
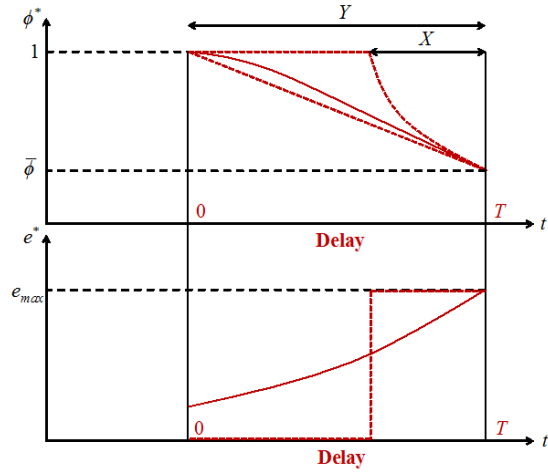Figure 1.A: Equilibrium for $T \leq X$
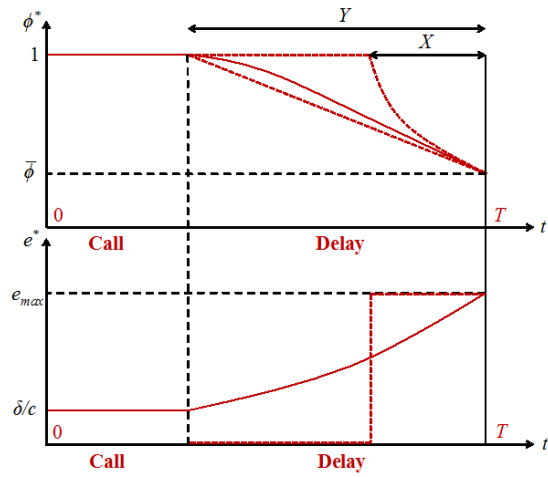

Figure 1.B: Equilibrium for $Y \geq T > X$

Figure 1.C: Equilibrium for $T > Y$

17

Next, in Panel 1.B the length of the deadline T is longer, specifically $Y \geq T > X$, and hence in response to the delay decision of informed team members, uninformed agents no longer exert maximum effort during the entire game. Instead, they choose to exert lower effort in such a way that the belief ϕ^* is equal to $\bar{\phi}$ at the end of the game. Note that since effort is not fully tied down in equilibrium, there are several ways in which uninformed agents can spread their effort. The solid and the dotted red lines depict two different equilibrium paths for effort e^* and the evolutions of the associated belief ϕ^*. Finally, in Panel 1.C we depict the equilibrium paths for loose deadlines where $T > Y$. As discussed before, at the beginning of the game uninformed agents exert effort $e^* = \frac{\delta}{c}$ and informed agents call a decision immediately. Thus, during this initial decision phase the belief ϕ^* remains constant at 1. However, once enough time has elapsed the delay phase begins and the game proceeds as in Panel 1.B.

4.1.2 Large Search Incentives ($\bar{\phi}_d \geq \bar{\phi}$)

We now briefly consider the opposite case, i.e., $\bar{\phi}_d \geq \bar{\phi}$, in which the incentives to exert effort for the uninformed agent exceed the incentives to delay for the informed agent at the deadline. In this case, there will be at most three different phases: no delay and low effort, followed by delay and high effort, and finally, mixing delay coupled with maximum effort.

In equilibrium, the belief $\phi^*(t)$ cannot drop below $\bar{\phi}_d$. Otherwise, an informed player would strictly prefer to call a decision as it is too likely that her partner is already informed. However, a belief $\phi(t) < 1$ cannot be consistent with an informed player's pure strategy to call a decision at time t.[10] Notice that by calling decisions at a rate $d(t)$ such that $(1 - \phi(t)) d(t) = \lambda e_{\max}$, an informed player keeps her partner's belief constant at $\phi(t)$. An informed player, however, is only indifferent about calling a decision when her belief equals exactly $\bar{\phi}_d$. This interplay introduces a new phase in equilibrium for games with length exceeding X_d where $\exp(-\lambda e_{\max} X_d) = \bar{\phi}_d$. After uninformed agents have exerted maximum effort for a length of time X_d and the belief $\phi^*(t)$ has decreased to $\bar{\phi}_d$, informed players start calling decisions at rate $d^*(t) = \frac{\lambda e_{\max}}{1 - \bar{\phi}_d}$ keeping the belief constant at $\bar{\phi}_d$.

The characterization of the equilibrium is thus very similar as before, with the exception of this final phase of mixing delay coupled with maximum effort exerted by uninformed agents. This final phase maximally lasts up to a length of time Z. The length of time $X_d + Z$ is the longest possible time for which maximum effort can be sustained throughout the game. When the length of the game exceeds $X_d + Z$, players will reduce their average effort during the first stage of the game such that the belief $\phi^*(t)$ equals exactly $\bar{\phi}_d$ for $t = T - Z$. The equilibrium strategies are formally characterized and discussed in detail in the web appendix.

Figure 1.D graphically illustrates the evolution of the strategies and beliefs over the course of the game. Equilibrium behavior of informed agents is divided into three distinct phases. Like in the baseline case, agents immediately call a decision upon becoming informed when deadline is far

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[10]If $d(t) = call$ and no player has called a decision at t, the belief should be reset at 1 and thus an informed player would again strictly prefer to delay the decision.

away. However, once the deadline is sufficiently close, i.e., $t \geq T - Y_d$, informed agents prefer to delay calling a decision and thus the equilibrium belief ϕ^* falls until it reaches $\bar{\phi}_d$. At that point, informed agents are indifferent between calling and delaying the decision and thus probabilistically choose one or the other until the conclusion of the game in such a way that ϕ^* remains constant at $\bar{\phi}_d$. Note again, that the dotted red lines for ϕ^* illustrate different equilibrium paths associated with different equilibrium paths for e^*. The evolution of effort in Figure 1.D is similar to the evolution of effort in Figure 1.C. Uninformed agents choose $e^* = \frac{\delta}{c}$ during the initial decision phase and then start increasing their effort until they exert effort $e_{\max}$ at the end of the game.
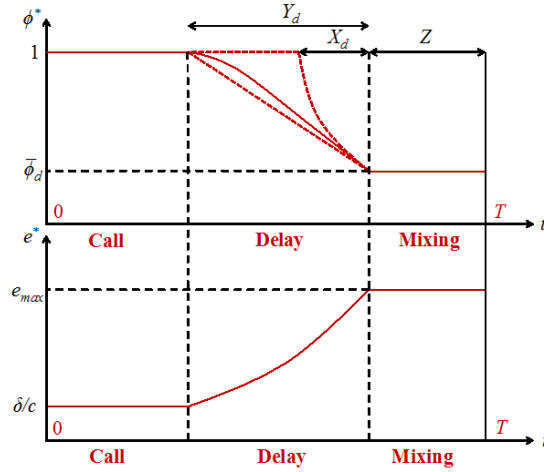
Figure 1.D: Equilibrium for $T > Y_d$ when $\bar{\phi} < \bar{\phi}_d$

The equilibrium description, both in the baseline case with small incentives for search and the case with large incentives for search, formally establish that even when committee members have perfectly aligned interests, decisions may be significantly delayed. This is due to two factors. First, when the deadline is far away, delay occurs due to a lack of information search. With no disciplining deadline, team members try to free-ride on each other's efforts and do not search very intensely. Hence, without the adequate information available to the group no decision can be taken. Second, when the deadline is close, delay occurs due to a lack of information sharing. Although uninformed agents search intensely for information and hence the group is likely to have valuable information at its disposal, no decision will be taken until the deadline. An informed agent will prefer not to divulge their information in order to keep any uniformed team member highly motivated to search for additional information.

4.2 Uniqueness

In this subsection we establish and discuss the uniqueness of the equilibrium described in Propositions 1 in the set of symmetric perfect Bayesian equilibria.[11] All proofs are in the web appendix.

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[11] Note, however, that in addition to the equilibrium discussed previously, there are also symmetric equilibria which involve strategies whereby both uninformed and informed agents call a decision at the same instant of time with

We denote an uninformed agent's strategy to call a decision by a function $\mu(t)$ and still refer to the probability that an informed agent calls a decision by $\rho(t)$. We allow agents to adopt piecewise continuous differentiable decision functions for $\rho(t)$ and $\mu(t)$. To this end define

$$D_\rho(t) = \lim_{s \to t^+} \rho(s) - \lim_{r \to t^-} \rho(r)$$

and

$$D_\mu(t) = \lim_{s \to t^+} \mu(s) - \lim_{r \to t^-} \mu(r)$$

to describe the probability mass of decisions at any points of discontinuity. Otherwise, the model is the same as in our earlier analysis. For a complete analysis of this augmented model we refer the interested reader to the appendix and here instead focus on the main steps of the proof. As before, we denote equilibrium strategies by a superscript *.

A symmetric perfect Bayesian equilibrium is described by a tuple $(e^*(t), \rho^*(t), \mu^*(t), \phi^*(t))$ if $\rho^*(t) + \mu^*(t) < 1$ for all $t < T$, where $\phi^*(t)$ is the Bayesian belief an agent has at time t that the other agent is uninformed conditional on no decision being called prior to that time. If $\exists t' < T : \rho^*(t') + \mu^*(t') = 1$, then it must also include off-equilibrium strategies and beliefs $(e^*(r|t), \rho^*(r|t), \mu^*(r|t), \phi^*(r|t))$ for all times t where $\rho^*(t) + \mu^*(t) = 1$ which themselves are perfect Bayesian equilibria of those subgames. $\phi^*(r|t)$ is the Bayesian belief an agent has at time r that the other agent is uninformed conditional on no decision being called prior to that time in a subgame starting at time t. We now rule out some types of strategies on the equilibrium path by the uninformed agent. The following lemma rules out a continuously increasing $\mu^*(t)$.

Lemma 1 $\nexists \mu^*(t), r > 0, \varepsilon > 0 : \frac{d\mu^*(t)}{dt} > 0 \ for \ t \in [r - \varepsilon, r]$.

Lemma 1 shows that in the set of symmetric equilibria there is no mixing in the decision strategy by an uninformed player at times that are reached on the equilibrium path. The following lemma rules out a jump in the decision function $\mu(t)$ on the equilibrium path if that jump does not occur when both types, informed and uninformed, call a decision with certainty at that instant.

Lemma 2 $\nexists \mu^*(t), 0 < s < T : D_{\mu^*}(s) > 0 \ and \ \mu^*(s) + \rho^*(s) < 1$.

Hence, the only equilibria involving $D_{\mu^*}(s) > 0$ also have $\mu^*(s) + \rho^*(s) = 1$ whereby beliefs at times later than s are off the equilibrium path. In this case, it may be possible to support an equilibrium where uninformed agents call a decision with appropriately specified off-equilibrium beliefs. These equilibria will have beliefs that $\mu(t) = 0$ until a time s at which point $\mu^*(s) + \rho^*(s) = 1$ which is equivalent to beliefs in a game where the deadline is $T = s$ and $\mu(t) = 0$. In this case our analysis characterizes the behavior at any time t on the equilibrium path and abstracts away from specifying behavior off the equilibrium path. We thus continue the analysis under the assumption

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probability 1 conditional on reaching that time. We do not discuss these equilibria where individuals are calling a decision with certainty at a point in time because they are in effect equivalent to deadlines which are enforced by appropriately specified off-equilibrium beliefs.

that $\mu^*(t) = 0$ for all t. This implies that all $t \leq T$ are reached with some non-zero probability in equilibrium. Therefore, there are no times t that are off the equilibrium path for which strategies and beliefs must be specified. An individual may find himself at a time t that can be reached on the equilibrium path, but where her own history of effort choices is inconsistent with the specified equilibrium. However, all costs are sunk and so the subgame is identical to the subgame on the equilibrium path. In these instances the strategies off the equilibrium path are the same as the on-equilibrium strategies at the corresponding time. The following proposition shows that the previously specified set of symmetric perfect Bayesian equilibria is unique.

Proposition 2 *Suppose $\mu^*(t) = 0$ then the set of equilibria described in Propositions 1 (and Proposition 9 in the Appendix) are the unique sets of symmetric perfect Bayesian equilibria under small and large incentives respectively.*

Thus, the withholding of information through delay in the lead-up to the deadline which we discussed in the previous subsections, is a characteristic of all symmetric equilibria. It further allows us to consider the welfare implications of (optimal) deadlines and comparative statics.

5 Setting Deadlines

In this section, we analyze the trade-offs groups face when setting deadlines. With a tight deadline the team members risk making a decision without information and have strong incentives to acquire information, but also to delay the decision. The expected decision time may therefore actually be larger when a short deadline is set. On the other hand, with a loose deadline the group members initially procrastinate and only begin gathering information in earnest when the deadline is close. When informed team members are concealing their information in the hope that other group members may acquire more information, a tighter deadline will reduce inefficient delay. We show how expected decision time and expected accuracy of the decision vary with the length of the deadline and determine the optimal deadline that maximizes the expected welfare for the group members at the start of the process.

5.1 Decision Time

In this subsection we examine the effect of the deadline on the expected time until a decision is made. The natural intuition is that tighter deadlines lead to shorter decision times. We show that this need not be the case and that instead the expected decision time may be non-monotonic in the length of the deadline. For loose deadlines, $T > Y$, the equilibrium is characterized by two phases: a phase of low effort and immediate decisions followed by a phase of pure delay and higher effort. In this case increasing the deadline T decreases the probability that agents reach the later period where decisions are delayed until the deadline. The overall effect of increasing the deadline is ambiguous as the combination of immediate decisions despite slow information acquisition may be a slower or faster process than incurring the fixed delay of Y upon reaching the later period. In contrast, for

tight deadlines, $T \leq Y$, there is no initial period of low effort and immediate decisions, but instead informed partners never disclose their information and the team always delays its decision until the deadline. The expected time until the action is taken equals T. Hence, a closer deadline will always reduce the expected decision time. This delay is strongly reminiscent of a widely accepted behavioral law called Parkinson's Law. This law, as stated in its original source (Parkinson 1955, 1958), posits that "the amount of time which one has to perform a task is the amount of time it will take to complete the task". In our context, this means that the amount of time in which the team has to make a decision is exactly the amount of time it will take to make a decision. The following proposition formalizes these ideas. The proof is in the Appendix.

Proposition 3 *For $T \leq Y$, the expected decision time is increasing in T. For $T > Y$, the expected decision time is decreasing in T if and only if $\frac{\alpha_2}{\alpha_1} > \frac{\bar{\phi}}{1 - \bar{\phi}}$ and increasing otherwise.*

For loose deadlines, the expected decision time equals

$$Et_c = \int_0^{T-Y} \tau_c f(\tau_c) \, d\tau_c + [1 - F(T-Y)] T,$$

where $f(t_c) = 2\lambda \frac{\delta}{c} \exp\left(-2\lambda \frac{\delta}{c} t_c\right)$ is the probability that a signal is acquired at time t_c, when the two agents are exerting the low equilibrium effort level $e^*(t_c) = \frac{\delta}{c}$. An increase in T reduces the probability that the stage without disclosure is reached, but increases the decision time if that stage is not reached. The derivative of the expected decision time with respect to the deadline is then given by

$$
\begin{aligned}
\frac{\partial Et_c}{\partial T} &= (1 - F(T-Y)) - f(T-Y)Y \\
&= f(T-Y)\left[\frac{1 - F(T-Y)}{f(T-Y)} - Y\right].
\end{aligned}
$$

The expected decision time Et_c is decreasing in the length of the game T only if $\frac{1 - F(T-Y)}{f(T-Y)} < Y$. The ratio $\frac{f(T-Y)}{1 - F(T-Y)} = 2\lambda \frac{\delta}{c}$ equals the rate at which decisions are called during the full-disclosure stage, which is the inverse of the expected decision time without a deadline. In other words, if the expected decision time without a deadline exceeds the length of the no-disclosure stage, lenghtening the deadline will have adverse effects on the expected decision time. Using the expression for Y, we find this is the case if and only if $\frac{\alpha_2}{\alpha_1} > \frac{\bar{\phi}}{1 - \bar{\phi}}$.

Our model predicts that committees should either take a decision relatively early on in the process or right at the deadline. Thus, Parkinson's Law applies for any deadline of length $T \leq Y$ for which decisions are only made exactly at the deadline. The relationship between delay, performance and deadlines has been extensively studied both in laboratory and field settings in psychology with a myriad of tasks (Bryan and Locke 1967; Peters et al. 1984, Locke et al. 1981 for an overview of this large literature). In addition to delay, their findings also indicate that with longer deadlines work intensity and performance may suffer. We address these issues in the next subsection.

22

5.2 Decision Precision

In addition to influencing how long it will take a group to make a decision, the choice of deadline also affects the expected precision that is available to the agents when a decision is made. When more signals are acquired, the agents have a more precise posterior distribution and thus incur a lower expected loss when making a decision. One would expect that a longer deadline would always allow for more information to be accumulated by the agents and hence lead to a more precise decision making process. However, this line of reasoning ignores that agents may choose to call a decision before the deadline.

Proposition 4 *The expected precision is increasing in T for $T \leq X$ and it is constant for $X < T \leq Y$. For $T > Y$, it is decreasing in T if and only if $\frac{\alpha_2}{\alpha_1} > \frac{\bar{\phi}^2}{\left(1-\bar{\phi}\right)^2}$ and increasing otherwise.*

For short deadlines, $T \leq X$, an increase in the length of the game strictly increases the expected precision of the decision made at the deadline. Agents exert maximum effort throughout the game as long as they are uninformed and thus have more time to become informed if the game lasts longer. The expected value of the decision taken equals

$$V_0 + \left(1 - \phi^*\left(T\right)^2\right)\alpha_1 + \left(1 - \phi^*\left(T\right)\right)^2\alpha_2,$$

where the probability that an agent is still uninformed at the deadline, $\phi^*\left(T\right) = \exp\left(-\lambda e_{\max}T\right)$, is decreasing in T. For intermediate deadlines, $X < T \leq Y$, an increase in the length of the game has no impact on the expected precision of the decision. The probability that an agent is still uninformed at the deadline equals $\phi^*\left(T\right) = \bar{\phi}$, regardless of the length of the game. This is an even stronger expression of Parkinson's Law, alternatively stated as: "Work expands so as to fill the time available for its completion." The additional time granted to the agents has no impact at all on their total effort. This result is also found in a plethora of field and experimental studies that find no change in task completion time and demonstrate that effort (or work pace) is adjusted to the time available or the difficulty of the task (Bassett 1979, Locke et al. 1981).

For long deadlines, $T > Y$, an increase in the length of the game may decrease the expected precision of the decision that is eventually taken. Agents may call a decision before the deadline is reached because they prefer to forego the delay that comes with waiting in the hope that the other agent may find a signal. When T is large, it becomes more likely that a decision is called by an informed agent who can rely only on one piece of information. The key comparison is therefore whether the expected precision for $T \leq Y$ is greater or smaller than the expected precision of a single signal, i.e.,

$$\left(1 - \phi^*\left(T\right)^2\right)\alpha_1 + \left(1 - \phi^*\left(T\right)\right)^2\alpha_2 \gtrless \alpha_1.$$

When α_2 is small compared to α_1 the second signal is of relatively little informational value and for $T \leq Y$ the agents run the risk of ending up with no signal at all. Thus, the expected precision is increasing in T. Conversely, when the second signal is sufficiently valuable, the expected precision

is higher for $T \leq Y$ and hence the expected precision is decreasing in T for $T > Y$. We find this is the case if and only if $\frac{\alpha_2}{\alpha_1} > \frac{\bar{\phi}^2}{\left(1-\bar{\phi}\right)^2}$.

Corollary 1 *When $\frac{\alpha_2}{\alpha_1} > \frac{\bar{\phi}^2}{\left(1-\bar{\phi}\right)^2}$ and $\left(2\lambda\alpha_2 - c\right)e_{\max} < \delta$, the team acquires too much information in expectation for $T \in [X, Y]$.*

Interestingly, when $\frac{\alpha_2}{\alpha_1} > \frac{\bar{\phi}^2}{\left(1-\bar{\phi}\right)^2}$ and $X \leq T \leq Y$, the expected precision of the decision can be inefficiently high. That is to say, agents may take decisions based on more information than is efficient from an ex-ante point of view. In particular, if it is socially efficient for the team to acquire only a single signal, i.e., $\left(2\lambda\alpha_2 - c\right)e_{\max} < \delta$, the agents will overacquire information in expectation. While it is not too surprising that agents may end up acquiring too much information ex-post given our assumption of private information acquisition, it is quite surprising that despite the agents' free-riding problem ex-ante overacquisition of information can occur in our model. As we show in the next subsection, ex-ante overacquisition of information may occur even when the deadline T is set optimally.

5.3 Welfare

In the previous subsections we analyzed how the length of the deadline may increase or decrease the expected decision time and the expected precision of the decision taken by the agents. In this subsection we turn our attention to the effect of the deadline on the welfare of the agents. We characterize the welfare of each agent as a function of the deadline and find that there exists a finite and unique welfare maximizing deadline for the group of agents. In the baseline case the optimal deadline is set such that agents always delay their decision until the deadline, but still engage in information acquisition. The following proposition formally characterizes the effect of the deadline on welfare.

Proposition 5 *The expected utility of the game is maximized for $T = X$. The expected utility is strictly increasing in the length of the deadline T for $0 \leq T \leq X$, stricly decreasing in T for $X < T \leq Y$ and independent of T for $T > Y$.*

The black line in Figure 2 graphically illustrates how expected welfare at the start of the game varies with the choice of the deadline T. As shown in the Proposition, there is a unique, finite welfare-maximizing deadline which is $T = X$. For very tight deadlines, $T \leq X$, agents exert maximum effort and make a decision only when the deadline arrives. Thus, increasing the deadline improves welfare because even though it increases the time until a decision is made, it also allows the agents more time to intensely search for valuable information prior to the deadline. However, once T is larger than X the aggregate effort exerted and the expected information acquired prior to the deadline remain unchanged. The team members simply choose their effort in such a way that they are equally well-informed at the deadline, no matter whether the deadline occurs at X or Y or at anytime in between. Consequently, any increase in the deadline over and above X only

24

introduces additional costly delay before a decision is made at the deadline. For T between X and Y, the expected utility decreases linearly in T at rate δ. Finally, for loose deadlines, $T > Y$, the welfare of the agents is independent of the deadline.[12] Shortening the length of the game reduces the probability that one's partner acquires a signal. Her effort level during the phase of no delay is such that the value of this lost opportunity, $\lambda e^*(0) \left[V_T^I(0) - V_T^U(0) \right]$, is exactly offset by the foregone delay δ.[13]
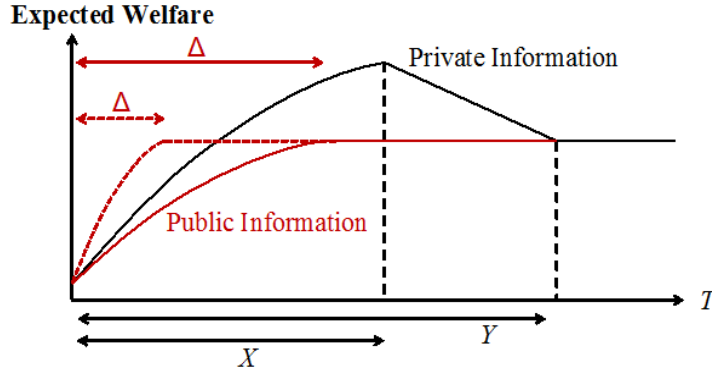
Figure 2: Expected welfare at $t = 0$ as a function of T under private and public information.

Related to our findings that emphasize the beneficial incentive effects of short deadlines as well as the lack of delay associated with them, are the influential studies of strategic decision making of executive teams in the microcomputer industry by Bourgeois and Eisenhardt (1988) and Eisenhardt (1989). The authors document that indecision and delay can cost firms their technical and market advantages and even lead to bankruptcy. They further show that management teams that make fast decisions due to strict deadlines also use high levels of information and develop many problem-solving alternatives. This fast and informed decision making avoids delay and is positively related to superior firm performance.

Corollary 2 *The optimal length of the deadline $T = X$ is decreasing in $e_{\max}$, c and increasing in α_1, α_2 and ambiguous with respect to λ.*

The optimal deadline is affected by changes in the underlying model parameters. Remember that the threshold X equals the time it takes when exerting the maximal effort level to reduce the

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[12]Notice that the expected utility in the long horizon game, $V_0 + \alpha_1 - \frac{c}{\lambda}$, exceeds the highest achievable expected utility in the one-player version, $V_0 + \alpha_1 - \frac{[ce_{\max} + \delta]}{\lambda e_{\max}}$. This is achieved when no deadline is imposed.

[13]Since a decision is immediately called when one agent is informed, the value of being informed, $V_T^I(0) = V_0 + \alpha_1$, does not depend on the deadline in this stage. Moreover, both agents are indifferent with respect to the effort level they exert,

$$\lambda \left[V_T^I(0) - V_T^U(0) \right] = c \text{ for any } T > Y.$$

Hence, the value of being uninformed, $V_T^U(0) = V_T^I(0) - \frac{c}{\lambda}$, does not depend on the deadline either.

probability of being still uninformed to $\bar{\phi}$,

$$\exp\left(-\lambda e_{\max} X\right) = \bar{\phi},$$

where $\bar{\phi}$ is the belief that makes an agent indifferent about the level of effort to exert,

$$\lambda\left[\bar{\phi}\alpha_1 + \left(1 - \bar{\phi}\right)\alpha_2\right] = c.$$

When the maximal effort $e_{\max}$ increases, it takes less time to reach this critical probability. Hence, the longest time that maximal effort can be induced decreases. An increase in the marginal productivity of effort for finding a signal λ has a similar effect in that it reduces the time needed for ϕ^* to reach $\bar{\phi}$. However, an increase in λ also has a second countervailing effect because it reduces $\bar{\phi}$ by making it more beneficial to exert effort. The aggregate effect on the optimal deadline of a change in λ is therefore ambiguous. In contrast, an increase in the marginal cost c unambiguously shortens the optimal deadline by increasing $\bar{\phi}$. When it is more costly to exert effort, an uninformed agent will be indifferent between exerting and not exerting effort even when it is less likely that the other agent is informed and hence maximal effort can be sustained for a shorter period of time. Conversely, increases in the value of the first α_1 and the second signal α_2 both raise the marginal benefit of effort and thus decrease $\bar{\phi}$ with the eventual effect of lengthening the optimal deadline. Finally, note that the optimal deadline is independent of the delay cost δ. This independence result is due to our initial assumption that δ is sufficiently small for informed agents to be willing to delay.[14]

6 Private Information: Incentives vs. Delay

In this section, we analyze how the private nature of acquired information affects welfare. When information is public, the acquisition of information discourages all partners from searching and thus leads to an immediate decision. When information is private, informed players can hide their information and delay calling a decision. The option to keep information private increases the returns to becoming informed relative to the case when information is public, which in turn increases a partner's incentives to search. We show that when the acquired information is private and the deadline is set optimally–trading off search incentives and inefficient delay–the expected welfare at the start of the game is higher than when the acquired information is public.

6.1 Public Information

We first characterize the equilibrium of the game when the information obtained by individual group members is immediately visible to the entire group. The equilibrium behavior of agents is

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[14]For the large incentives case, $\bar{\phi}_d \geq \bar{\phi}$, there is still a unique finite deadline that maximizes the agents' welfare, equal to $T = X_d + Z$, where Z is decreasing in δ. As in the baseline case it is again optimal to set the deadline exactly at the greatest length of time during which uninformed agents will exert maximum effort at any point in time.

straightforward in this case. First, whenever an agent succeeds in finding a signal, a decision is called immediately. The reason is simply that no player would find it privately optimal to search for a second signal. Second, both agents choose low effort when the deadline is still far away and they switch to exerting maximal effort when the deadline is close. We formally characterize equilibrium behavior in the following proposition.

Proposition 6 *When information is public, the strategies in the unique, symmetric equilibrium are as follows:*
i) If $T \leq \Delta$, any uninformed player chooses to exert maximum effort, $e^{pub}(t) = e_{\max}$, for all t.
ii) If $T > \Delta$, any uninformed player chooses to exert $e^{pub}(t) = \frac{\delta}{c}$ for $t \leq T - \Delta$ and chooses $e^{pub}(t) = e_{\max}$ for $t > T - \Delta$, where

$$\exp\left(-2\lambda e_{\max}\Delta\right) = \frac{c - \frac{\delta}{e_{\max}}}{2\lambda\alpha_1 - \left(c + \frac{\delta}{e_{\max}}\right)}.$$

The public nature of information affects the agents' incentives to search for information. The value of being informed is constant throughout the course of the game, since agents can be sure that a decision is called as soon as one signal is obtained. For tight deadlines, $T \leq \Delta$, the value of being uninformed decreases over time because the likelihood of having to take an uninformed decision at the deadline increases. As a result, the incentive to exert effort increases as the deadline approaches. When the time left until the deadline is exactly Δ, a player is indifferent about how much effort to exert if her partner exerts maximum effort until the deadline. Now, consider increasing the deadline T above the threshold Δ. At any time t before $T - \Delta$ is reached, a player is indifferent about how much effort to exert only if her partner exerts $e^{pub}(t) = \frac{\delta}{c}$ and the value of being uninformed is independent of the remaining time until the deadline. Not surprisingly, this corresponds to the initial phase of low effort and full disclosure for long deadlines in the case of private information.

6.2 Welfare Comparison

We now compare the welfare achieved when acquired information is public or private. We find that the option to conceal information increases the incentives to become informed and thus mitigates the free-rider problem. The important consequence is that private information leads to higher welfare.

Proposition 7 *The highest welfare achieved by optimally setting the deadline when information is private exceeds the highest welfare when information is public.*

As long as the deadline is sufficiently far away, the effort strategies coincide, since agents fully disclose their acquired information, whether they are required to or not. For long games, welfare does not depend on the deadline, nor does it depend on whether the acquired information is public or private. For close deadlines, the incentives to search are different in the two cases, since informed agents no longer disclose their information when they are not required to. A priori, it may seem

that the change in the observability of information can increase or decrease the search incentives. The private nature of information provides more incentives for uninformed players by allowing informed players to 'rest on their laurels'. However, the fact that other players may be already informed, decreases the value of additional information and thus the incentives to search. The first effect dominates the second effect in equilibrium. When information is private, maximum incentives for search can be sustained throughout for games with longer deadlines than when information is public, i.e., $X > \Delta$. This also implies that for games with long deadlines, the initial stage of low effort $e(t) = \frac{\delta}{c}$ lasts longer when information is public, since $Y > X$.[15]

Corollary 3 *Incentives for maximal search can be sustained for a longer period of time when information is private than when it is public.*

Clearly, the higher search incentives due to the private nature of information increase the team partners' welfare by mitigating the inefficiency due to free-riding. However, for short deadlines, $T \leq \Delta$, incentives are sufficiently strong for players to exert maximum effort, regardless of whether information is public or private.

The option to hold back information also affects welfare through the delay of decisions. Informed players stop searching and may delay a decision in the hope that their partner becomes informed. Hence, both players may have stopped searching, but still delay a decision not knowing that their partner is already informed. This is clearly inefficient ex post. However, as any uninformed player would stop searching if the information were disclosed, any opportunity to acquire a second signal is lost when information is public. This opportunity can only be valuable if it is socially efficient for one player to search for a second signal, $(2\lambda\alpha_2 - c) e_{\max} > \delta$. Hence, when the deadline is short and it is socially inefficient to acquire a second signal, the team partners' welfare is increased by committing to disclose any information they acquire.

Corollary 4 *Welfare under public information exceeds welfare under private information if $T \leq \Delta$ and $(2\lambda\alpha_2 - c) e_{\max} < \delta$.*

In Figure 2, we also plot the expected welfare at $t = 0$ for different lengths of the deadline T under public information. The full red line shows the case where welfare under public information is always lower than under private information. In contrast, when searching for a second signal is socially inefficient and the deadline is set inefficiently short, welfare under public information can be higher than under private information as illustrated by the dashed red line. However, keeping information private is always preferable for the team when the deadline can be set optimally. That is, the stronger search incentives due to the private nature information can be exploited at a relatively low cost of inefficient delay by setting the appropriate deadline.

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[15]Notice that equilibria exist for which $e^*(t) \geq e^{pub}(t)$ for any t. However, $e^*(t) = 0$ ($< e^{pub}(t)$) for some t may well be part of an equilibrium strategy for an uninformed player.

7 Discussion

Our basic framework identified a strong tension between the incentives to reveal and search for information. In equilibrium, this resulted in two potential phases depending on the proximity of the deadline: a first phase of low effort and full disclosure and a second phase of high effort and no disclosure. In this section we discuss the robustness of the qualitative equilibrium characteristics to modifications in the signal structure and the number of players. We also briefly consider some extensions of our basic framework that are relevant in group decisions, like the introduction of explicit contracts and reputational rewards to being informed, the presence of communication frictions, and the potential role of a third party intermediary such as a committee chairperson. To simplify the presentation we discuss each new element separately and keep the mathematical formalities to a minimum.

7.1 Signal Structure

In this section, we investigate the robustness of the qualitative characteristics of the earlier model to a setting where agents have incentives to acquire more than one signal. We consider a setting where an individual is prepared to continue searching for a second but not a third signal, $\alpha_2 > \frac{c}{\lambda} > \alpha_3$. In the previous setting, calling a decision is a dominant strategy, once a signal is disclosed. Hence, we did not distinguish between the two actions. Here agents may want to communicate that they have found a signal without calling a decision. Therefore, it is natural to distinguish between the two actions. We do this by allowing agents to take an action $w_i \in N$ at any instance of time. The action $w_i = n$ verifiably reveals that an agent has found n signals. This has the potentially appealing characteristic that an agent with one signal, although not willing to call a decision unilaterally, may nevertheless communicate this to the other agent.

We describe the main results here and refer the interested reader to the appendix for an explicit characterization of the setting and propositions. We consider two extreme cases: a short deadline and an infinite deadline. We show that a symmetric equilibrium of a game with a short deadline (below a threshold level) exhibits no information revelation $w_i^* = 0$, pure delay $d_i^* = 0$, and high effort provision having found zero or one signal, $e_i^* (t, n) = e_{\max}$ for $n = 0, 1$, and no effort after acquiring two signals, $e_i^* (t, n) = 0$ for $n = 2$. These are characteristics shared by our earlier model where an agent exerts high effort provision while having a low number of signals and zero effort after acquiring a sufficient number of signals. Similarly pure delay (and hence lack of timely information sharing) is also observed in our earlier model close to a deadline.

To gain some intuition for what may occur a long way from the deadline we also consider an equilibrium of the infinite horizon game. This is done to avoid the complications in a game with a finite horizon, of specifying the changes in behaviour for the subgames as we transition from behavior far away from the deadline to close to the deadline. The infinite horizon case allows one to focus on a setting where there is no effect of a future deadline. We show that the symmetric equilibrium strategies are similar to the equilibrium strategies in our earlier model far away from

the deadline (i.e., for $t < T - Y$). We find immediate revelation of information by each individual, $w_i^* = n$. Hence, a decision is called whenever the number of signals reaches two, $d_i^* = call$ if $n + w_{-i} \geq 2$. This is similar to the earlier model whereby the agents immediately call a decision upon acquiring information. Individuals also exert less than the maximum effort level $e_i^* = \frac{\delta}{c} < e_{\max}$ as in the earlier model which trades off free riding incentives with incentives to bring forward the decision. Although we do not have a complete characterization of this setting we conjecture on the basis of these two cases that the qualitative characteristics of our earlier model close to the deadline and far away from a deadline remain.

7.2 Number of Players

We now consider how the number of players influences the interactions of the members of the group. We find that adding more players decreases X, the longest deadline which can sustain maximum effort by all players as an equilibrium. However, the effect on Y, the longest deadline for which no decision is taken until the deadline, is ambiguous.

The intuition for the first result is that as more team members are added to the group and are exerting maximum effort in the face of a close deadline $(T < X)$ it is more likely that other team members have already found pieces of information. As a result, an uninformed player searching for information will only be able to discover less valuable additional information and this mutes the incentives for effort. That is to say maximum effort can only be sustained for a shorter period before the deadline. We illustrate this in the two player case where the relationship which determines X is given by

$$\lambda \exp\left(-\lambda e_{\max} X\right) \alpha_1 + \lambda \left(1 - \exp\left(-\lambda e_{\max} X\right)\right) \alpha_2 = c.$$

With an additional third player the relevant threshold X is defined by the following equation

$$\lambda \left(\exp\left(-\lambda e_{\max} X'\right)\right)^2 \alpha_1 + \lambda 2 \exp\left(-\lambda e_{\max} X'\right) \left(1 - \exp\left(-\lambda e_{\max} X'\right)\right) \alpha_2$$
$$+ \lambda \left(1 - \exp\left(-\lambda e_{\max} X'\right)\right)^2 \alpha_3 = c.$$

Since the probability weights on α_1, α_2 and α_3 sum to one in both equations and $\alpha_1 > \alpha_2 > \alpha_3$, it must be the case that $\left(\exp\left(-\lambda e_{\max} X'\right)\right)^2 > \exp\left(-\lambda e_{\max} X\right)$ and hence $X' < X$.[16]

While the relationship between the amount of time during which uninformed members exert maximum effort and the number of group members is unambiguous, the relationship between the

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[16] More generally, consider any group of n members and add an additional group member. The relevant equations are now

$$\lambda \sum_{k=0}^{n-1} \binom{n-1}{k} \left(1 - \exp\left(-\lambda e_{\max} X^n\right)\right)^{n-1-k} \left(\exp\left(-\lambda e_{\max} X^n\right)\right)^k \alpha_{n-k} = c$$

and

$$\lambda \sum_{k=0}^{n} \binom{n}{k} \left(1 - \exp\left(-\lambda e_{\max} X^{n+1}\right)\right)^{n-k} \left(\exp\left(-\lambda e_{\max} X^{n+1}\right)\right)^k \alpha_{n+1-k} = c$$

Note that $\lambda \sum_{k=0}^{n} \binom{n}{k} \left(1 - \phi\right)^{n-k} \phi^k \alpha_{n+1-k}$ is strictly decreasing in n since $\alpha_1 > \alpha_2 > ... > \alpha_n$. Note further that this expression is increasing in ϕ. For both indifference conditions to be satisfied we again require that $X^{n+1} < X^n$.

length of time for which no decision is taken until the deadline and the number of group members is ambiguous: the delay phase may increase or decrease when the group grows larger. Consider the case of two and three group members where the thresholds are defined in the following way

$$Y = \frac{(1 - \exp(-\lambda e_{\max} X)) \alpha_2}{\delta}$$

and

$$Y' = \frac{\left(1 - \exp(-\lambda e_{\max} X')^2\right) \alpha_2 + (1 - \exp(-\lambda e_{\max} X'))^2 \alpha_3}{\delta}$$

Let $\alpha_3 = 0$ then from the indifference conditions for X and X' we know that $(\exp(-\lambda e_{\max} X'))^2 > \exp(-\lambda e_{\max} X)$ and hence $Y' < Y$. The introduction of additional players makes it more likely that uninformed players are looking for one of many signals. When the value of these additional signals is very low, search incentives are only maintained if the probability that one is searching for a valuable first signal is increased. Hence, the aggregate search effort by the group members must decrease if the number of team members increases. This, in turn, decreases the incentives to delay by informed agents. On the other hand, when $\alpha_3 = \alpha_2$ then from the indifference conditions for X and X' we have $\exp(-\lambda e_{\max} X) = (\exp(-\lambda e_{\max} X'))^2$ and so $Y' > Y$. This is the flipside of the previous result. When obtaining a third signal is almost as valuable as finding a second signal, the search incentives for each uninformed group member do not decrease by much, but the presence of an additional team member who is searching for information makes it more attractive for an informed agent to delay a decision.

7.3   Alternative Mechanisms

Our model captures settings in which there are typically severe restrictions on the types of mechanisms and contracts that are available. There is a broad range of assumptions one can make about what can be contracted on which can help in part to address the incentive distortions of our model. However, our analysis has identified a strong tension between incentives for effort provision and incentives for information revelation. By fixing the incentives in one dimension, the incentives in the other dimension may be reduced. The incentives for an agent to reveal information in our model are determined by the trade-off between the benefit of waiting and the associated costs of delay, or more formally

$$\lambda \tilde{e}^*(t) \phi(t) \alpha_2 \gtrless \delta.$$

All else equal the higher equilibrium effort $\tilde{e}^*(t)$ of the other agent when uninformed, the greater are the incentives to withhold information for an informed agent. A lesson we draw from this is that bonuses or reputational rewards to bringing information to the table induce team members to exert more effort, but do not necessarily result in faster decisions. They may simply shift the source of inefficiency from one of limited effort provision to limited information sharing. Consider a bonus b paid to any team member who possesses information at the time a decision is taken. This bonus would increase each team member's incentives to exert effort and thus decreases the belief

$\bar{\phi}_b < \bar{\phi}$ which makes a member indifferent about how much effort to exert,

$$\lambda \left[\bar{\phi}_b \alpha_1 + \left(1 - \bar{\phi}_b\right) \alpha_2 + b \right] = c.$$

Hence, the longest time that maximum effort can be induced in equilibrium increases,

$$X_b = \frac{-\log \bar{\phi}_b}{\lambda e_{\max}} > X.$$

Since more information is expected to be acquired when deadlines loom large, team members are more willing to delay a decision. In equilibrium they stop disclosing information earlier on,

$$Y_b = \frac{\left(1 - \bar{\phi}_b\right) \alpha_2}{\delta} > Y.$$

A second lesson is that the ability of team members not to disclose their information may be beneficial for welfare since it improves their incentives to become informed. Communication frictions or organizational hierarchies that inhibit communication at the time of decision may induce the disclosure of information immediately after acquisition, but thus may well decrease the incentives for search. Consider a situation where team members take decisions unilaterally and can only send information, but not ask for information due to communication frictions. That is, when taking a decision, a team member cannot consult her partner and is limited to the information previously revealed. Team members will disclose information immediately to avoid that information ends up being unused. However, just like with public disclosure rules, this reduces the incentives to become informed. The analysis is identical to the public information case analyzed earlier. As we showed before this is not necessarily welfare-improving and with an optimally set deadline is welfare-decreasing.

To guarantee a welfare increase, contracts or mechanisms should increase incentives in the one dimension, without hurting incentives in the other dimension. One type of contract or mechanism that may be reasonable in our setting is a payment that depends on the order that relevant information is revealed or that simply depends on time. An example is a decreasing payment (increasing punishment) to the agents depending on when the decision is taken. Essentially, this type of scheme will increase the delay cost δ. If we take the case of a long deadline then the free-riding effort level is $\frac{\delta}{c}$. A relatively simple way to obtain maximum effort and immediate information revelation would be to set the rate of decrease (increase) of a reward (punishment) for making a decision at a particular time equal to $ce_{\max} - \delta$. The effective discount rate an agent then faces is $ce_{\max}$. Hence, this contract fixes the free-riding effort at $e_{\max}$ and information is revealed immediately as $ce_{\max} > \lambda e_{\max} \alpha_2$.[17]

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[17]However, in implementing this type of scheme one must also check that agents still have an incentive to search for information at all. This is satisfied in our original model by the assumption that $\alpha_1 > \frac{c}{\lambda} + \frac{\delta}{\lambda e_{\max}}$. In adjusting the effective delay cost this constraint may no longer hold and agents will simply call an immediate decision. If $\alpha_1 > \frac{2c}{\lambda}$, this will not occur.

7.4 Moderator

A clear inefficiency due to the private nature of information occurs when both players have acquired a signal prior to the deadline and thus have stopped searching for additional information, but the decision is nonetheless delayed as neither player shares their information. There are potential gains from the introduction of a moderator who cannot exert effort (or whose effort decision is unaffected by any information found by the other agents), but is able to facilitate communication between the team members. The role of this moderator would be to avoid situations in which both team members acquired a signal, but neither is willing to reveal it to the other agent.

We previously showed that there is a unique optimal deadline $T = X$ which maximizes the ex-ante welfare of the players. It is straightforward to show that for this optimal deadline ex-ante welfare can be further increased by introducing a moderator to whom information can be revealed. The moderator will choose when to call a decision, maximizing her private value of the decision accounting for the delay cost. Team members are willing to disclose their information to the moderator, since the moderator will call a decision when both players have revealed their information, but will not call a decision beforehand. The reason is that the expected informational gain for the moderator when an uninformed partner exerts maximum effort, outweighs the cost of delaying the decision. In the game of length X without a moderator, uninformed players are exerting maximum effort throughout in equilibrium. With a moderator, the incentive to exert effort for an uninformed player is even higher because finding and revealing information may induce the moderator to call a decision before the deadline and thus reduces the expected decision time.[18] Notice that a benevolent moderator who tries to maximize the social surplus, may not convince team members to reveal their information. If it is socially inefficient to search for a second signal, the benevolent moderator will call a decision after only one team member discloses her information. Hence, informed team members are unwilling to reveal their information. A privately motivated moderator is therefore more effective at inducing information revelation in our setting.

8 Conclusion

In private and public organizations, teams are often allocated the dual task of finding and taking a decision. In this paper we have investigated the link between the incentive to search and the incentive to share decision-relevant information. One clear lesson that emerges from our analysis is that team members are reluctant to disclose information that undermines the incentives of fellow team members to search for further information. As a result, although a strict deadline provides strong incentives for agents to gather information, it also mutes the incentives to reveal information

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[18]In the presence of a moderator, the optimal deadline is no longer equal to X. Without a deadline, an equilibrium exists in which both players exert maximum effort from the start until they acquire information which they immediately reveal to the moderator. The moderator calls a decision only when both players have acquired information. The option to free-ride on the information acquisition efforts of the other agent is effectively eliminated if a moderator is committed not to take a decision until both agents have each found a piece of information. Surprisingly, in the presence of a moderator, an approaching deadline could undermine incentives to exert maximum effort, since the deadline provides an alternative way to end the game.

and to make a fast decision. In this light it may therefore not be too surprising that strict deadlines may sometimes be counterproductive when immediate decisions are required, in particular in settings where "Parkison's Law" applies. In our setting the optimal deadline strikes a balance between providing strong incentives to search for information and limiting delay. Furthermore, we have shown that mutual monitoring in teams is not a panacea to solving incentive problems. In fact, the non-observability of information in conjunction with joint control of the length of the deadline is precisely what allows agents to circumvent the moral hazard in teams problem. Finally, we emphasized that when structuring group decision making processes one has to be careful that features that sharpen search incentives may blunt disclosure incentives and vice versa.

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A Appendix: Proofs

Proof of Proposition 1.

We consider the three different cases separately.

i) Case 1: $T \leq X$.

We start by writing out the implied continuation values of informed and uninformed agents on the equilibrium path for the proposed equilibrium strategies. At the deadline, $t = T$, these are equal to the expected value of a decision at that time,

$$
\begin{aligned}
V^I(T) &= V_0 + \alpha_1 + (1 - \exp(-\lambda e_{\max} T)) \alpha_2, \\
V^U(T) &= V_0 + (1 - \exp(-\lambda e_{\max} T)) \alpha_1.
\end{aligned}
$$

For any $t \in [0, T]$, the continuation values are

$$
\begin{aligned}
V^I(t) &= V^I(T) - \delta(T - t), \\
V^U(t) &= [1 - \exp(-\lambda e_{\max}(T - t))] V^I(T) + \exp(-\lambda e_{\max}(T - t)) V^U(T) \\
&\quad - \frac{c}{\lambda} [1 - \exp(-\lambda e_{\max}(T - t))] - \delta(T - t).
\end{aligned}
$$

The difference in continuation values at and before the deadline equals

$$
\begin{aligned}
V^I(T) - V^U(T) &= \exp(-\lambda e_{\max} T) \alpha_1 + [1 - \exp(-\lambda e_{\max} T)] \alpha_2, \\
V^I(t) - V^U(t) &= \frac{c}{\lambda} + \exp(-\lambda e_{\max}(T - t)) \left[V^I(T) - V^U(T) - \frac{c}{\lambda} \right].
\end{aligned}
$$

Informed strategy: Check that the informed individual's decision strategy $d^*(t) = 0$ is optimal by noting:

$$
V^I(t) > V_0 + \alpha_1 + (1 - \exp(-\lambda e_{\max} t)) \alpha_2 \text{ for all } t < X,
$$

since $X < Y = \frac{[(1 - \bar{\phi}) \alpha_2]}{\delta}$. The right-hand side above is the value of calling a decision at a time t for an informed player.

Uninformed strategy: We check that the uninformed agent's choice of effort $e^*(t) = e_{\max}$ is optimal by noting that

$$
\begin{aligned}
V^I(t) - V^U(t) - \frac{c}{\lambda} &= \exp(-\lambda e_{\max}[T - t]) \left[\exp(-\lambda e_{\max} T) \alpha_1 + (1 - \exp(-\lambda e_{\max} T)) \alpha_2 - \frac{c}{\lambda} \right] \\
&> 0 \text{ for all } t < T \leq X,
\end{aligned}
$$

since $\phi^*(T) = \exp(-\lambda e_{\max} T) > \bar{\phi}$. We then check that the uninformed individual will not call a decision. Note that because exerting effort is optimal, we have $V^U(t) \geq V^U(T) - \delta(T - t)$, where the right hand side

36

equals the expected utility when not exerting any effort and not calling a decision either. Moreover,

$$
\begin{aligned}
V^U\left(T\right) - \delta\left(T-t\right) &= V_0 + \left(1 - \exp\left(-\lambda e_{\max}T\right)\right)\alpha_1 - \delta\left(T-t\right) \\
&> V_0 + \left(1 - \exp\left(-\lambda e_{\max}t\right)\right)\alpha_1 \text{ for all } t < X,
\end{aligned}
$$

since $X < Y = \frac{\left(1-\bar{\phi}\right)\alpha_2}{\delta}$ and $\alpha_1 > \alpha_2$. The right-hand side of the inequality above is the value of calling a decision at a time t for an uninformed player.

ii) Case 2: $X < T \leq Y$.

We again start by writing out the implied continuation values of informed and uninformed agents on the equilibrium path for the proposed equilibrium strategies. At the deadline, we find

$$
\begin{aligned}
V^I\left(T\right) &= V_0 + \alpha_1 + \left(1 - \bar{\phi}\right)\alpha_2, \\
V^U\left(T\right) &= V_0 + \left(1 - \bar{\phi}\right)\alpha_1.
\end{aligned}
$$

For general $t \in [0,T]$, the continuation values are

$$
V^I\left(t\right) = V^I\left(T\right) - \delta\left(T-t\right),
$$

$$
\begin{aligned}
V^U\left(t\right) = {}&\left(1 - \exp\left(-\lambda\int_t^T e^*\left(s\right)ds\right)\right)V^I\left(T\right) + \exp\left(-\lambda\int_t^T e^*\left(s\right)ds\right)V^U\left(T\right) \\
&- \frac{c}{\lambda}\left(1 - \exp\left(-\lambda\int_t^T e^*\left(s\right)ds\right)\right) - \delta\left(T-t\right).
\end{aligned}
$$

Informed strategy: We check that the informed individual's decision strategy $d^*\left(t\right) = 0$ is optimal by noting that $V^I\left(t\right) \geq V_0 + \alpha_1 + \left(1 - \phi^*\left(t\right)\right)\alpha_2$ when $\phi^*\left(t\right) = \exp\left(-\lambda\int_0^t e^*\left(s\right)ds\right) \geq \frac{\delta}{\alpha_2}\left(T-t\right) + \bar{\phi}$, which is true given the effort strategy specified.

Uninformed strategy: We check that the uninformed agent is indifferent about the level of effort to exert for all $t \in [0,T]$ by noting that for $\phi^*\left(T\right) = \bar{\phi}$:

$$
\begin{aligned}
V^I\left(t\right) - V^U\left(t\right) - \frac{c}{\lambda} &= \exp\left(-\lambda\int_t^T e^*\left(s\right)ds\right)\left[V^I\left(T\right) - V^U\left(T\right) - \frac{c}{\lambda}\right] \\
&= \exp\left(-\lambda\int_t^T e^*\left(s\right)ds\right)\left[\bar{\phi}\alpha_1 + \left(1 - \bar{\phi}\right)\alpha_2 - \frac{c}{\lambda}\right] \\
&= 0.
\end{aligned}
$$

Finally, the argument for the uninformed individual not to call a decision is the same as in Case 1.

iii) Case 3: $T > Y$.

Since for $t \in [0, T-Y]$ immediate decisions are called, the belief is $\phi^*\left(t\right) = 1$ for the subgame starting at $t = T - Y$. Hence, the proof for the strategies being equilibria of that subgame are encompassed in case 2. It remains to show that the strategies specified for $t \in [0, T-Y]$ also constitute an equilibrium. We write out the implied continuation values of informed and uninformed agents on the equilibrium path for the proposed equilibrium strategies for $t \in [0, T-Y]$,

$$
\begin{aligned}
V^I\left(t\right) &= V_0 + \alpha_1, \\
V^U\left(t\right) &= V_0 + \alpha_1 - \frac{c}{\lambda},
\end{aligned}
$$

where the continuation value of the uninformed player follows from

$$V^U(t) = \int_t^{T-Y} \left[V^I(s) - c \int_t^s e^*(r)\,dr - \delta(s-t) \right] 2\lambda e^*(s) \exp(-2\lambda \int_t^s e^*(r)\,dr)\,ds$$

$$+ \exp(-2\lambda \int_t^{T-Y} e^*(r)\,dr) \left[V^U(T-Y) - c \int_t^{T-Y} e^*(r)\,dr - \delta(T-Y-t) \right].$$

Using the fact that $e^*(s) = \frac{\delta}{c}$ and $V^I(s) = V^I(0)$ for $s \in [0, T-Y]$, we can further simplify the expression and find

$$V^U(t) = [1 - \exp(-2\lambda e^*(t)(T-Y-t))]\left(V^I(t) - \frac{c}{\lambda} \right)$$

$$+ \exp(-2\lambda e^*(t)(T-Y-t)) V^U(T-Y).$$

From case 2, we know that

$$V^U(T-Y) = V^I(T-Y) - \frac{c}{\lambda} - \exp\left(-\lambda \int_t^T e^*(s)\,ds \right)\left[\bar{\phi}\alpha_1 + (1-\bar{\phi})\alpha_2 - \frac{c}{\lambda} \right]$$

$$= V^I(T-Y) - \frac{c}{\lambda}.$$

Hence,

$$V^U(t) = V^I(t) - \frac{c}{\lambda} \text{ for } t \in [0, T-Y].$$

Informed strategy: We prove that the informed individual's decision strategy $d^*(t) = call$ is optimal. The payoff from waiting until $\hat{t} < T - Y$ upon becoming informed at time t which we denote by $V^I(\hat{t}|t)$ is:

$$V^I(\hat{t}|t) = \int_t^{\hat{t}} (V_0 + \alpha_1 + \alpha_2 - \delta(s-t))\lambda e^*(s) \exp(-\lambda e^*(s)(s-t))\,ds$$

$$+ \exp(-\lambda e^*(s)(\hat{t}-t))(V_0 + \alpha_1 - \delta(\hat{t}-t)),$$

where $e^*(s) = \frac{\delta}{c}$. Using partial integration to solve the integral, this simplifies to

$$V^I(\hat{t}|t) = V_0 + \alpha_1 + \left[1 - \exp\left(-\lambda \frac{\delta}{c}(\hat{t}-t) \right) \right]\left(\alpha_2 - \frac{c}{\lambda} \right).$$

This is strictly decreasing in $\hat{t}$ since $\frac{c}{\lambda} > \alpha_2$. Conditional on not waiting beyond $T - Y$, the optimal decision time is therefore to call at

$$t = \arg\max_{\hat{t} \geq t} V^I(\hat{t}|t).$$

Moreover, given the results in case 2, the agent will not prefer to wait beyond $T - Y$. Hence, an informed agent optimally makes an immediate decision, $d^*(t) = call$ for $t \leq T - Y$.

Uninformed strategy: Note that the uninformed agent is indifferent about the level of effort to exert for all $t \in [0, T-Y]$ by noting that $V^U(t) = V^I(t) - \frac{c}{\lambda}$. Furthermore, $V^I(t) - \frac{c}{\lambda} > V_0$, so it is never optimal for the uninformed agent to call a decision.■

Proof of Proposition 2.
See Web Appendix.■

Proof of Proposition 3.

Denote by t_c the time at which a decision is called. For $T \leq Y$, $Et_c = T$. Hence, $\frac{dEt_c}{dT} > 0$. For $T > Y$,

$$
\begin{aligned}
Et_c &= \int_0^{T-Y} t 2\lambda \frac{\delta}{c} \exp\left(-2\lambda \frac{\delta}{c} t\right) dt + \exp\left(-2\lambda \frac{\delta}{c} (T - Y)\right) T \\
&= \frac{1}{2\lambda \frac{\delta}{c}} \left[1 - \exp\left(-2\lambda \frac{\delta}{c} (T - Y)\right)\right] + \exp\left(-2\lambda \frac{\delta}{c} (T - Y)\right) Y.
\end{aligned}
$$

Hence,

$$
\frac{dEt_c}{dT} = 2\lambda \frac{\delta}{c} \exp\left(-2\lambda \frac{\delta}{c} (T - Y)\right) \left(\frac{1}{2\lambda \frac{\delta}{c}} - Y\right).
$$

It follows that $\frac{dEt_c}{dT} < 0$ if and only if $\frac{1}{2\lambda \frac{\delta}{c}} < Y$. Substituting for $Y = \frac{(1-\bar{\phi})\alpha_2}{\delta}$, this condition can be restated as $\frac{\alpha_2}{\alpha_1} > \frac{\bar{\phi}}{1-\bar{\phi}}$, or, in terms of the primitives of the model, as $\alpha_1 - \frac{c}{\lambda} > \frac{\alpha_1}{\alpha_2}\left(\frac{c}{\lambda} - \alpha_2\right)$. Notice that since $\alpha_1 > \frac{c}{\lambda} > \alpha_2$, by assumption, both sides of the inequality are positive and the relationship is satisfied for $\frac{c}{\lambda}$ close to α_2 and violated for $\frac{c}{\lambda}$ close to α_1.∎

Proof of Proposition 4.

If $T \leq X$, agents only call a decision at the deadline and uninformed agents are exerting maximum effort until the deadline. Hence, the expected number of acquired signals is strictly increasing in T. If $X < T \leq Y$, agents still only call a decision at the deadline and uninformed agents are exerting the same aggregate amount of effort until the deadline. Thus, the expected number of acquired signals is the same for any T in this range. Finally, if $T > Y$ agents may call a decision before the deadline. They exert effort $\frac{\delta}{c}$ and immediately call a decision when informed. Thus, for large T the expected number of signals when a decision is called is approximately equal to 1 and the associated expected decision payoff is $V_0 + \alpha_1$. In contrast, for $T \leq Y$, the team may acquire a second signal, but end up without any signal as well. The associated expected utility loss when a decision is called is equal to

$$
\left(1 - \bar{\phi}\right)^2 (V_0 + \alpha_1 + \alpha_2) + 2\left(1 - \bar{\phi}\right) \bar{\phi} (V_0 + \alpha_1) + \bar{\phi} V_0.
$$

Thus, for $T > Y$, the expected value of the decision is a weighted sum of the expression above and $V_0 + \alpha_1$. By increasing T, weight is shifted from the first to the second term. Hence, the expected precision is increasing in T for $T > Y$ if and only if

$$
\left(1 - \bar{\phi}\right)^2 (V_0 + \alpha_1 + \alpha_2) + 2\left(1 - \bar{\phi}\right) \bar{\phi} (V_0 + \alpha_1) + \bar{\phi} V_0 < V_0 + \alpha_1.
$$

This simplifies to $\frac{\alpha_2}{\alpha_1} < \frac{\bar{\phi}^2}{\left(1-\bar{\phi}\right)^2}$. This condition can be satisfied given the restrictions on the parameters. For, $\alpha_1 = \frac{2c}{\lambda}$ and $\alpha_2 = \frac{c}{2\lambda}$, we have $\frac{\alpha_2}{\alpha_1} = \frac{1}{4}$ and $\bar{\phi} = \frac{\frac{c}{\lambda} - \alpha_2}{\alpha_1 - \alpha_2} = \frac{1}{3}$. Hence, $\frac{\bar{\phi}^2}{\left(1-\bar{\phi}\right)^2} = \frac{1}{4} = \frac{\alpha_2}{\alpha_1}$. Thus, for slightly smaller (larger) values of α_1 the expected value of information when a decision is made will be decreasing (increasing) in T for $T > Y$.[19]∎

Proof of Proposition 5.

Denote by $V_T^U(0)$ the expected utility (when uninformed) at the start $t = 0$ of the game with a deadline at T,

$$
V_T^U(0) = \begin{cases} V_0 + \alpha_1 - \frac{c}{\lambda} \text{ for } T \geq Y, \\ V_0 + \alpha_1 + \left(1 - \bar{\phi}\right) \alpha_2 - \delta T - \frac{c}{\lambda} \text{ for } Y \geq T > X, \\ V_0 + \alpha_1 + \alpha_1 \left(1 - 2\phi^*(T)\right) - (\alpha_1 - \alpha_2)\left(1 - \phi^*(T)\right)^2 + \frac{c}{\lambda}\phi^*(T) - \delta T - \frac{c}{\lambda} \text{ for } 0 \leq T \leq X, \\ \text{where } \phi^*(T) = \exp\left(-\lambda e_{\max} T\right) \text{ and } \bar{\phi} = \exp\left(-\lambda e_{\max} X\right). \end{cases}
$$

The linear decrease in expected utility $V_T^U(0)$ when increasing T for $X < T \leq Y$ is immediate since the aggregate effort and hence expected precision of the decision are independent of the deadline, but the delay

---

[19]Note that $\bar{\phi}_d = \frac{\delta}{\lambda e_{\max}\alpha_2}$, so let δ be small such that $\bar{\phi}_d < \bar{\phi}$ is satisfied.

cost increases with the length of the deadline. For $T \geq Y$, the expected utility is independent of T, hence the result is immediate as well. Now, for $0 \leq T \leq X$, we find that

$$\frac{dV_T^U(0)}{dT} = 2\lambda e_{\max} \left[\alpha_1 \phi^*(T) + \alpha_2 (1 - \phi^*(T)) - \frac{c}{2\lambda}\right] \exp(-\lambda e_{\max} T) - \delta.$$

At $T = X$, we have $\phi^*(T) = \bar{\phi}$. Hence, we find that $\lim_{T \to X} \frac{dV_T^U(0)}{dT} = e_{\max} c \bar{\phi} - \delta$. Moreover, since $e_{\max} \bar{\phi}_d \alpha_2 > \frac{\delta}{\lambda} > \frac{\delta}{c} \alpha_2$, we find $e_{\max} c \bar{\phi}_d > \delta$. By assumption, in the case of small incentives, we have $\bar{\phi} > \bar{\phi}_d$ and thus $\lim_{T \to X} \frac{dV_T^U(0)}{dT} > 0$. Note further that

$$\frac{\partial}{\partial T}\left(\frac{dV_T^U(0)}{dT}\right) = 2\lambda e_{\max} \times$$

$$\left\{-\lambda e_{\max} \phi^*(T) \left[\alpha_1 \phi^*(T) + \alpha_2 (1 - \phi^*(T)) - \frac{c}{2\lambda}\right] - (\lambda e_{\max} \phi^*(T))^2 (\alpha_1 - \alpha_2)\right\} < 0.$$

Hence, $\frac{dV_T^U(0)}{dT} > 0$ for all $T \in [0, X)$. ∎

Proof of Proposition 6.

Since $\lambda \alpha_2 < c$, no player searches for a second signal and hence the game ends as soon as one player becomes informed. At time t, the value of being informed equals

$$V^I(t) = V_0 + \alpha_1.$$

The value of being uninformed at t, if all agents exert maximum effort until the deadline, equals

$$V^U(t) = V_0 + \left[\alpha_1 - \left(\frac{c}{2\lambda} + \frac{\delta}{2\lambda e_{\max}}\right)\right] \{1 - \exp[-2\lambda e_{\max}(T - t)]\}.$$

Notice that $\alpha_1 > \frac{c}{\lambda} > \frac{c}{2\lambda} + \frac{\delta}{2\lambda e_{\max}}$. Hence, $\frac{dV^U(t)}{dt} < 0$. Since $V^I(t)$ is constant, this implies that $V^I(t) - V^U(t)$ is increasing in t. Define Δ by

$$V^I(T - \Delta) - V^U(T - \Delta) = \frac{c}{\lambda}.$$

At any time $t > \max\{T - \Delta, 0\}$, uninformed agents exert maximum effort, since $V^I(t) - V^U(t) > \frac{c}{\lambda}$. Notice that these equilibrium strategies are unique, since for a lower effort level by the other agent, the incentives to exert effort increase further. At any time t before the threshold $T - \Delta$, each agents exerts $e^{pub}(t) = \frac{\delta}{c}$, which makes every agent indifferent about how much effort to exert. This equilibrium strategy corresponds to the first stage in the private information case for long deadlines when informed players disclose their information. The proof for uniqueness of these equilibrium strategies is the same as for the private information case. ∎

Proof of Proposition 7.

For the case of public information, the continuation values at the beginning of the game are given by

$$V_T^{U,pub}(0) = \begin{cases} V_0 + \alpha_1 - \frac{c}{\lambda} & \text{for } T \geq \Delta, \\ V_0 + \left[\alpha_1 - \left(\frac{c}{2\lambda} + \frac{\delta}{2\lambda e_{\max}}\right)\right][1 - \exp(-2\lambda e_{\max} T)] & \text{for } T < \Delta. \end{cases}$$

Together with the results in Proposition 5, this implies the following relations for any $T' > \max\{Y, \Delta\}$:

$$\max_T V_T^U(0) > V_{T'}^U(0) = V_{T'}^{U,pub}(0) = \max_T V_T^{U,pub}(0).$$

Hence, $\max_T V_T^U(0) > \max_T V_T^{U,pub}(0)$. ∎

B   WEB APPENDIX

B.1   Proofs of Corollaries

Proof of Corollary 1.

The condition $(2\lambda\alpha_2 - c)\,e_{\max} < \delta$ implies that it is socially efficient for the team to acquire only a single signal. The efficient precision of the decision is thus bounded above by $V_0 + \alpha_1$. However, for $T \in [X, Y]$, the expected precision of the decision equals $V_0 + \left(1 - \bar{\phi}\right)^2 (\alpha_1 + \alpha_2) + 2\left(1 - \bar{\phi}\right)\bar{\phi}\,(V_0 + \alpha_1)$. When $\frac{\alpha_2}{\alpha_1} > \frac{\bar{\phi}^2}{\left(1-\bar{\phi}\right)^2}$, this exceeds $V_0 + \alpha_1$. $\blacksquare$

Proof of Corollary 2.

This follows immediately from

$$\exp\left(-\lambda e_{\max}X\right) = \bar{\phi} \text{ and } \lambda\left[\bar{\phi}\alpha_1 + \left(1 - \bar{\phi}\right)\alpha_2\right] = c.$$

If either c decreases, α_1 increases or α_2 increases, $\bar{\phi}$ decreases and thus X increases. If $e_{\max}$ increases, clearly X decreases. Finally, using $X = -\frac{1}{\lambda e_{\max}}\log\frac{\frac{c}{\lambda} - \alpha_2}{\alpha_1 - \alpha_2}$, we find that $\frac{dX}{d\lambda} > 0$ if and only if $-\log\bar{\phi} < \frac{c}{\bar{\phi}\lambda} = e_{\max}\alpha_2$. $\blacksquare$

Proof of Corollary 3.

For $T > X$, an equilibrium with private information exists in which $e^*(t) = 0$ for $t < T - X$ and $e^*(t) = e_{\max}$ for $t \geq T - X$. In the unique equilibrium with public information $e^{pub}(t) = e_{\max}$ if and only if $t \geq T - \Delta$. We show that $X > \Delta$ by contradiction. By definition, X is the deadline T solving

$$V_T^I(0) - V_T^U(0) = \frac{c}{\lambda} \Leftrightarrow \alpha_1\phi^*(T) + \alpha_2\left(1 - \phi^*(T)\right) = \frac{c}{\lambda},$$

where $\phi^*(T) = \exp\left(-\lambda e_{\max}T\right)$. Also, by definition, Δ is the deadline T solving

$$V_T^{I,pub}(0) - V_T^{U,pub}(0) = \frac{c}{\lambda} \Leftrightarrow \alpha_1\phi^*(T)^2 + \left(\frac{c}{2\lambda} + \frac{\delta}{2\lambda e_{\max}}\right)\left(1 - \phi^*(T)^2\right) = \frac{c}{\lambda},$$

where we use $\exp\left(-2\lambda e_{\max}T\right) = \phi^*(T)^2$. Since $\phi^*(T)^2 \leq \phi^*(T)$, a necessary condition for $X < \Delta$ is

$$\alpha_2 < \frac{c}{2\lambda} + \frac{\delta}{2\lambda e_{\max}}, \tag{8}$$

which implies that searching for a second signal is socially inefficient. We now show that when this inequality holds, welfare under public information exceeds the welfare under private information for short deadlines,

$$V_T^{U,pub}(0) - V_T^U(0) > 0 \text{ for } T \leq \min\{X, \Delta\}. \tag{9}$$

However, from Proposition 7, we have

$$V_X^U(0) \geq V_\Delta^{U,pub}(0) = \max_T V_T^{U,pub}(0) \geq V_X^{U,pub}(0).$$

Hence, this implies that $X > \Delta$, which is a contradiction. To establish the inequality (9), we use that for $T \leq \Delta$,

$$V_T^{U,pub}(0) = V_0 + \left(\alpha_1 - \frac{c}{2\lambda} - \frac{\delta}{2\lambda e_{\max}}\right)\left(1 - \exp\left(-2\lambda e_{\max}T\right)\right),$$

and for $T < X$,

$$V_T^U(0) = V_0 + \alpha_1\left(1 - \phi^*(T)^2\right) + \alpha_2\left(1 - \phi^*(T)\right)^2 - \frac{c}{\lambda}\left(1 - \phi^*(T)\right) - \delta T.$$

Using $\exp\left(-2\lambda e_{\max}T\right) = \phi^*\left(T\right)^2$, we find that higher welfare is achieved in the public information case if

$$\alpha_2\left(1-\phi^*\left(T\right)\right)^2 - \frac{c}{\lambda}\left(1-\phi^*\left(T\right)\right) - \delta T \leq -\frac{c}{2\lambda}\left(1-\phi^*\left(T\right)^2\right) - \frac{\delta}{2\lambda e_{\max}}\left(1-\phi^*\left(T\right)^2\right).$$

Rearranging, we find

$$\left(\alpha_2 - \frac{c}{2\lambda} - \frac{\delta}{2\lambda e_{\max}}\right)\left(1-\phi^*\left(T\right)\right)^2 \leq \delta\left(T - \frac{1-\phi^*\left(T\right)}{\lambda e_{\max}}\right).$$

The term $\frac{1-\phi^*\left(T\right)}{\lambda e_{\max}}$ corresponds to the expected duration of a game with maximum length T when a decision is called at rate $\lambda e_{\max}$ and is thus smaller than T. Hence, the right-hand side has a positive sign. Moreover, from inequality (8), we know that the left-hand side has a negative sign. This establishes the inequality.∎

Proof of Corollary 4.

Knowing that $T < \Delta$ implies $T < X$ by Corollary **3**, this follows immediately from the second part of the proof of that Corollary.∎

B.2   Simple Model

In this section, we formally state the equilibrium strategies and beliefs for the simple model introduced in Section 3. We also provide a proof for both the described equilibrium and two corollary results discussed in the main text.

Proposition 8 *For $\delta > \lambda e_{\max}\phi\alpha_2$, an informed first player calls a decision, while an uninformed second player exerts $e\left(1,0\right) = e_{\max}$. Otherwise, the unique equilibrium strategies and beliefs are as follows:*

(i) if $\phi \geq \bar{\phi}$, an informed first player does not call a decision, $d\left(\phi,1\right) = 0$. An uninformed second player chooses to exert maximum effort $e\left(\hat{\phi},0\right) = e_{\max}$. Her belief remains unchanged $\hat{\phi} = \phi$.

(ii) if $\phi < \bar{\phi}$, an informed first player calls a decision with positive probability, $d\left(\phi,1\right) = \frac{\bar{\phi}-\phi}{\bar{\phi}\left(1-\phi\right)}$. An uninformed second player chooses to exert effort $e\left(\hat{\phi},0\right) = \bar{e}\left(\phi\right)$. Her belief increases to $\hat{\phi} = \bar{\phi}$.

Proof. If $\delta > \lambda e_{\max}\phi\alpha_2$, the informed player calls a decision for any $e \in [0,e_{\max}]$. If no decision is called, the second player updates her prior belief to $\hat{\phi} = 1$. She exerts maximal effort $e\left(1,0\right) = e_{\max}$, since $\lambda\alpha_1 > c$. This equilibrium is unique.

If $\delta \leq \lambda e_{\max}\phi\alpha_2$, the informed player is willing to delay a decision only if $e \geq \bar{e}\left(\phi\right)$. The second player exerts maximum effort only if $\hat{\phi} \geq \bar{\phi}$. If the first player never calls a decision, the second player's belief remains unchanged $\hat{\phi} = \phi$. We distinguish between two cases.

In the first case with $\phi > \bar{\phi}$, the equilibrium strategies are as described in (i). The uninformed second player exerts maximum effort as $\hat{\phi} = \phi > \bar{\phi}$. The informed first player delays as $e_{\max} \geq \bar{e}\left(\phi\right)$. Moreover, the equilibrium is unique. If the informed player calls a decision with $d\left(\phi,1\right) > 0$, the second player would update her belief to $\hat{\phi} > \phi \left(\geq \bar{\phi}\right)$ and exert maximum effort $e_{\max} \geq \bar{e}\left(\phi\right)$. Hence, the informed player is not willing to call a decision. This constitutes a contradiction. The strategy is thus unique except when $\bar{e}\left(\phi\right) = e_{\max}$. In this case, the first agent is also willing to call a decision, despite the maximum effort of the second agent.

In the second case with $\phi \leq \bar{\phi}$, the equilibrium strategies are as described in (ii).[20] The uninformed second player is willing to exert effort $e\left(\hat{\phi},0\right) = \bar{e}\left(\phi\right)$ as her updated belief equals $\hat{\phi} = \bar{\phi}$ when $d\left(\phi,1\right) = \frac{\bar{\phi}-\phi}{\phi\left(1-\phi\right)} \geq 0$.

---

[20]Notice that in a model where all players start with the same information as in the next section, no player will ever conceal information up to the point that the other player becomes discouraged to exert effort. Hence, in equilibrium, ϕ never drops below $\bar{\phi}_e$.

The informed first player is willing to call a decision with positive probability as $e\left(\hat{\phi}, 1\right) = \bar{e}\left(\phi\right)$. Also this equilibrium is unique. If the second player would exert a lower effort, the informed first player would call a decision. Hence, the second player would update her belief to $\hat{\phi} = 1$, and thus be unwilling to exert low effort. If the first player would exert a higher effort level, the informed first player would delay a decision. Hence, the second player would keep the same belief $\hat{\phi} = \phi < \bar{\phi}$, and thus be unwilling to exert high effort. ∎

Corollary 5 *No equilibrium exists in which informed players always disclose their information when the cost of delay is small.*

Proof. When $\delta \leq \lambda e_{\max}\phi\alpha_2$, the equilibrium described in the Proposition involves the informed agent delaying the decision and thus not disclosing information with positive probability d. ∎

Corollary 6 *The first player's gain from being informed is higher if her information is private rather than public.*

Proof. When information is public, the values of being informed and uninformed are

$$
\begin{aligned}
V^{I,Pub}\left(\phi\right) &= V_0 + \alpha_1 + \left(1 - \phi\right)\alpha_2 \\
V^{U,Pub}\left(\phi\right) &= V_0 + \lambda e_{\max}\phi\alpha_1 + \left(1 - \phi\right)\alpha_1 - \delta.
\end{aligned}
$$

When information is private and $\phi > \bar{\phi}$, the value of being informed increases to

$$
V^{I,Priv}\left(\phi\right) = V_0 + \alpha_1 + \left(1 - \phi\right)\alpha_2 + \lambda e_{\max}\phi\alpha_2 - \delta,
$$

while the value of being uinformed remains unchanged. Hence, $V^{I,Pub}\left(\phi\right) - V^{U,Pub}\left(\phi\right) \leq V^{I,Priv}\left(\phi\right) - V^{U,Priv}\left(\phi\right)$ for $\phi > \bar{\phi}$, since $\lambda e_{\max}\phi\alpha_2 - \delta \geq 0$. When information is private and $\phi \leq \bar{\phi}$, the value of being informed remains unchanged, but the value of being uninformed decreases to

$$
V^{U,Priv} = V_0 - \delta + \lambda\bar{e}\left(\phi\right)\phi\alpha_1 + \left(1 - \phi\right)\alpha_1.
$$

Hence, $V^{I,Pub}\left(\phi\right) - V^{U,Pub}\left(\phi\right) \leq V^{I,Priv}\left(\phi\right) - V^{U,Priv}\left(\phi\right)$ for $\phi \leq \bar{\phi}$, since $\lambda\left[e_{\max} - \bar{e}\left(\phi\right)\right]\phi\alpha_1 \geq 0$. ∎

B.3 Large Search Incentives ($\bar{\phi}_d \geq \bar{\phi}$)

In this section, we describe the case where search incentives are large ($\bar{\phi}_d \geq \bar{\phi}$, see Section 4.1.2) in more detail. We also state the equilibrium strategies and beliefs formally and provide a proof.

We consider deadlines of different length T of which there are four distinct cases. We define two thresholds X_d and Y_d, similar to X and Y, and an additional threshold Z. The threshold X_d solves

$$
\exp\left(-\lambda e_{\max}X_d\right) = \bar{\phi}_d.
$$

The threshold Y_d solves

$$
\left(1 - \bar{\phi}_d\right)\alpha_2 = \delta Y_d.
$$

The characterization of the equilibrium is very similar as before, with the exception of a final stage which lasts up to Z for games with length exceeding X_d. Once the length of the game exceeds X_d and uninformed agents have exerted maximum effort until $t = X_d$, informed players will call decisions at a rate $d^*\left(t\right)$ such that $\left(1 - \bar{\phi}_d\right)d^*\left(t\right) = \lambda e_{\max}$ keeping the belief constant at $\bar{\phi}_d$. The threshold Z is the maximum length of this mixing stage which maintains maximum incentives to exert effort throughout the game,

$$
Z = -\frac{1}{2\lambda e_{\max}}\log\left(-\frac{\left(1 - \bar{\phi}_d\right)\alpha_2 - \frac{ce_{\max} - \delta}{\lambda e_{\max}}}{\bar{\phi}_d 2\alpha_1 + \left(1 - \bar{\phi}_d\right)\alpha_2 - \frac{ce_{\max} + \delta}{\lambda e_{\max}}}\right).
$$

Proposition 9 *If $\bar{\phi}_d \geq \bar{\phi}$, then the equilibrium strategies and beliefs are as follows:*

i) If $T \leq X_d$, any informed player chooses not to call a decision, $d^*(t) = 0$, for all t while any uninformed player chooses to exert maximum effort, $e^*(t) = e_{\max}$, for all t. The agents' beliefs evolve according to $\phi^*(t) = \exp(-\lambda e_{\max} t)$.

ii) If $X_d < T \leq X_d + Z$, any informed player chooses not to call a decision up for $t < X_d$ and to call for a decision at the mixing rate $d^*(t) = \frac{(\lambda e_{\max})^2 \alpha_2}{\lambda e_{\max} \alpha_2 - \delta}$ for $t \geq X_d$. Any uninformed player chooses to exert maximum effort, $e^*(t) = e_{\max}$, for all t. The agents' beliefs evolve according to $\phi^*(t) = \exp(-\lambda e_{\max} t)$ for $t \leq X_d$ and $\phi^*(t) = \bar{\phi}_d$ for $t > X_d$.

iii) If $X_d + Z < T \leq Y_d + Z$, any informed agent chooses not to call a decision for $t < t_d \equiv T - Z$, and to call for a decision at the mixing rate $d^*(t) = \frac{(\lambda e_{\max})^2 \alpha_2}{\lambda e_{\max} \alpha_2 - \delta}$ for $t \geq t_d$. Any uninformed player chooses to exert effort $e^*(t)$ for $0 \leq t < t_d$ which is not uniquely determined, but the effort choice must satisfy the following conditions:

$$\phi^*(t) = \exp\left(-\lambda \int_0^t e^*(s)\, ds\right) \leq \frac{\delta}{\alpha_2}[t - t_0] \ \ for \ t \in [t_0, t_d] \tag{10}$$

and

$$\phi(t_d) = \exp\left(-\lambda \int_{t_0}^{t_d} e^*(s)\, ds\right) = \frac{\delta}{\lambda \alpha_2}, \tag{11}$$

for $t_0 = 0$. For $t \geq t_d$ the uninformed agent exerts maximal effort $e^*(t) = e_{\max}$. The agents' beliefs evolve according to $\phi^*(t) = \exp\left(-\lambda \int_0^t e^*(s)\, ds\right)$ for $0 \leq t \leq t_d$ and $\phi^*(t) = \bar{\phi}_d$ for $t > t_d$.

iv) If $T > Y_d + Z$, any informed player chooses to call for an immediate decision for $t < t_d - Y_d$, not to call a decision, $d^*(t) = 0$ for $t_d - Y_d \leq t < t_d$, and to call for a decision at the mixing rate $d^*(t) = \frac{(\lambda e_{\max})^2 \alpha_2}{\lambda e_{\max} \alpha_2 - \delta}$ for $t \geq t_d$. Any uninformed agent chooses to exert effort $e^*(t) = \frac{\delta}{c}$ for $t < t_d - Y_d$, and to exert effort $e^*(t)$ for $t_d - Y_d \leq t < t_d$ which is not uniquely determined but must satisfy the following conditions (10) and (6) for $t_0 = t_d - Y_d$, and to exert maximal effort $e^*(t) = e_{\max}$ for $t \geq t_d$.

Proof. i) Case 1: $T \leq X_d$

The proof is here exactly the same as for Case 1 in Proposition 1, since $\bar{\phi}_d = \exp(-\lambda e_{\max} X_d) > \bar{\phi}$ in this case with large incentives for search.

ii) **Case 2: $X_d < T \leq X_d + Z$**

We again start by writing out the implied continuation values of informed and uninformed agents on the equilibrium path for the proposed equilibrium strategies.

At the deadline, the continuation values equal

$$\begin{aligned} V^I(T) &= V_0 + \alpha_1 + (1 - \bar{\phi}_d)\alpha_2, \\ V^U(T) &= V_0 + (1 - \bar{\phi}_d)\alpha_1. \end{aligned}$$

For $t \in [X_d, T]$, the continuation values equal

$$V^I(t) = V^I(T)$$

and

$$V^U(t) = \int_t^T \left(V_0 + \alpha_1 + \frac{(1 - \bar{\phi}_d)\alpha_2}{2} - (ce_{\max} + \delta)(s - t)\right) 2\lambda e^{-2\lambda e_{\max}(s-t)} ds$$

$$+ e^{-2\lambda e_{\max}(T-t)}\left(V^U(T) - (ce_{\max} + \delta)(T - t)\right).$$

Note that the 2 in the density function comes from the probability of one of two events occuring, "agent finds information" or "the other agent calls a decision". In this time interval, the rate of information acquisition is the same as the rate at which the other agent is calling decisions. The payoff contains the term $\frac{(1 - \bar{\phi}_d)\alpha_2}{2}$

44

because with $\frac{1}{2}$ probability the agent will find information before the other agent calls a decision in which case the payoff is increased by $\left(1 - \bar{\phi}_d\right) \alpha_2$. The continuation value simplifies further to

$$V^U(t) = V_0 + \alpha_1 + \frac{\left(1 - \bar{\phi}_d\right) \alpha_2}{2} - (ce_{\max} + \delta) \frac{1}{2\lambda e_{\max}}$$
$$- e^{-2\lambda e_{\max}(T-t)} \left[\bar{\phi}_d \alpha_1 + \frac{\left(1 - \bar{\phi}_d\right) \alpha_2}{2} - (ce_{\max} + \delta) \frac{1}{2\lambda e_{\max}} \right].$$

The difference between being informed and uninformed is given by

$$V^I(t) - V^U(t) = \frac{\left(1 - \bar{\phi}_d\right) \alpha_2}{2} + (ce_{\max} + \delta) \frac{1}{2\lambda e_{\max}}$$
$$+ e^{-2\lambda e_{\max}(T-t)} \left[\bar{\phi}_d \alpha_1 + \frac{\left(1 - \bar{\phi}_d\right) \alpha_2}{2} - (ce_{\max} + \delta) \frac{1}{2\lambda e_{\max}} \right].$$

We define Z such that this difference equals exactly $\frac{c}{\lambda}$ when $t = T - Z$. Notice that $\frac{d\left(V^I(t) - V^U(t)\right)}{dt}|_{t=T-Z} > 0$, since $\frac{dV^I(t)}{dt}|_{t=T-Z} = 0$ and $\frac{dV^U(t)}{dt}|_{t=T-Z} < 0$.

For $t \in [0, X_d]$, the continuation values equal

$$V^I(t) = V_0 + \alpha_1 + \left(1 - \bar{\phi}_d\right) \alpha_2 - \delta\left(X_d - t\right),$$
$$V^U(t) = \left(1 - \exp\left(-\lambda e_{\max}\left(X_d - t\right)\right)\right) \left(V^I(X_d) - \frac{c}{\lambda}\right) + \exp\left(-\lambda e_{\max}\left(X_d - t\right)\right) V^U(X_d) - \delta\left(\hat{t} - t\right).$$

The difference between the two equals

$$V^I(t) - V^U(t) = \frac{c}{\lambda} + e^{-\lambda(X_d - t)} \left(V^I(X_d) - V^U(X_d) - \frac{c}{\lambda}\right).$$

Informed strategy: Check first that the informed individual's decision strategy $d^*(t) = 0$ is optimal for $t \in [0, X_d]$ by verifying whether $V^I(t) \geq V_0 + \alpha_1 + (1 - \phi^*(t)) \alpha_2$, where the right hand side equals the payoff from an immediate decision. Rearranging , we get

$$\phi^*(t) - \bar{\phi}_d \geq \frac{\delta}{\alpha_2}\left(X_d - t\right),$$

which holds with equality for $t = X_d$. Furthermore, the derivative of the LHS is strictly less than the RHS, $-\lambda e_{\max} \phi^*(t) < -\frac{\delta}{\alpha_2}$, for $t < X_d$. Hence, the relation holds for $t < X_d$. Check second that the informed individual is indifferent about a decision now versus delaying a decision any amount Δt into the future for $t \in (X_d, T]$, so that she is willing to call a decision at a positive rate. The expected utility of an immediate decision at t equals

$$V(t|t) = V_0 + \alpha_1 + \left(1 - \bar{\phi}_d\right) \alpha_2.$$

This is exactly the same as the expected utility of waiting until $t + \Delta t$ to call a decision, when $d^*(t) = \frac{\lambda e_{\max}}{1 - \bar{\phi}_d}$ for $t \geq X_d$,

$$V(t + \Delta t|t) = V_0 + \alpha_1 + \int_t^{t+\Delta t} \left(\alpha_2 - \delta(s - t)\right) d^*(s) \left(1 - \phi(s)\right) \exp\left(-\int_t^s d^*(r) \left(1 - \phi^*(r)\right) dr\right) ds$$
$$+ \exp\left(-\int_t^{t+\Delta t} d^*(r) \left(1 - \phi^*(r)\right) dr\right) \left(\left(1 - \bar{\phi}_d\right) \alpha_2 - \delta \Delta t\right).$$

Using that $\phi^*(r) = \bar{\phi}_d$ for $r \geq X_d$, this simplifies further to

$$V(t + \Delta t|t) = V_0 + \alpha_1 + \left(1 - \exp\left(-d^*(t)\left(1 - \bar{\phi}_d\right)\Delta t\right)\right)\left(\alpha_2 - \frac{\delta}{d^*(t)\left(1 - \bar{\phi}_d\right)}\right)$$
$$+ \exp\left(-d^*(t)\left(1 - \bar{\phi}_d\right)\Delta t\right)\left(\left(1 - \bar{\phi}_d\right)\alpha_2\right),$$

where

$$\frac{\delta}{d^*(t)\left(1 - \bar{\phi}_d\right)} = \frac{\delta}{\lambda e_{\max}} = \bar{\phi}_d \alpha_2.$$

Hence, we find indeed that

$$V(t + \Delta t|t) = V_0 + \alpha_1 + \left(1 - \bar{\phi}_d\right)\alpha_2.$$

and thus the mixing strategy is an equilibrium strategy.

Uninformed strategy: Check that the uninformed agent's choice of effort $e^*(t) = e_{\max}$ is optimal by noting that $V^I(t) - V^U(t) > \frac{c}{\lambda}$ for all t provided $T < X_d + Z$ and $V^I(t) - V^U(t) = \frac{c}{\lambda}$ for $t \in [0, X_d]$ when $T = X_d + Z$. The argument that an uninformed agent will not call a decision is again the same as in Case 1 of Proposition 1. The expected utility when following the equilibrium strategy exceeds the expected utility when exerting no effort, but delaying a decision, which exceeds the expected utility of calling a decision immediately.

iii) **Case 3:** $X_d + Z < T \leq Y_d + Z$

Notice first that the subgames for $t \geq T - Z$ are identical to those described above in case 2 for $t \geq X_d$ and the proof is identical. Turning to $t < T - Z$. For $t \in [0, T - Z]$, the continuation value for the informed individual is

$$V^I(t) = V_0 + \alpha_1 + \left(1 - \bar{\phi}_d\right)\alpha_2 - \delta(T - Z - t)$$

and for the uninformed agent

$$V^U(t) = \left(1 - \exp\left(-\lambda \int_t^{T-Z} e^*(s)\,ds\right)\right)V^I(T - Z) + \exp\left(-\lambda \int_t^{T-Z} e^*(s)\,ds\right)V^U(T - Z)$$
$$- \frac{c}{\lambda}\left(1 - \exp\left(-\lambda \int_t^{T-Z} e^*(s)\,ds\right)\right) - \delta(T - Z - t).$$

Hence, the difference equals

$$V^I(t) - V^U(t) = \frac{c}{\lambda} \text{ for } 0 \leq t \leq T - Z.$$

Informed strategy: This corresponds to the proof in Case 2 of Proposition 1. Check that the informed individual's decision strategy $d^*(t) = 0$ is optimal by noting that $V^I(t) \geq V_0 + \alpha_1 + (1 - \phi^*(t))\alpha_2$ when $\phi^*(t) = \exp\left(-\lambda \int_0^t e^*(s)\,ds\right) \geq \bar{\phi}_d + \frac{\delta}{\alpha_2}(X_d - t)$, which is true given the equilibrium effort strategy specified.

Uninformed strategy: Check that the uninformed agent is indifferent about the level of effort to exert for all $t \in [0, T - Z]$, since $V^I(t) - V^U(t) = \frac{c}{\lambda}$. An uninformed agent will not call a decision provided that

$$V^U(t) > V_0 + (1 - \phi^*(t))\alpha_1$$

$\Leftrightarrow$

$$V_0 + \alpha_1 + \left(1 - \bar{\phi}_d\right)\alpha_2 - \frac{c}{\lambda} - \delta(X_d - t) >$$
$$V_0 + \alpha_1 + (1 - \phi^*(t))\alpha_2 - (\phi^*(t)\alpha_1 + (1 - \phi^*(t))\alpha_2).$$

We know from the informed player's strategy that

$$\left(1 - \bar{\phi}_d\right) \alpha_2 - \delta \left(X_d - t\right) \geq \left(1 - \phi^*\left(t\right)\right) \alpha_2.$$

Hence, it remains to show that

$$\phi^*\left(t\right) \alpha_1 + \left(1 - \phi^*\left(t\right)\right) \alpha_2 > \frac{c}{\lambda}$$

which is true because $\bar{\phi}\alpha_1 + \left(1 - \bar{\phi}\right) \alpha_2 = \frac{c}{\lambda}$ and $\phi^*\left(t\right) > \bar{\phi}_d > \bar{\phi}$.

iv) **Case 4:** $T > Y_d + Z$

Here again we note that all subgames starting from $t = T - Y_d + Z$ are encompassed by the proof of Case 3 above, and the continuation values at $t = T - Y_d + Z$ are

$$\begin{aligned} V^I\left(t\right) &= V_0 + \alpha_1 \\ V^U\left(t\right) &= V_0 + \alpha_1 - \frac{c}{\lambda} \end{aligned}$$

which are exactly the same continuation values as in **Case 3** of Proposition 1 for $t = T - Y$. In this case the strategies and the proof for the subgames $t < T - \left(Y_d + Z\right)$ are identical to that for **Case 3** of Proposition 1 for $t < T - Y$. ∎

For $T < X_d$, the equilibrium strategies are exactly like before. For $T \geq X_d$, the marginal value of information at and close to the deadline is strictly greater than $\frac{c}{\lambda}$, unlike in the small incentives case. This also continues to be the case for all longer deadlines. As the length of the game T increases, however, the incentives for effort at a given time decrease. To see this, consider the incentives for effort at $t = 0$ which are given by

$$V^I\left(0\right) - V^U\left(0\right) = \frac{c}{\lambda} + \bar{\phi}_d \left(V^I\left(X_d\right) - V^U\left(X_d\right) - \frac{c}{\lambda}\right).$$

The only part of the expression which changes with T, is $V^U\left(X_d\right)$ since all the other terms above are constants and

$$V^I\left(X_d\right) = V^I\left(T\right) = V_0 + \alpha_1 + \left(1 - \bar{\phi}_d\right) \alpha_2.$$

$V^U\left(X_d\right)$ can be rewritten as

$$V^U\left(X_d\right) = \exp\left(-2\lambda e_{\max}\left(T - X_d\right)\right) V^U\left(T\right)$$
$$+ \left(1 - \exp\left(-2\lambda e_{\max}\left(T - X_d\right)\right)\right) \left[V_0 + \alpha_1 + \frac{\left(1 - \bar{\phi}_d\right) \alpha_2}{2} - \left(c e_{\max} + \delta\right) \frac{1}{2\lambda e_{\max}}\right].$$

This is a weighted sum of the expected payoff conditional on either finding information or the other agent calling a decision prior to the deadline and the payoff from being uninformed at the deadline. Both of these payoffs are independent of T and it is only the relative likelihood of each which is affected by T. The likelihood of being uninformed at the deadline $\exp\left(-2\lambda e_{\max}\left(T - X_d\right)\right)$ decreases in T. Hence, the continuation value of being uninformed at X_d is increasing in T. There exists a deadline $T = X_d + Z$ where $V^I\left(X_d\right) - V^U\left(X_d\right) = \frac{c}{\lambda}$ and an agent is indifferent about exerting effort at $t = 0$, so $V^I\left(0\right) - V^U\left(0\right) = \frac{c}{\lambda}$. For T larger than $X_d + Z$, that is case iii) above, maximal effort by uninformed agents can no longer be sustained throughout the entire game. In equilibrium, an uninformed agent reduces her average effort intensity before $t_d = T - Z$ such that $\phi\left(t_d\right) = \bar{\phi}_d$, while the informed agent fully delays. For T larger than $Y_d + Z$, informed agents will no longer prefer to delay their decision at the beginning of the game, exactly as in the case with small incentives.

B.4 Uniqueness

In this section, we prove uniqueness of our proposed equilibrium. We first provide a more general description of our model and then state a sequence of lemmas that we use to prove uniqueness of our equilibrium.

B.4.1 General Model Description

Define the decision strategy of the uninformed agent by $\mu(t)$ which is the conditional probability of calling an uninformed decision by time t given that the other agent does not call a decision prior to t. Define the decision strategy of the informed agent by $\rho(t)$ as the conditional probability of calling an informed decision by time t given that the other agent does not call a decision prior to t. Define $e(t)$ as the effort strategy of the uninformed agent. Define $\sigma(t)$ as the conditional probability of being uninformed by time t given that the other agent does not call a decision prior to t. Hence, $1 - \sigma(t)$ is the conditional probability of being informed by that time. $\sigma(t)$ changes over time according to

$$\frac{d\sigma}{dt} = -\lambda e(t)(\sigma(t) - \mu(t))$$

The effort strategy of the uninformed agent influences $\frac{d\sigma}{dt}$. The decision strategy of an uninformed agent influences both $\frac{d\sigma}{dt}$ and $\mu(t)$. The decision strategy of the informed agent controls $\rho(t)$. The following relationships between these functions hold

$$\mu(t) + \rho(t) \leq 1$$

and

$$\mu(t) \leq \sigma(t)$$

and

$$\rho(t) \leq 1 - \sigma(t)$$

We will use $\tilde{\ }$ to denote the strategies of the other player. The Bayesian belief $\phi(t)$ at a time t that the other agent is uninformed conditional on no decision being called prior to that time is

$$\phi(t) = \frac{\tilde{\sigma}(t) - \tilde{\mu}(t)}{1 - \tilde{\mu}(t) - \tilde{\rho}(t)}$$

A strategy for an agent maps into a path for $e(t), \sigma(t), \mu(t), \rho(t)$. We restrict our attention to strategies which result in piecewise continuously differentiable functions of $e(t)$, $\sigma(t)$, $\mu(t)$ and $\rho(t)$. Clearly, given the nature of the model $\frac{d\sigma}{dt} \leq 0$ since agents do not lose or forget signals and $\frac{d\mu}{dt}, \frac{d\rho}{dt} \geq 0$ since calling a decision is irreversible. The upper bound on $e(t)$ also ensures that $\sigma(t)$ is continuous.

We have assumed that $\rho(t)$ and $\mu(t)$ are continuous and differentiable at all but a finite number of points. Denote the set of points where the strategy is discontinuous by $\chi_\rho = \{t_1^\rho, ...t_\nu^\rho\}$, $\chi_\mu = \{t_1^\mu, ...t_\nu^\mu\}$, $\tilde{\chi}_\rho = \{\tilde{t}_1^\rho, ...\tilde{t}_\nu^\rho\}$, $\tilde{\chi}_\mu = \{\tilde{t}_1^\mu, ...\tilde{t}_\nu^\mu\}$ and define $\chi = \chi_\rho \cup \chi_\mu$ and $\tilde{\chi} = \tilde{\chi}_\rho \cup \tilde{\chi}_\mu$. Also, define

$$D_\rho(t) = \lim_{s \to t^+} \rho(s) - \lim_{r \to t^-} \rho(r)$$

and

$$D_\mu(t) = \lim_{s \to t^+} \mu(s) - \lim_{r \to t^-} \mu(r)$$

These are non-zero only at points in χ_ρ and χ_μ respectively and represent the probability that a decision is called at that moment conditional on the other agent not calling a decision prior to that time. The objective

function of the agent is:

$$\max_{e(t),\mu(t),\rho(t)} V_0 + \int_0^T \left[\alpha_1 + \left(\frac{1-\tilde{\sigma}(t)-\tilde{\rho}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}\right)\alpha_2\right]\frac{d\rho}{dt}\left[1-\tilde{\mu}(t)-\tilde{\rho}(t)\right]dt$$

$$+\int_0^T \left(\frac{1-\tilde{\sigma}(t)-\tilde{\rho}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}\right)\alpha_1\frac{d\mu}{dt}\left[1-\tilde{\mu}(t)-\tilde{\rho}(t)\right]dt$$

$$+\int_0^T \left[\alpha_1 + \left(\frac{1-\sigma(t)-\rho(t)}{1-\mu(t)-\rho(t)}\right)\alpha_2\right]\frac{d\tilde{\rho}}{dt}\left[1-\mu(t)-\rho(t)\right]dt$$

$$+\int_0^T \left(\frac{1-\sigma(t)-\rho(t)}{1-\mu(t)-\rho(t)}\right)\alpha_1\frac{d\tilde{\mu}}{dt}\left[1-\mu(t)-\rho(t)\right]dt$$

$$+\sum_{t\in\chi\cap\tilde{\chi}}\left[1-\tilde{\mu}(t)-\tilde{\rho}(t)\right]\left\{\begin{array}{c}D_\rho(t)\left[\alpha_1+\left(\frac{1-\tilde{\sigma}(t)-\tilde{\rho}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}\right)\alpha_2\right]\\+D_\mu(t)\left(\frac{1-\tilde{\sigma}(t)-\tilde{\rho}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}\right)\alpha_1\end{array}\right\}$$

$$+\sum_{t\in\chi\cap\tilde{\chi}}\left[1-\mu(t)-\rho(t)-D_\rho(t)-D_\mu(t)\right]\left\{\begin{array}{c}D_{\tilde{\rho}}(t)\left[\alpha_1+\left(\frac{1-\sigma(t)-\rho(t)-D_\rho(t)}{1-\mu(t)-\rho(t)-D_\rho(t)-D_\mu(t)}\right)\alpha_2\right]\\D_{\tilde{\mu}}(t)\left(\frac{1-\sigma(t)-\rho(t)-D_\rho(t)}{1-\mu(t)-\rho(t)-D_\rho(t)-D_\mu(t)}\right)\alpha_1\end{array}\right\}$$

$$+\sum_{t\in\chi\setminus\tilde{\chi}}\left[1-\tilde{\mu}(t)-\tilde{\rho}(t)\right]\left\{D_\rho(t)\left[\alpha_1+\left(\frac{1-\tilde{\sigma}(t)-\tilde{\rho}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}\right)\alpha_2\right]+D_\mu(t)\left(\frac{1-\tilde{\sigma}(t)-\tilde{\rho}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}\right)\alpha_1\right\}$$

$$+\sum_{t\in\tilde{\chi}\setminus\chi}\left[1-\mu(t)-\rho(t)\right]\left\{D_{\tilde{\rho}}(t)\left[\alpha_1+\left(\frac{1-\sigma(t)-\rho(t)}{1-\mu(t)-\rho(t)}\right)\alpha_2\right]+D_{\tilde{\mu}}(t)\left(\frac{1-\sigma(t)-\rho(t)}{1-\mu(t)-\rho(t)}\right)\alpha_1\right\}$$

$$+(1-\mu(T)-\rho(T))(1-\tilde{\mu}(T)-\tilde{\rho}(T))\left[\alpha_1+\left(\frac{1-\tilde{\sigma}(T)-\tilde{\rho}(T)}{1-\tilde{\mu}(T)-\tilde{\rho}(T)}\right)\alpha_2\right]$$

$$-\delta T(1-\mu(T)-\rho(T))(1-\tilde{\mu}(T)-\tilde{\rho}(T))$$

$$-\int_0^T \delta(1-\mu(t)-\rho(t))(1-\tilde{\mu}(t)-\tilde{\rho}(t))\,dt$$

$$-c(1-\tilde{\mu}(T)-\tilde{\rho}(T))(\sigma(T)-\mu(T))\int_0^T e(t)\,dt$$

$$-c\int_0^T e(t)(1-\tilde{\mu}(t)-\tilde{\rho}(t))(\sigma(t)-\mu(t))\,dt$$

$$\begin{aligned}s.t.\ \rho(t) &\leq 1-\sigma(t),\ \mu(t)\leq\sigma(t)\\ \sigma(0) &= 1,\ \frac{d\sigma}{dt}=-\lambda e(t)(\sigma(t)-\mu(t))\end{aligned}$$

where $\tilde{\mu}(t),\tilde{\rho}(t),\tilde{\sigma}(t)$ denote the strategy of the other player. To simplify notation we will continue using the definition of $\phi(t)=\frac{\tilde{\sigma}(t)-\tilde{\mu}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}$.

We are interested in perfect Bayesian equilibria of the model so strategies must be an equilibrium for all subgames starting at each time t. We describe these by writing out the problem in terms of continuation values for the informed $V^I(t)$ and uninformed agent $V^U(t)$ upon reaching time t. Also, write $\tilde{\rho}(s|t)\ \tilde{\mu}(s|t)$ for the perceived probabilities that an informed and uninformed agent calls a decision at $s\geq t$ given the agent is at t. At on-equilibrium times these are $\tilde{\rho}(s|t)=\frac{\tilde{\rho}(s)-\tilde{\rho}(t)}{1-\tilde{\rho}(t)}$ and $\tilde{\mu}(s|t)=\frac{\tilde{\mu}(s)-\tilde{\mu}(t)}{1-\tilde{\mu}(t)}$ however if an off-equilibrium time is reached then this is no longer the case and separate equilibrium strategies and beliefs must be specified for these subgames. As we will show below the set of symmetric perfect Bayesian equilibria we focus on involve all times being reached with positive probability.

The continuation value from being informed is just the payoff from implementing the optimal stopping

policy $\widehat{t}_\rho$ from that moment forward:

$$V^I(t) = \max_{\widehat{t}\in[t,T]} V_0 + \alpha_1 + \int_t^{\widehat{t}} (\alpha_2 - \delta(r-t)) \frac{d\tilde{\rho}(r|t)}{dt} dr + \int_t^{\widehat{t}} (-\delta(r-t)) \frac{d\tilde{\mu}(r|t)}{dt} dr \qquad (12)$$

$$+ \sum_{\substack{r\in\tilde{\chi} \\ t<r<\widehat{t}}} D_{\tilde{\rho}}(r|t)(\alpha_2 - \delta(r-t)) + D_{\tilde{\mu}}(r|t)(-\delta(r-t))$$

$$+ \left(1 - \tilde{\mu}\left(\widehat{t}|t\right) - \check{\rho}\left(\widehat{t}|t\right)\right)\left(\left(1 - \phi^*\left(\widehat{t}|t\right)\right)\alpha_2 - \delta\left[\widehat{t}-t\right]\right)$$

where $\widehat{t}_\rho^*(t)$ is the optimizer or set of optimizers of equation 12. The decision strategy is optimal provided that:

$$\lim_{s\to r+} \rho^*(s|t) = \sigma(r) \text{ if } r = \widehat{t}_\rho^*(t)$$

$$\frac{d\rho^*(r|t)}{dt} \geq 0 \text{ or } D_{\rho^*}(r|t) \geq 0 \text{ if } r \in \widehat{t}_\rho^*(t)$$

$$\frac{d\rho^*(r|t)}{dt} = 0 \text{ if } r \notin \widehat{t}_\rho^*(t)$$

and these conditions ensure ρ satisfies the adding up constraint:

$$\int_{\widehat{t}_\rho^*(t)} \frac{d\rho^*(t)}{dt} dt = \sigma\left(\max\left(\widehat{t}^*(t)\right)\right) - \rho(t) + (\sigma(T) - \rho(T)) \times \mathbf{1}\left(\max\left(\widehat{t}^*(t)\right) = T\right)$$

The payoff from being uninformed is defined by a joint effort and stopping problem given by:

$$V^U(t) = \max_{\substack{\widehat{t}\in[t,T] \\ e(r|t) \text{ for } r\in[t,\widehat{t}]}} V_0 + \int_t^{\widehat{t}} \left[\alpha_1 - \delta(r-t) - c\int_t^r e(w|t)dw\right] \frac{d\tilde{\rho}(r|t)}{dt} \left[1 - \exp\left(-\lambda\int_t^r e(w|t)dw\right)\right] dr$$

$$+ \int_t^{\widehat{t}} \left[-\delta(r-t) - c\int_t^r e(w|t)dw\right] \frac{d\tilde{\mu}(r)}{dt} \left[1 - \exp\left(-\lambda\int_t^r e(w|t)dw\right)\right] dr$$

$$+ \int_t^{\widehat{t}} \left[V^I(t) - \delta(r-t) - c\int_t^r e(w|t)dw\right] \lambda e(r) \exp\left(-\lambda\int_t^r e(w|t)dw\right)(1 - \tilde{\mu}(r|t) - \check{\rho}(r|t)) dr$$

$$\text{(13)}$$

$$+ \sum_{\substack{r\in\tilde{\chi}\backslash\chi \\ t<r<\widehat{t}_\mu(t)}} \frac{1 - \mu(r) - \sigma(r)}{1 - \mu(t) - \sigma(t)} \left[\begin{array}{l} D_{\tilde{\rho}}(r|t)\left(\alpha_1 - \delta(r-t) - c\int_t^r e(w|t)dw\right) \\ +D_{\tilde{\mu}}(r|t)\left(-\delta(r-t) - c\int_t^r e(w|t)dw\right) \end{array}\right]$$

$$+ \left(1 - \exp\left(-\int_t^{\widehat{t}} \lambda e(r)dr\right)\right)(1 - \tilde{\mu}(\widehat{t}|t) - \check{\rho}(\widehat{t}|t))\left[\left(1 - \phi^*(\widehat{t}|t)\right)\alpha_1 - \delta(\widehat{t}-t) - c\int_t^{\widehat{t}} e(r|t)dr\right]$$

where $\widehat{t}_\mu^*(t)$ is the optimizer or set of optimizers of equation 13. The condition for the effort strategy profile to be an equilibrium satisfies

$$e^*(t) = \arg\max_{e\in[0,1]} \lambda e\left(V^I(t) - V^U(t)\right) - ce$$

and the decision strategy is an equilibrium provided that

$$\lim_{s \to r^+} \mu^*\left(s|t\right) = 1 - \sigma\left(r\right) \text{ if } r = \hat{t}_\mu^*\left(t\right)$$

$$\frac{d\mu^*\left(r|t\right)}{dt} \geq 0 \text{ or } D_{\rho^*}\left(r|t\right) \geq 0 \text{ if } r \in \hat{t}_\mu^*\left(t\right)$$

$$\frac{d\mu^*\left(r|t\right)}{dt} = 0 \text{ if } r \notin \hat{t}_\mu^*\left(t\right)$$

where $\hat{t}^*\left(t\right)$ solves (13) the uninformed agent's effort and stopping problem. These conditions also ensure that it satisfies the adding up constraint

$$\int_{\hat{t}_\mu^*(t)} \frac{d\mu^*\left(t\right)}{dt} dt = 1 - \sigma\left(\max\left(\hat{t}_\mu^*\left(t\right)\right)\right) - \mu\left(t\right) + \left(1 - \sigma\left(T\right) - \mu\left(T\right)\right) \times \mathbf{1}\left(\max\left(\hat{t}_\mu^*\left(t\right)\right) = T\right)$$

A symmetric perfect Bayesian equilibrium may be described by a tuple $\left(e^*\left(t\right), \rho^*\left(t\right), \mu^*\left(t\right), \phi^*\left(t\right)\right)$ if $\rho^*\left(t\right) + \mu^*\left(t\right) < 1$ for all $t < T$, where $\phi^*\left(t\right)$ is the Bayesian belief an agent has at time t that the other agent is uninformed conditional on no decision being called prior to that time. If $\exists t' < T : \rho^*\left(t'\right) + \mu^*\left(t'\right) = 1$ then it must also include off-equilibrium strategies and beliefs $\left(e^*\left(r|t\right), \rho^*\left(r|t\right), \mu^*\left(r|t\right), \phi^*\left(r|t\right)\right)$ for all times t where $\rho^*\left(t\right) + \mu^*\left(t\right) = 1$ which themselves are equilibria of those subgames, where $\phi^*\left(r|t\right)$ is the Bayesian belief an agent has at time r that the other agent is uninformed conditional on no decision being called prior to that time in a subgame starting at time t. We now rule out some types of decision strategies at on-equilibrium times by the uninformed agent. The following lemma rules out a continuously increasing $\mu^*\left(t\right)$.

Lemma 3 $\nexists \mu^*\left(t\right), r > 0, \varepsilon > 0 : \frac{d\mu^*(t)}{dt} > 0$ for $t \in \left[r - \varepsilon, r\right]$.

Proof. Suppose not and $\exists \mu^*\left(t\right) : \frac{d\mu^*(t)}{dt} > 0$ for $t \in \left[r - \varepsilon, r\right]$. Then

$$\rho^*\left(t\right) = \sigma^*\left(t\right) \text{ for } t \in \left(r - \varepsilon, r\right)$$

and hence

$$\phi^*\left(t\right) = 1 \text{ for } t \in \left(r - \varepsilon, r\right)$$

if not then $\exists r' > t$ such that

$$V_0 + \alpha_1 + \left(1 - \phi^*\left(t\right)\right)\alpha_2 \leq V_0 + \alpha_1 + \left(1 - \tilde{\mu}^*\left(r'|t\right) - \tilde{\rho}^*\left(r'|t\right)\right)\left[\left(1 - \phi^*\left(r'\right)\right)\alpha_2 - \delta\left(r' - t\right)\right]$$

$$- \delta \left[\int_t^{r'} \left(y - t\right)\left[\frac{d\tilde{\mu}^*\left(y|t\right)}{dy} + \frac{d\tilde{\rho}^*\left(y|t\right)}{dy}\right] dy + \sum_{\substack{y \in \tilde{\chi} \\ t < y < r'}} \left(y - t\right)\left[D_{\tilde{\mu}}\left(y|t\right) + D_{\tilde{\rho}}\left(y|t\right)\right] \right]$$

$$+ \alpha_2 \left[\int_t^{r'} \frac{d\tilde{\rho}^*\left(r'|t\right)}{dy} dy + \sum_{\substack{y \in \tilde{\chi} \\ t < y < r'}} D_{\tilde{\rho}}\left(y|t\right) \right]$$

This can be rewritten as

$$\delta \left[\int_t^{r'} (y-t) \left[\frac{d\tilde{\mu}^*(y|t)}{dy} + \frac{d\tilde{\rho}^*(y|t)}{dy} \right] dy + \sum_{\substack{y \in \tilde{\chi} \\ t < y < r'}} [D_{\tilde{\mu}}(y|t) + D_{\tilde{\rho}}(y|t)] (-\delta(y-t)) + (r'-t)(1 - \tilde{\mu}^*(r'|t)) \right]$$

$$\leq \alpha_2 \left[(1 - \tilde{\mu}^*(r'|t) - \tilde{\rho}^*(r'|t))(1 - \phi^*(r')) - (1 - \phi^*(t)) + \int_t^{r'} \frac{d\tilde{\rho}^*(r'|t)}{dy} dy + \sum_{\substack{y \in \tilde{\chi} \\ t < y < r'}} D_{\tilde{\rho}}(y|t) \right].$$

However, if this inequality holds then an uninformed agent could do strictly better by delaying until r' since the comparison of payoffs would result in the same expression except with α_1 replacing α_2. The inequality would then be strict and this would be a contradiction of the uninformed agent mixing at t. Hence, $\phi^*(t) = 1$ for $t \in (r - \varepsilon, r)$. However, if $\phi^*(t) = 1$ then an uninformed agent can do strictly better by delaying and putting in effort $e_{\max}$ over a period of time since

$$\lambda e_{\max} \alpha_1 > c e_{\max} + \delta$$

by assumption. ∎

The following lemma rules out a jump in the decision function $\mu^*(t)$ at a time on the equilibrium path as long as that jump does not occur when both types, informed and uninformed, call a decision with certainty at that instant.

Lemma 4 $\not\exists \mu^*(t), 0 < s < T : D_{\mu^*}(s) > 0$ and $\mu^*(s) + \rho^*(s) < 1$

Proof. Suppose not and $\exists s : D_{\mu^*}(s) > 0$ and $\mu^*(s) + \rho^*(s) < 1$. As above, this implies

$$\lim_{r \to s^+} \phi^*(r) = 1$$

by the same reasoning as before. Hence, an uninformed agent at s can do better than calling an immediate decision by delaying and putting in effort since

$$\lambda e_{\max} \alpha_1 > c e_{\max} + \delta$$

by assumption. ∎

The only equilibria involving $D_{\mu^*}(s) > 0$ also have $\mu^*(s) + \rho^*(s) = 1$ whereby beliefs at times later than s are off the equilibrium path. In this case it may be possible to support uninformed agents calling a decision with appropriately specified off-equilibrium-path beliefs. However, we will exclude this type of equilibrium as we feel for all intents and purposes that it is equivalent to imposing a deadline at time s. We thus continue the analysis under the assumption that $\mu^*(t) = 0$ for all t. This also means that all times are reached on the equilibrium path. The following lemma rules out jumps in the decision function of the informed type $\rho^*(t)$.

Lemma 5 $\not\exists \rho^*(t), 0 < \hat{t} < T : D_{\rho^*}(\hat{t}) > 0$.

Proof. We proceed with a proof by contradiction. Say there is an equilibrium with a mass point at a time $\hat{t}$ where a mass of $D_{\rho^*}(\hat{t}) = (1 - \phi^*(\hat{t}))\beta > 0$ decisions are called. For this to be the case then $\phi < 1$. If $\phi^* = 1$ $\rho^*(t) = \sigma^*(t)$ informed agents may only call decision at the rate at which uninformed agents are becoming informed. Consider $\lim_{t \to \hat{t}^-} e^*(t)$ and $\lim_{t \to \hat{t}^+} e^*(t)$. For there to be a mass point the following conditions need to hold for an agent not to call an earlier or later decision.

$$\lim_{t \to \hat{t}^-} \lambda e^*(t) \alpha_2 (1 - \phi^*(t)) > \delta - \varepsilon \text{ for any } \varepsilon > 0$$

and

$$\lim_{t \to t\hat{\ }+} \lambda e^* (t) \alpha_2 (1 - \phi^* (t)) < \delta + \varepsilon \text{ for any } \varepsilon > 0$$

The first of these inequalities implies that an informed agent will be willing not to call a decision in the neighborhood immediately prior to $t\hat{\ }$. The second guarantees that an informed agent cannot do better by waiting at time $t\hat{\ }$. Also note that $\lim_{t \to t\hat{\ }+} \phi^* (t\hat{\ }) > \lim_{t \to t\hat{\ }-} \phi^* (t\hat{\ })$ due to the mass point. This implies there is a discontinuous change in the effort level at $t\hat{\ }$ if the above two conditions are to be satisfied. We show that this cannot be maintained in equilibrium. We can write the continuation value from being uninformed at time $t = t\hat{\ } - \Delta t$ as:

$$V^U (t\hat{\ } - \Delta t) = \int_{t\hat{\ } - \Delta t}^{t\hat{\ }-} \left(\frac{\frac{d\sigma^*(s|t\hat{\ }-\Delta t)}{ds}}{1 - \sigma^* (s|t\hat{\ } - \Delta t)} V^I (t) + (V_0 + \alpha_1) \frac{\frac{d\tilde{\rho}^*(s|t\hat{\ }-\Delta t)}{ds}}{1 - \tilde{\rho}^* (s|t\hat{\ } - \Delta t)} - ce^* (s) - \delta \right) \times$$
$$(1 - \sigma^* (s|t\hat{\ } - \Delta t)) (1 - \tilde{\rho}^* (s|t\hat{\ } - \Delta t)) ds$$

$$+ (1 - \sigma^* (t\hat{\ }|t\hat{\ } - \Delta t)) D_{\tilde{\rho}^*} (t\hat{\ }|t\hat{\ } - \Delta t) \left(V_0 + \alpha_1 - \delta \Delta t - c \int_{t\hat{\ } - \Delta t}^{t\hat{\ }} e^* (s) ds \right)$$

$$\left(\begin{array}{c} \int_{t\hat{\ }+}^{t\hat{\ }+\Delta t} \left(\frac{\frac{d\sigma^*(s|t\hat{\ }-\Delta t)}{ds}}{1-\sigma^*(s|t\hat{\ }-\Delta t)} V^I (t) + (V_0 + \alpha_1) \frac{\frac{d\tilde{\rho}^*(s|t\hat{\ }-\Delta t)}{ds}}{1-\tilde{\rho}^*(s|t\hat{\ }-\Delta t)} - ce^* (s) - \delta \right) \times \\ (1 - \sigma^* (s|t\hat{\ } - \Delta t)) (1 - \tilde{\rho}^* (s|t\hat{\ } - \Delta t)) ds \end{array} \right)$$
$$+ (1 - \sigma^* (t\hat{\ } + \Delta t|t\hat{\ } - \Delta t)) (1 - \tilde{\rho}^* (t\hat{\ } + \Delta t|t\hat{\ } - \Delta t)) V^U (t\hat{\ } + \Delta t)$$

Where Δt may always be chosen small enough such that there are no other points of discontinuity of $\rho^* (t)$ for $t \in [t\hat{\ } - \Delta t, t\hat{\ } + \Delta t]$ other than at $t = t\hat{\ }$. Now consider moving a unit of effort from $t\hat{\ } - \varepsilon$ to $t\hat{\ } + \varepsilon$ by augmenting the strategy $e^* (t)$ as follows:

$$e^{**} (t) = e^* (t) - \varepsilon \text{ for } t \in [t\hat{\ } - \Delta t, t\hat{\ })$$
$$e^{**} (t) = e^* (t) + \varepsilon \text{ for } t \in [t\hat{\ }, t\hat{\ } + \Delta t]$$

The strategies are piecewise continuous so we can always find a Δt such that they are continuous over the intervals $[t - \Delta t, t)$ and $(t, t + \Delta t]$. Using a Taylor series expansion

$$\lim_{\Delta t \to 0} \frac{V^U (t\hat{\ } - \Delta t|e^{**}) - V^U (t\hat{\ } - \Delta t|e^*)}{\Delta t} = -\varepsilon \left(\lambda \left(\lim_{t \to t\hat{\ }-} V^I (t) \right) - c \right)$$
$$+ \lambda \varepsilon D_{\rho^*} (t\hat{\ }|t\hat{\ } - \Delta t) (V_0 + \alpha_1)$$
$$+ \varepsilon (1 - D_{\rho^*} (t\hat{\ }|t\hat{\ } - \Delta t)) \left(\lambda \left(\lim_{t \to t\hat{\ }+} V^I (t) \right) - c \right) - O (\Delta t)$$

$$\lim_{\Delta t \to 0} \frac{V^U (t\hat{\ } - \Delta t|e^{**}) - V^U (t\hat{\ } - \Delta t|e^*)}{\Delta t} \geq -\varepsilon \left(\lambda (V_0 + \alpha_1 + (1 - \phi (t\hat{\ })) \alpha_2) - c \right)$$
$$+ \lambda \varepsilon D_{\rho^*} (t\hat{\ }|t\hat{\ } - \Delta t) (V_0 + \alpha_1)$$
$$+ \varepsilon (1 - D_{\rho^*} (t\hat{\ }|t\hat{\ } - \Delta t)) (\lambda (V_0 + \alpha_1 + (1 - \phi (t\hat{\ }) - D_{\rho^*} (t\hat{\ }|t\hat{\ } - \Delta t)) \alpha_2) - c) - O (\Delta t)$$

Define the right-hand side by R then

$$R = -D_{\rho^*} (t\hat{\ }|t\hat{\ } - \Delta t) \lambda \alpha_2 \varepsilon + D_{\rho^*} (t\hat{\ }|t\hat{\ } - \Delta t) c\varepsilon - O (\Delta t)$$
$$= \varepsilon D_{\rho^*} (t\hat{\ }|t\hat{\ } - \Delta t) (c - \lambda \alpha_2) - O (\Delta t)$$
$$> 0$$

Hence, there exists $\Delta t > 0$ such that this change in strategy is profitable which is a contradiction that the original effort $e^* (t)$ is optimal and can be part of an equilibrium. ∎

This along with the earlier lemmas that uninformed individuals do not call decisions implies that $\phi^*(t)$, $\rho^*(t)$ and $\sigma^*(t)$ are all continuous.

Lemma 6 $V^I(t)$ *is continuous.*

Proof. The continuity of $\phi^*(t)$, $\rho^*(t)$ and $\sigma^*(t)$ ensures that

$$f\left(t,\widehat{t}\right) = V_0 + \alpha_1 + \int_t^{\widehat{t}} (\alpha_2 - \delta(r - t)) \frac{d\tilde{\rho}^*(r|t)}{dt} dr$$

$$+ \left(1 - \tilde{\rho}^*\left(\widehat{t}|t\right)\right)\left(\left(1 - \phi^*\left(\widehat{t}|t\right)\right)\alpha_2 - \delta\left[\widehat{t} - t\right]\right)$$

is continuous in $\widehat{t}$. Hence, $V^I(t) = \max_{\widehat{t} \in [t,T]} f(t)$ is continuous in t by the theorem of the maximum (Berge 1963). ∎

Lemma 7 $V^U(t)$ *is continuous.*

Proof. The continuity of $\phi^*(t)$, $\rho^*(t)$ and $\sigma^*(t)$ ensures that

$$f(t, e(r|t)) = V_0 + \int_t^T \left(\alpha_1 - \delta(r - t) - c\int_t^r e(w|t)\,dw\right) \frac{d\tilde{\rho}^*(r|t)}{dt}\left[1 - \exp\left(-\lambda\int_t^r e(w|t)\,dw\right)\right] dr$$

$$+ \int_t^T \left(V^I(t) - \delta(r - t) - c\int_t^r e(w|t)\,dw\right)\lambda e(r)\exp\left(-\lambda\int_t^r e(w|t)\,dw\right)[1 - \tilde{\rho}^*(r|t)]\,dr$$

$$+ \left(1 - \exp\left(-\int_t^T e(r)\,dr\right)\right)(1 - \tilde{\rho}^*(T|t))\left[(1 - \phi^*(T|t))\alpha_1 - \delta(T - t) - c\int_t^T e(r|t)\,dr\right]$$

is continuous in $e(r|t)$. Hence,

$$V^I(t) = \max_{e(r|t) \in C_1([t,T],[0,e_{\max}])} f(t, e(r|t))$$

where $C_1([t,T],[0,e_{\max}])$ are piecewise continuous functions with domain $[t,T]$ and range $[0, e_{\max}]$ that are continuous in t by the theorem of the maximum (Berge 1963). ∎

Lemma 8 *Suppose* $\rho^*(t)$ *and* $e^*(t)$ *constitute equilibrium strategies and* $\exists s, \Delta s > 0 : \frac{d\rho^*(t)}{dt} > 0$ *and* $0 < e^*(t) \leq e_{\max}$ *for* $t \in [s, s + \Delta s]$, *then* $e^*(t) = \frac{\delta}{\lambda\phi(t)\alpha_2}$ *for* $t \in [s + \Delta s]$.

Proof. We use the relation $d^*(t)(1 - \phi^*(t)) = \frac{\frac{d\rho^*(t)}{dt}}{1 - \rho^*(t)}$. Also, note that for $\mu^*(t) = 0$

$$\phi(t) = \frac{\tilde{\sigma}(t)}{1 - \tilde{\rho}(t)}$$

$$\frac{d\phi}{dt} = \frac{\frac{d\tilde{\sigma}}{dt} + \frac{\frac{d\tilde{\rho}}{dt}}{1 - \tilde{\rho}(t)}\tilde{\sigma}(t)}{1 - \tilde{\rho}(t)}$$

$$= \frac{-\lambda\tilde{e}(t)\tilde{\sigma}(t) + \tilde{d}(t)(1 - \phi(t))\tilde{\sigma}(t)}{1 - \tilde{\rho}(t)}$$

$$= \left[\tilde{d}(t)(1 - \phi(t)) - \lambda\tilde{e}(t)\right]\frac{\tilde{\sigma}(t)}{1 - \tilde{\rho}(t)}$$

$$= \left[\tilde{d}(t)(1 - \phi(t)) - \lambda\tilde{e}(t)\right]\phi(t)$$

The incentives for delaying rather than taking a decision are equal if the agent is mixing, that is

$$V_0 + \alpha_1 + (1 - \phi^*(t))\, \alpha_2 = \int_t^{t+\Delta t} -\delta\,(r - t)\,\tilde{d}^*(r)\,(1 - \phi^*(r))\exp\left(-\int_t^r \tilde{d}^*(s)\,(1 - \phi^*(s))\,ds\right) dr$$

$$+ (V_0 + \alpha_1 + \alpha_2)\left(1 - \exp\left(-\int_t^{t+\Delta t} \tilde{d}^*(s)\,(1 - \phi^*(s))\,ds\right)\right)$$

$$+ \exp\left(-\int_t^r \tilde{d}^*(s)\,(1 - \phi^*(s))\,ds\right)(V_0 + \alpha_1 + (1 - \phi^*(t + \Delta t))\,\alpha_2 - \Delta t \delta)$$

This can be rearranged to obtain

$$\delta \int_t^{t+\Delta t} (r - t)\,\tilde{d}^*(r)\,(1 - \phi^*(r))\exp\left(-\int_t^r \tilde{d}^*(s)\,(1 - \phi^*(s))\,ds\right) dr$$

$$+ \delta \Delta t \exp\left(-\int_t^{t+\Delta t} \tilde{d}^*(s)\,(1 - \phi^*(s))\,ds\right)$$

$$= \alpha_2\left(\phi(t) - \phi(t + \Delta t)\exp\left(-\int_t^{t+\Delta t} \tilde{d}^*(s)\,(1 - \phi^*(s))\,ds\right)\right)$$

Apply a Taylor series expansion to $\phi^*(t + \Delta t)$ and $\exp\left(-\int_t^{t+\Delta t} \tilde{d}^*(s)\,(1 - \phi^*(s))\,ds\right)$

$$\phi^*(t + \Delta t) = \phi^*(t) - \int_t^{t+\Delta t}\left[\lambda \tilde{e}^*(s) - \tilde{d}^*(s)\,(1 - \phi^*(s))\right]\phi^*(s)\,ds$$

$\Leftrightarrow$

$$\phi^*(t + \Delta t) = \phi^*(t)\left\{1 - \Delta t\left[\lambda \tilde{e}^*(t) - \tilde{d}^*(t)\,(1 - \phi^*(t))\right]\right\} + O\left((\Delta t)^2\right)$$

and apply it also to $\exp\left(-\int_t^{t+\Delta t} d(s)\,(1 - \phi(s))\,ds\right)$:

$$\exp\left(-\int_t^{t+\Delta t} \tilde{d}^*(s)\,(1 - \phi^*(s))\,ds\right) = 1 - \Delta t \tilde{d}^*(t)\,(1 - \phi^*(t)) + O\left((\Delta t)^2\right)$$

We combine these expressions and denote the expression inside of the brackets on the right-hand side by $R = \phi^*(t) - \phi^*(t + \Delta t)\exp\left(-\int_t^{t+\Delta t} \tilde{d}^*(s)\,(1 - \phi^*(s))\,ds\right)$

$$R = \phi^*(t)\,\alpha_2\left\{\Delta t\left[\lambda \tilde{e}^*(t) - \tilde{d}^*(t)\,(1 - \phi^*(t))\right]\right\} + \Delta t \tilde{d}^*(t)\,(1 - \phi^*(t)) + O\left((\Delta t)^2\right)$$

Simplifying this expression yields

$$R = \Delta t\left[\lambda \tilde{e}^*(t)\,\phi^*(t)\,\alpha_2\right] + O\left((\Delta t)^2\right)$$

Denote the left-hand side by L and apply a Taylor series expansion:

$$L = \delta \int_t^{t+\Delta t} (r - t)\,\tilde{d}^*(r)\,(1 - \phi^*(r))\exp\left(-\int_t^r \tilde{d}^*(s)\,(1 - \phi^*(s))\,ds\right) dr$$

$$+ \delta \Delta t \exp\left(-\int_t^{t+\Delta t} \tilde{d}^*(s)\,(1 - \phi^*(s))\,ds\right)$$

$$L = \delta \Delta t + O\left(\Delta t^2\right)$$

Equating the Δt terms from the left- and right-hand sides leads one to conclude that $\delta = \lambda \tilde{e}^*(t)\,\phi^*(t)\,\alpha_2$ ∎

55

The indifference condition implies $\lambda \tilde{e}^* (t) \phi^* (t) \alpha_2 = \delta$ when $\tilde{d}^* (t) > 0$ for all t.

Lemma 9 *Suppose $\rho^* (t)$, $e^* (t)$ and $\phi^* (t)$ constitute equilibrium strategies and beliefs, and $\exists s, \Delta s > 0:$ $\frac{d\rho^* (t)}{dt} > 0, \phi^* (t) < 1$ and $0 < e^* (t) \leq e_{\max}$ for $t \in [s, s + \Delta s]$, then $e^* (t) = e_{\max}, \phi^* (t) = \bar{\phi}_d, d^* (t) = \frac{\lambda^2 \alpha_2}{\lambda \alpha_2 - \delta}$ for $t \in [s + \Delta s]$.*

Proof. Suppose not. This implies $e^* (t) < e_{\max}$, $\phi^* (t) > \bar{\phi}_d$ from lemma 8. The following condition must hold

$$V^I (s) - V^U (s) = \frac{c}{\lambda} \text{ for all } s \in [t, t + \Delta t]$$

for effort to be optimal. Hence, we also require

$$
\begin{aligned}
V^I (t) - V^I (t + \Delta t) &= V^U (t) - V^U (t + \Delta t) \\
[\phi^* (t + \Delta t) - \phi^* (t)] \alpha_2 &= V^U (t) - V^U (t + \Delta t)
\end{aligned}
$$

since this is true for all s. Recall

$$
\begin{aligned}
V^I (t) = \int_t^{t+\Delta t} -\delta (r - t) \tilde{d}^* (r) (1 - \phi^* (r)) \exp \left(-\int_t^r \tilde{d}^* (s) (1 - \phi^* (s)) ds \right) dr \\
+ (V_0 + \alpha_1 + \alpha_2) \left[1 - \exp \left(-\int_t^{t+\Delta t} \tilde{d}^* (s) (1 - \phi^* (s)) ds \right) \right] \\
+ \exp \left(-\int_t^{t+\Delta t} \tilde{d}^* (s) (1 - \phi^* (s)) ds \right) \left[V^I (t + \Delta t) - \Delta t \delta \right]
\end{aligned}
$$

and

$$
\begin{aligned}
V^I (t) - V^I (t + \Delta t) = \int_t^{t+\Delta t} -\delta (r - t) \tilde{d}^* (r) (1 - \phi^* (r)) \exp \left(-\int_t^r \tilde{d}^* (s) (1 - \phi^* (s)) ds \right) dr \\
+ \left(V_0 + \alpha_1 + \alpha_2 - V^I (t + \Delta t) \right) \left[1 - \exp \left(-\int_t^{t+\Delta t} \tilde{d}^* (s) (1 - \phi^* (s)) ds \right) \right] \\
- \Delta t \delta \exp \left(-\int_t^{t+\Delta t} \tilde{d}^* (s) (1 - \phi^* (s)) ds ds \right)
\end{aligned}
$$

Now we can write $V^U (t)$ as follows

$$
\begin{aligned}
V^U (t) = \int_t^{t+\Delta t} -\delta (r - t) \tilde{d}^* (r) (1 - \phi^* (r)) \exp \left(-\int_t^r \tilde{d}^* (s) (1 - \phi^* (s)) ds \right) dr \\
+ (V_0 + \alpha_1) \left(1 - \exp \left(-\int_t^{t+\Delta t} \tilde{d}^* (s) (1 - \phi^* (s)) ds \right) \right) \\
+ \exp \left(-\int_t^{t+\Delta t} \tilde{d}^* (s) (1 - \phi^* (s)) ds \right) \left[V^U (t + \Delta t) - \Delta t \delta \right]
\end{aligned}
$$

since the agent is indifferent about exerting effort we can write it out assuming $e_i(t) = 0$. We can further calculate $V^U(t) - V^U(t + \Delta t)$ using $V^U(t + \Delta t) = V^I(t + \Delta t) - \frac{c}{\lambda}$

$$
V^U(t) - V^U(t + \Delta t) = \int_t^{t+\Delta t} -\delta(r - t)\tilde{d}^*(r)(1 - \phi^*(r)) \exp\left(-\int_t^r \tilde{d}^*(s)(1 - \phi^*(s))\,ds\right) dr
$$

$$
+ \left(V_0 + \alpha_1 - V^I(t + \Delta t) + \frac{c}{\lambda}\right)\left[1 - \exp\left(-\int_t^{t+\Delta t} \tilde{d}^*(s)(1 - \phi^*(s))\,ds\right)\right]
$$

$$
- \Delta t \delta \exp\left(-\int_t^{t+\Delta t} \tilde{d}^*(s)(1 - \phi^*(s))\,ds\right)
$$

$$
V^U(t) - V^U(t + \Delta t) = V^I(t) - V^I(t + \Delta t) + \left(\frac{c}{\lambda} - \alpha_2\right)\left[1 - \exp\left(-\int_t^{t+\Delta t} \tilde{d}^*(s)(1 - \phi^*(s))\,ds\right)\right]
$$

Hence, we have a contradiction $V^U(t) - V^U(t + \Delta t) > V^I(t) - V^I(t + \Delta t)$ for $\tilde{d}^*(s) > 0$ and $\phi^*(s) < 1$. ∎

Lemma 10 $\phi^*(t) \geq \max\{\bar{\phi}_d, \bar{\phi}\}$

Proof. Suppose $\exists \check{t} : \phi^*(\check{t}) < \bar{\phi}_d$ then $\exists \Delta \check{t} > 0 : \phi^*(s) < \bar{\phi}_d$ for $s \in [\check{t} - \Delta \check{t}, \check{t}]$. However, this is a contradiction as an informed player will strictly prefer to call a decision for all $s \in [\check{t} - \Delta \check{t}, \check{t}]$ and hence $\phi^*(\check{t}) > \bar{\phi}_d$ which is a contradiction that $\exists \check{t} : \phi^*(\check{t}) < \bar{\phi}_d$. Suppose $\exists \check{t} : \phi^*(\check{t}) < \bar{\phi}$ then $\exists \Delta \check{t} > 0 : \phi^*(\check{t} - \Delta \check{t}) < \bar{\phi}$. This implies an upper bound on the value of information is $(1 - \phi^*(\check{t}))\alpha_2 + \phi^*(\check{t})\alpha_1$. Thus, for any time s such that $\phi^*(s) < \bar{\phi} \Rightarrow e^*(s) = 0$ therefore $\frac{d\phi^*(s)}{ds} = 0$ so $\lim_{r \to} \phi^*(\check{t}^-) > \bar{\phi}(\check{t})$ which is a violation of the continuity of $\phi^*(t)$. This is a contradiction given the upper bound on the arrival rate of information. ∎

Lemma 11 *Suppose $e^*(t) < e_{\max}$ for some t then under large incentives $\phi^*(T) = \bar{\phi}_d$ and under small incentives $\phi^*(T) = \bar{\phi}$*

Proof. Suppose not, then $\phi^*(T) > \max\{\bar{\phi}_d, \bar{\phi}\}$. Let $\hat{t} = \inf\{t | e^*(t) < e_{\max}\}$. Being informed at time $\hat{t}$ has a continuation value given by

$$
V^I(\hat{t}) = V_0 + \alpha_1 + (1 - \phi^*(T))\alpha_2 - \delta(T - \hat{t})
$$

since the optimal strategy for an informed individual is to delay until the deadline which is due to $e^*(t) = e_{\max}$ and $\phi^*(t) > \bar{\phi}_d$ for $t \geq \hat{t}$. The continuation value for the uninformed individual is

$$
V^U(\hat{t}) = \left(V_0 + \alpha_1 + (1 - \phi^*(T))\alpha_2 - \frac{c}{\lambda}\right)[1 - \exp(-\lambda e_{\max}(T - \hat{t}))]
$$

$$
+ \exp(-\lambda e_{\max}(T - \hat{t}))[V_0 + (1 - \phi^*(T))\alpha_1] - \delta(T - \hat{t})
$$

Thus, incentives for effort are given by

$$
V^I(\hat{t}) - V^U(\hat{t}) = \frac{c}{\lambda} + \exp(-\lambda e_{\max}(T - \hat{t}))\left[(1 - \phi^*(T))\alpha_2 + \phi^*(T)\alpha_1 - \frac{c}{\lambda}\right].
$$

Further, by definition of $\bar{\phi}_d$ and $\bar{\phi}$.we have

$$
(1 - \phi^*(T))\alpha_2 + \phi^*(T)\alpha_1 > ((1 - \bar{\phi}_d)\alpha_2 + \bar{\phi}_d\alpha_1)
$$

$$
> ((1 - \bar{\phi})\alpha_2 + \bar{\phi}\alpha_1) = \frac{c}{\lambda}
$$

Hence, $V^I\left(\hat{t}\right) - V^U\left(\hat{t}\right) > \frac{c}{\lambda}$ and by the continuity of V^I and V^U $\exists \omega > 0 : e^*\left(\hat{t} - \omega\right) = e_{\max}$ which is a contradiction of $\hat{t} = \inf\left\{t | e^*\left(t\right) < e_{\max}\right\}$. $\quad\blacksquare$

The previous lemmas restrict the set of potential equilibria to those where $\phi^*\left(t\right)$ is continuous, decreasing and bounded below by $\max\left\{\bar{\phi}_d, \bar{\phi}\right\}$. Furthermore, if decisions are taken prior to the deadline, then $\frac{d\phi^*}{dt} = 0$ and either $\phi^* = 1$ or $\phi^* = \bar{\phi}_d$ during those times.

B.4.2 Proof for Uniqueness of Symmetric Equilibria Set for Large Incentives Case

Define $V^{I*}\left(t\right)$ and $V^{U*}\left(t\right)$

$$
\begin{aligned}
V^{I*}\left(t\right) &= V_0 + \alpha_1 + \left(1 - \bar{\phi}_d\right)\alpha_2 \\
V^{U*}\left(t\right) &= V_0 + \alpha_1 + \frac{\left(1 - \bar{\phi}_d\right)\alpha_2}{2} - \left(c + \frac{\delta}{e_{\max}}\right)\frac{1}{2\lambda} \\
&\quad - \exp\left(-2\lambda e_{\max}\left(T - t\right)\right)\left[\bar{\phi}_d\alpha_1 + \frac{\left(1 - \bar{\phi}_d\right)\alpha_2}{2} - \left(c + \frac{\delta}{e_{\max}}\right)\frac{1}{2\lambda}\right]
\end{aligned}
$$

Note further that

$$
\begin{aligned}
V^{I*}\left(t\right) - V^{U*}\left(t\right) &= \frac{\left(1 - \bar{\phi}_d\right)\alpha_2}{2} + \left(c + \frac{\delta}{e_{\max}}\right)\frac{1}{2\lambda} \\
&\quad + e^{-2\lambda(T-t)}\left[\bar{\phi}_d\alpha_1 + \frac{\left(1 - \bar{\phi}_d\right)\alpha_2}{2} - \left(c + \frac{\delta}{e_{\max}}\right)\frac{1}{2\lambda}\right]
\end{aligned}
$$

and

$$
V^{I*}\left(t\right) - V^{U*}\left(t\right)
\begin{cases}
> \frac{c}{\lambda} & \text{for } T - t < Z \\
= \frac{c}{\lambda} & \text{for } T - t = Z \\
< \frac{c}{\lambda} & \text{for } T - t > Z
\end{cases}
$$

Also, define $\bar{t}_d\left(\phi\right)$, $V^{I*}\left(t, \phi\right)$ and $V^{U*}\left(t, \phi\right)$

$$
\bar{t}_d\left(\phi\right) = \frac{1}{\lambda}\ln\frac{\phi}{\bar{\phi}_d}
$$

$$
V^{I*}\left(t, \phi\right) =
\begin{cases}
V_0 + \alpha_1 + \left(1 - \phi\left(t\right)\exp\left(-\lambda e_{\max}\left(T - t\right)\right)\right)\alpha_2 - \delta\left(T - t\right) & \text{for } T - \bar{t}_d\left(\phi\right) < t \leq T \\
V^{I*}\left(t + \tilde{t}\right) - \delta\tilde{t} & \text{for } T - \bar{t}_d\left(\phi\right) - Z \leq t \leq T - \bar{t}_d\left(\phi\right),
\end{cases}
$$

$$
V^{U*}\left(t, \phi\right) =
\begin{cases}
\left[1 - \exp\left(-\lambda e_{\max}\left(T - t\right)\right)\right]\left(V_0 + \alpha_1 + \left(1 - \phi\left(t\right)\exp\left(-\lambda e_{\max}\left(T - t\right)\right)\right)\alpha_2 - \frac{c}{\lambda}\right) \\
\quad + \exp\left(-\lambda e_{\max}\left(T - t\right)\right)\left(V_0 + \left[1 - \phi\left(t\right)\exp\left(-\lambda e_{\max}\left(T - t\right)\right)\right]\alpha_1\right) - \delta\left(T - t\right) \\
\hspace{8cm}\text{for } T - \bar{t}_d\left(\phi\right) < t \leq T \\
\left(1 - \exp\left(-\lambda e_{\max}\tilde{t}\right)\right)\left(V^{I*}\left(t + \tilde{t}\right) - \frac{c}{\lambda}\right) + \exp\left(-\lambda e_{\max}\tilde{t}\right)V^{U*}\left(t + \tilde{t}\right) - \delta\tilde{t} \\
\hspace{8cm}\text{for } T - \bar{t}_d\left(\phi\right) - Z \leq t \leq T - \bar{t}_d\left(\phi\right).
\end{cases}
$$

Note that

$$
V^{I*}\left(t, \phi\right) - V^{U*}\left(t, \phi\right) =
\begin{cases}
\frac{c}{\lambda} + \exp\left(-\lambda e_{\max}\left(T - t\right)\right)\left[\phi\left(t\right)\exp\left(-\lambda e_{\max}\left(T - t\right)\right)\alpha_1\right. \\
\hspace{2cm}\left. + \left(1 - \phi\left(t\right)\exp\left(-\lambda e_{\max}\left(T - t\right)\right)\right)\alpha_2 - \frac{c}{\lambda}\right] \\
\hspace{6cm}\text{for } T - \bar{t}_d\left(\phi\right) < t \leq T \\
\frac{c}{\lambda} + \exp\left(-\lambda e_{\max}\tilde{t}\right)\left(V^{I*}\left(t + \tilde{t}\right) - V^{U*}\left(t + \tilde{t}\right) - \frac{c}{\lambda}\right) \\
\hspace{6cm}\text{for } T - \bar{t}_d\left(\phi\right) - Z \leq t \leq T - \bar{t}_d\left(\phi\right).
\end{cases}
$$

Hence,

$$V^{I*}(t,\phi) - V^{U*}(t,\phi) \begin{cases} > \frac{c}{\lambda} \text{ for } T - \bar{t}_d(\phi) - Z < t \\ = \frac{c}{\lambda} \text{ for } T - \bar{t}_d(\phi) - Z = t \\ < \frac{c}{\lambda} \text{ for } T - \bar{t}_d(\phi) - Z > t \end{cases}.$$

Lemma 12 *The unique equilibrium strategies in any subgame starting at t with beliefs $\phi(t) \geq \bar{\phi}_d$ such that $t \geq T - Z - \bar{t}_d(\phi)$ are $e^*(s) = e_{\max}$ for $t \leq s \leq T$ and $d^*(s) = \begin{cases} 0 \text{ for } t \leq s \leq \bar{t}_d(\phi) \\ \frac{\lambda^2 \alpha_2}{\lambda \alpha_2 - \delta} \text{ for } \bar{t}_d(\phi) < s \leq T \end{cases}$*

Proof. Suppose $\exists s \geq t, \varepsilon > 0$ such that $e^*(r) < e_{\max}$ for $r \in [s - \varepsilon, s)$. If this is the case, we can check the continuation values at $\hat{s}$ where

$$\hat{s} = \sup\{r | e^*(r) < 1\}.$$

Given that $e^*(s) = e_{\max}$ for $r \geq \hat{s}$ then the unique decision strategy is

$$d^*(s) = \begin{cases} 0 \text{ for } \hat{s} \leq s \leq \min\{T, \hat{s} + \bar{t}_d(\phi(\hat{s}))\} \\ \frac{\lambda^2 \alpha_2}{\lambda \alpha_2 - \delta} \text{ for } \hat{s} + \bar{t}_d(\phi(\hat{s})) < s \leq T \text{ if } \hat{s} + \hat{t}(\phi(\hat{s})) < T \end{cases}$$

since the only belief at which an informed individual will call a decision is $\phi = \bar{\phi}_d$ when the uninformed agent is exerting maximum effort. We can therefore write the continuation values as $V^{I*}(\hat{s}, \phi)$ and $V^{U*}(\hat{s}, \phi)$. The contradiction now comes from noting that for $t > T - Z - \bar{t}_d(\phi)$ and $V^{I*}(t, \phi) - V^{U*}(t, \phi) > \frac{c}{\lambda}$. Therefore, $\exists \zeta : V^{I*}(r, \phi(r)) - V^{U*}(r, \phi(r)) > \frac{c}{\lambda}$ and $e^*(r) < e_{\max}$ for $r \in [\hat{s} - \zeta, \hat{s})$ which means $e^*(r)$ is not an equilibrium strategy. ∎

Lemma 13 *Suppose $T \geq X_d + Z$, then an upper bound on $\phi^*(t)$ is given by*

$$\phi^*(t) \leq \begin{array}{ll} \bar{\phi}_d \exp(\lambda(T - Z - t)) & \text{for } T - X_d - Z \leq t < T - Z \\ \bar{\phi}_d & \text{for } T - Z \leq t \leq T \end{array}$$

Proof. Suppose $\exists t', \phi^*(t') > \bar{\phi}_d \exp(\lambda(T - Z - t'))$ for $T - X_d - Z \leq t' < T - Z$ or $\phi^*(t) > \bar{\phi}_d$ for $T - Z \leq t' \leq T$ then

$$\exists s < t' : (s, \phi^*(s)) \in \left\{ \begin{array}{l} (r(\phi), \phi) | r(\phi) = T - X_d - Z + \frac{1}{\lambda e_{\max}} \ln \frac{1}{\phi} \\ +\gamma\left(t' - \left(T - X_d - Z + \frac{1}{\lambda e_{\max}} \ln \frac{1}{\phi}\right)\right), \gamma \in (0, 1), \phi \in [\phi(t), 1] \end{array} \right\}.$$

Now $s > T - Z - \bar{t}_d(\phi^*(s))$ and thus the unique equilibrium of the subgame starting from $(s, \phi^*(s))$ is given by Lemma 12. However, the Bayesian belief $\widehat{\phi}^*(r)$ in this subgame reaches

$$\widehat{\phi}^*(r) = \phi^*(t') = (T - X_d - Z) + \frac{1}{\lambda e_{\max}} \ln \frac{1}{\phi^*(t')} + \gamma(s)\left(t' - (T - X_d - Z) + \frac{1}{\lambda e_{\max}} \ln \frac{1}{\phi^*(t')}\right)$$

where $\gamma(s) = \frac{s - (T - X_d - Z) + \frac{1}{\lambda e_{\max}} \ln \frac{1}{\phi^*(s)}}{t' - (T - X_d - Z) + \frac{1}{\lambda e_{\max}} \ln \frac{1}{\phi^*(t')}} < 1$. Thus, $r < t$ and $\widehat{\phi}^*(t') > \phi^*(t')$ and hence $\phi^*(t')$ is not part of a perfect Bayesian equilibrium. ∎

Together with lemma 10 this uniquely determines $\phi(t) = \bar{\phi}_d$ for $t \geq T - Z$ if $T \geq X_d + Z$.

Lemma 14 *Suppose $T \geq X_d + Z$ then a lower bound on $\phi^*(t)$ is given by*

$$\phi^*(t) \geq \begin{cases} \bar{\phi}_d \text{ for } t \geq T - Z \\ \bar{\phi}_d + \delta(T - Z - t) \text{ for } T - Y_d - Z \leq t < T - Z \\ 1 \text{ for } t \leq T - Y_d - Z \end{cases}$$

Proof. Lemma 13 and lemma 10 pin down $\phi^*(t) = \bar{\phi}_d$ for $t \geq T - Z$. Now suppose $\exists s < T - Y_d - Z : \phi^*(s) < 1$ or $\exists s : T - Y_d - Z \leq s < T - Z, \phi^*(s) < \bar{\phi}_d + \delta(T - Z - s)$. If $\exists r < T - Z : d^*(t) = 0$ for $t \in [r, T - Z]$ there is an immediate contradiction as informed individuals would strictly prefer to call an immediate decision. If not then using lemma 9 if $d^*(t) > 0$ for $s \leq t < T - Z$ then $e^*(t) = e_{\max}, \phi^*(t) = \bar{\phi}_d, d^*(t) = \frac{\lambda^2 \alpha_2}{\lambda \alpha_2 - \delta}$ for $t \in [r, T - Z]$. However in this case we also have a contradiction as $V^I(t) - V^U(t) = V^{I*}(t) - V^{U*}(t) < \frac{c}{\lambda}$ since $t < T - Z$ and the effort strategy $e^*(t) = e_{\max}$ is not optimal and cannot be part of an equilibrium. ∎

These two lemmas provide an upper and lower bound on the values of $\phi^*(t)$ in equilibrium. The proof for uniqueness now proceeds by showing that the only equilibrium strategies which support values of ϕ between these bounds are the ones given in the propositions.

Proof. i) Case 1: $T < X_d$

Informed strategy: $d^*(t) = 0$ for all t. Uninformed strategy: $e^*(t) = e_{\max}$ for all t. Beliefs: $\phi^*(t) = \exp(-\lambda e_{\max} t)$ for all t.

ii) Case 2: $X_d \leq T < X_d + Z$

Informed strategy: $d^*(t) = 0$ for $t < X_d$, $d^*(t) = \frac{\lambda^2 \alpha_2}{\lambda \alpha_2 - \delta}$ for $t \geq X_d$. Uninformed strategy: $e^*(t) = e_{\max}$ for all t. Beliefs: $\phi^*(t) = \exp(-\lambda e_{\max} t)$ for $t < X_d$.and $\phi(t) = \bar{\phi}_d$ for $t \geq X_d$.

Lemma 12 covers Case 1 and 2.

iii) Case 3: $X_d + Z < T < Y_d + Z$

Informed strategy: $d^*(t) = 0$ for $t < T - Z$ and $d^*(t) = \frac{\lambda^2 \alpha_2}{\lambda \alpha_2 - \delta}$ for $t \geq T - Z$. Uninformed strategy: $d^*(t) = 0$ for all t and $e^*(t)$ satisfies

$$\exp\left(-\lambda \int_0^t e^*(s)\, ds\right) \geq \bar{\phi}_d + (T - Z - t)\frac{\delta}{\alpha_2}$$

and

$$\exp\left(-\lambda \int_0^{T-Z} e^*(s)\, ds\right) = \bar{\phi}_d.$$

for $t < T - Z$ and $e^*(t) = e_{\max}$ for all $t \geq T - Z$. Beliefs: $\phi^*(t) = \exp\left(-\lambda \int_0^t e^*(s)\, ds\right)$ for $t < T - Z$ and $\phi^*(t) = \bar{\phi}_d$ for $t \geq T - Z$.

Lemmas 12, 14 and 13 determine the bounds on $\phi^*(t)$ and that $\phi^*(t) = \bar{\phi}_d, e^*(t) = e_{\max}$ for $t \geq T - Z$. Lemma 9 implies that $d^*(t) = 0$ for $t < T - Z$ which suffices along with the earlier lemmas for the equilibrium strategy set.

iv) Case 4: $T > Y_d + Z$

Informed strategy: $d^*(t) = call$ for $0 \leq t \leq T - Y_d - Z$ and $d^*(t) = 0$ for $T - Y_d - Z < t < T - Z$ and $d^*(t) = \frac{\lambda^2 \alpha_2}{\lambda \alpha_2 - \delta}$ for $t \geq T - Z$. Uninformed strategy: $d^*(t) = 0$ for all t. $e^*(t) = \frac{\delta}{c}$ for $0 \leq t \leq T - Y_d - Z$ and $e^*(t)$ satisfies

$$\exp\left(-\lambda \int_{T-Y_d-Z}^t e^*(s)\, ds\right) \geq \bar{\phi}_d + (T - Z - t)\frac{\delta}{\alpha_2}$$

and

$$\exp\left(-\lambda \int_{T-Y_d-Z}^{T-Z} e^*(s)\, ds\right) = \bar{\phi}_d.$$

for $T - Y_d - Z < t < T - Z$ and $e^*(t) = e_{\max}$ for $t \geq T - Z$.

Beliefs: $\phi^*(t) = 1$ for $0 \leq t \leq T - Y_d - Z$, $\phi^*(t) = \exp\left(-\lambda \int_{T-Y_d-Z}^t e^*(s)\, ds\right)$ for $T - Y_d - Z < t < T - Z$ and $\phi^*(t) = \bar{\phi}_d$ for $t \geq T - Z$.

Case 3 above covers the subgames for $t > T - Z - \widehat{T}$. It remains to show that the above strategies are unique for $t \leq T - Z - \widehat{T}$. For $t \leq T - Z - \widehat{T}$ we have shown that $\phi^*(t) = 1$. First, rule out that $e(t) = 0$. If this were the case the continuation payoffs would be $V^I(t) = V_0 + \alpha_1$ and $V^U(t) = V_0 + \alpha_1 - \frac{c}{\lambda} - (\check{t} - t)\delta$ where

60

$\tilde{t} = \inf\{s > t : e(s) > 0\}$, therefore the strategy $e^*(t) = 0$ is not optimal as $V^I - V^U > \frac{c}{\lambda}$. Implying that $0 < e^*(t) \le \frac{\delta}{\lambda\alpha_2}$ and individuals call a decision immediately. We have $V^I(t) = V_0 + \alpha_1$ so $V^U(t) = V_0 + \alpha_1 - \frac{c}{\lambda}$ for $t \le T - Z - \widehat{T}$.

$$V^U(t) = \int_t^{t+\Delta t} \left(V_0 + \alpha_1 - c\int_t^s e^*(r)\,dr - \delta(s-t) \right) 2\lambda e^*(s) \exp\left(-2\lambda \int_t^s e^*(r)\,dr \right) ds$$

$$+ \exp\left(-2\lambda \int_t^{t+\Delta t} e^*(r)\,dr \right) \left(V^U(t+\Delta t) - \delta\Delta t - c\int_t^{t+\Delta t} e^*(r)\,dr \right)$$

$$V^U(t) = \left(V_0 + \alpha_1 - \frac{c}{2\lambda} \right)\left(1 - \exp\left(-2\lambda \int_t^{t+\Delta t} e^*(r)\,dr \right) \right)$$

$$- \int_t^{t+\Delta t} \delta(s-t)\, 2\lambda e^*(s) \exp\left(-2\lambda \int_t^s e^*(r)\,dr \right) ds$$

$$+ \exp\left(-2\lambda \int_t^{t+\Delta t} e^*(r)\,dr \right)\left(V^U(t+\Delta t) - \delta\Delta t \right)$$

$$V^U(t) = V_0 + \alpha_1 - \frac{c}{\lambda} + \frac{c}{2\lambda}\left(1 - \exp\left(-2\lambda \int_t^{t+\Delta t} e^*(r)\,dr \right) \right)$$

$$- \int_t^{t+\Delta t} \delta(s-t)\, 2\lambda e^*(s) \exp\left(-2\lambda \int_t^s e^*(r)\,dr \right) ds$$

$$- \exp\left(-2\lambda \int_t^{t+\Delta t} e^*(r)\,dr \right)\delta\Delta t$$

hence

$$\frac{c}{2\lambda}\left(1 - \exp\left(-2\lambda \int_t^{t+\Delta t} e^*(r)\,dr \right) \right) = \int_t^{t+\Delta t} \delta(s-t)\, 2\lambda e^*(s) \exp\left(-2\lambda \int_t^s e^*(r)\,dr \right) ds$$

$$+ \exp\left(-2\lambda \int_t^{t+\Delta t} e^*(r)\,dr \right)\delta\Delta t$$

$$ce^*(t)\Delta t + O\left(\Delta t^2\right) = \delta\Delta t + O\left(\Delta t^2\right)$$

We therefore require that $e^*(t) = \frac{\delta}{c}$. ∎

B.4.3 Proof for Uniqueness of Symmetric Equilibria Set for Baseline Case

Define

$$\bar{t}_e(\phi) = \frac{1}{\lambda}\ln\frac{\phi}{\check{\phi}}$$

$$V_e^{I*}(t,\phi) = V_0 + \alpha_1 + (1 - \phi\exp[-\lambda e_{\max}(T-t)])\alpha_2 - \delta\bar{t}_e(\phi)$$

as well as

$$V_e^{U*}(t,\phi) = (1 - \exp[-\lambda e_{\max}(T-t)])\left(V_0 + \alpha_1 + (1 - \phi\exp[-\lambda e_{\max}(T-t)])\alpha_2 - \frac{c}{\lambda} \right)$$

$$+ \exp[-\lambda e_{\max}(T-t)](V_0 + (1 - \phi\exp[-\lambda e_{\max}(T-t)])\alpha_1) - \delta\bar{t}_e(\phi)$$

and

$$V_e^{I*}(t,\phi) - V_e^{U*}(t,\phi) = \frac{c}{\lambda}$$
$$+ \exp\left[-\lambda e_{\max}(T-t)\right]\left(\phi \exp\left[-\lambda e_{\max}(T-t)\right]\alpha_1 + (1 - \phi \exp\left[-\lambda e_{\max}(T-t)\right])\alpha_2 - \frac{c}{\lambda}\right)$$

Note that

$$V_e^{I*}(t,\phi) - V_e^{U*}(t,\phi) > \frac{c}{\lambda}$$

provided that $t < \bar{t}_e(\phi)$.

Lemma 15 *The unique equilibrium strategy in any subgame starting at t with beliefs $\phi(t)$ such that $t \geq T - \bar{t}_e(\phi)$ is $e^*(s) = e_{\max}$ for $t \leq s \leq T$ and $d^*(s) = 0$.*

Proof. Suppose $\exists s \geq t, \varepsilon > 0$ such that $e^*(r) < e_{\max}$ for $r \in [s - \varepsilon, s)$. If this is the case, we can check the continuation values at $\widehat{s}$ where

$$\widehat{s} = \sup\{r | e^*(r) < 1\}.$$

Given that $e^*(r) = e_{\max}$ for $r \geq \widehat{s}$ then the unique decision strategy is $d^*(r) = 0$ since the only belief at which an informed individual will call a decision is $\bar{\phi}_d < \bar{\phi}$ when the uninformed agent is exerting maximum effort. We can therefore write the continuation values as $V_e^{I*}(\widehat{s}, \phi(\widehat{s})), V_e^{U*}(\widehat{s}, \phi(\widehat{s}))$. The contradiction now comes from noting that

$$t > T - \bar{t}_e(\phi) \Rightarrow \widehat{s} > T - \bar{t}_e(\phi(\widehat{s}))$$

hence $V_e^{I*}(\widehat{s}, \phi(\widehat{s})) - V_e^{U*}(\widehat{s}, \phi(\widehat{s})) > \frac{c}{\lambda}$. Thus $\exists \zeta : V_e^{I*}(r, \phi(r)) - V_e^{U*}(r, \phi(r)) > \frac{c}{\lambda}$ and $e^*(r) < e_{\max}$ for $r \in [\widehat{s} - \zeta, \widehat{s})$ which means that $e^*(r)$ is not an equilibrium strategy. ∎

Lemma 16 *Suppose $T \geq X$, then an upper bound on $\phi^*(t)$ is given by*

$$\phi^*(t) \leq \begin{cases} 1 \text{ for } t < T - X \\ \bar{\phi}\exp(\lambda(T-t)) \text{ for } T - X \leq t < T \end{cases}$$

Proof. Suppose $\exists t'$ such that $\phi^*(t') > \bar{\phi}\exp(\lambda(T-t))$ for $T - X \leq t' < T$ then

$$\exists (s, \phi^*(s)) \in \left\{ \begin{array}{c} (r(\phi), \phi) | r(\phi) = T - X + \frac{1}{\lambda e_{\max}}\ln\frac{1}{\phi} \\ +\gamma\left(t' - \left(T - X + \frac{1}{\lambda e_{\max}}\ln\frac{1}{\phi}\right)\right), \gamma \in (0,1), \phi \in [\phi(t'), 1] \end{array} \right\}$$

Now $s > T - \bar{t}_e(\phi^*(s))$, so the unique equilibrium of the subgame starting from $(s, \phi^*(s))$ is given by Lemma 16. However, the Bayesian belief $\widehat{\phi}^*(r)$ in this subgame reaches $\widehat{\phi}^*(r) = \phi^*(t')$ at $r = (T - X) + \frac{1}{\lambda e_{\max}}\ln\frac{1}{\phi^*(t')} + \gamma(s)\left[t' - (T - X) + \frac{1}{\lambda e_{\max}}\ln\frac{1}{\phi^*(t')}\right]$ where

$$\gamma(s) = \frac{s - (T - X) + \frac{1}{\lambda e_{\max}}\ln\frac{1}{\phi^*(s)}}{t' - (T - X) + \frac{1}{\lambda e_{\max}}\ln\frac{1}{\phi^*(t')}} < 1$$

Thus, $r < t'$ and $\widehat{\phi}^*(t') > \phi^*(t')$ and hence $\phi^*(t')$ is not part of the perfect Bayesian equilibrium. ∎
This uniquely determines $\phi^*(T) = \bar{\phi}$ for $T \geq X$.

Lemma 17 *Suppose $T \geq X$ then a lower bound on $\phi^*(t)$ is given by*

$$\phi^*(t) \geq \begin{cases} \bar{\phi} + \delta(T - t) \text{ for } T - Y \leq t < T \\ 1 \text{ for } t \leq T - Y \end{cases}$$

Proof. As noted above $\phi^*(T) = \bar{\phi}$, if $T \geq X$. Now suppose $\exists s : \phi^*(s) < 1$ for $s < T - Y$ or $\phi^*(s) < \bar{\phi} + \delta(T - s)$ for $T - Y \leq t < T$. If this is the case then using lemma 9 $d^*(t) = 0$ for $t \in [s, T]$ and there is

an immediate contradiction as informed individuals will strictly prefer to call a decision at s than wait until T. ■

These lemmas provide an upper and lower bound on the values of $\phi^*(t)$ in equilibrium. As before, the proof for uniqueness now proceeds by showing that the only equilibrium strategies which support values of ϕ between these bounds are the ones given in the propositions.

Proof. i) Case 1: $T < X$

Informed strategy: $d^*(t) = 0$ for all t. Uninformed strategy: $d^*(t) = 0$ for all t and $e^*(t) = e_{\max}$ for all t. Beliefs: Beliefs evolve according to $\phi^*(t) = \exp\left(-\lambda \int_0^t e^*(s)\,ds\right)$ for all t.

This follows immediately from Lemma 15.

ii) Case 2: $X < T < Y$

Informed strategy: $d^*(t) = 0$ for all t. Uninformed strategy: $d^*(t) = 0$ for all t and $e^*(t)$ satisfies

$$\exp\left(-\lambda \int_0^t e^*(s)\,ds\right) \geq \bar{\phi} + (T-t)\frac{\delta}{\alpha_2}$$

and

$$\exp\left(-\lambda \int_0^T e^*(s)\,ds\right) = \bar{\phi}.$$

Beliefs: Beliefs evolve according to $\phi^*(t) = \exp\left(-\lambda \int_0^t e^*(s)\,ds\right)$ for all t.

Lemma 9 and $\phi^*(T) = \bar{\phi}$ (as shown above from Lemmas 15 and 16) imply that $d^*(t) = 0$ for all t. The restriction on $e^*(t)$ comes from Lemma 17. Beliefs are given by Bayesian updating.

iii) Case 3: $T > \frac{1}{\delta}(1-\bar{\phi})\alpha_2$

Informed strategy: $d^*(t) = call$ for $t < T - Y$ and $d(t) = 0$ for $t \geq T - Y$
Uninformed strategy: $d^*(t) = 0$ for all t and $e^*(t) = \frac{\delta}{c}$ for $t < T - Y$. $e(t)$ satisfies

$$\exp\left(-\lambda \int_{T-Y}^t e^*(s)\,ds\right) \geq \bar{\phi} + (T-t)\frac{\delta}{\alpha_2}$$

and

$$\exp\left(-\lambda \int_{T-Y}^T e^*(s)\,ds\right) = \bar{\phi}.$$

for $t \geq T - Y$. Beliefs: $\phi(t) = 1$ for $t < T - Y$ and $\phi^*(t) = \exp\left(-\lambda \int_0^t e^*(s)\,ds\right)$ for $t \geq T - Y$.

The proof for Case 2 encompasses the subgames for $t \geq T - Y$. The uniqueness for $t < T - Y$ is completely analogous to the proof in case 4 for large incentives of the uniqueness of equilibrium strategies for $t < T - Y_d - Z$. ■

B.5 Signal Stucture

In this section, we provide an explicit characterization of the setup and the results when extending the signal structure as discussed in Section 7.1.

B.5.1 Setup

We assume that an individual is prepared to search for a second signal, $\alpha_2 > \frac{c}{\lambda} + \frac{\delta}{\lambda e_{\max}}$. We also assume that an individual is prepared to delay a decision if the other player is exerting maximum effort to obtain a third signal, $\lambda \alpha_3 e_{\max} > \delta$, and that this is no longer true for a fourth signal, $\lambda \alpha_4 e_{\max} < \delta$.

From a modeling standpoint it is also reasonable to draw a distinction between calling a decision and communicating that a signal has been found. Hence we allow at each instance agents to reveal that they have found a signal without calling a decision. We denote this action by $w_i(t,n) : [0,T] \times \{0,1,2\} \to \{0,1\}$ where

n is the number of signals an agent has acquired. We restrict this to be non-decreasing such that an agent may only reveal that a signal was received but cannot subsequently conceal this having already revealed it. To simplify notation we also assume that if the other agent has revealed that they have received a signal, $w_{-i} = 1$ then the action w_i is no longer available. Common knowledge of 2 signals will lead to a decision in a similar way to common knowledge of a single signal in the earlier model would also lead to a decision. Hence in the event that the other agent has previously announced a signal then announcing and calling are equivalent for an agent.

The history of the game at a given point in time is given by (μ, ν). $\mu \in \{0, 1, 2\}$ indicates whether no signal has been announced, $\mu = 0$, or if one has been anounced which player announced it, $\mu = 1, 2$. $\nu \in [0, t]$ indicates at what time the player announced a signal $\nu \in (0, t]$. If no signal has been announced, $\nu = 0$.

B.5.2 Short Deadline

We first consider a short deadline. The equilibrium exhibits no revelation of information $w_i(t, n) = 0$, no decisions on-equilibrium prior to the deadline, maximum effort by the agents having only acquired one or two signals and zero effort having acquired two signals.

We define $\tilde{X}$ as

$$\lambda \left[\begin{array}{c} \exp(-\lambda e_{\max} \tilde{X}) \alpha_2 + \lambda e_{\max} \tilde{X} \exp(-\lambda e_{\max} \tilde{X}) \alpha_3 \\ + \left(1 - \exp(-\lambda e_{\max} \tilde{X}) - \lambda e_{\max} \tilde{X} \exp(-\lambda e_{\max} \tilde{X})\right) \alpha_4 \end{array} \right] = c.$$

Note that

$$\exp(-\lambda e_{\max} \tilde{X}) \alpha_2 + \left(1 - \exp(-\lambda e_{\max} \tilde{X})\right) \alpha_3 > \frac{c}{\lambda}.$$

Also, we assume parameter values such that $\tilde{X} < \tilde{X}_d$, where $\tilde{X}_d$ is given by

$$\lambda e_{\max} \left[\exp(-\lambda e_{\max} \tilde{X}_d) \alpha_3 + \lambda e_{\max} \tilde{X}_d \exp(-\lambda e_{\max} \tilde{X}_d) \alpha_4 \right] = \delta.$$

Proposition 10 $\exists T > 0$ *such that a symmetric Perfect Bayesian Equilibrium is*

$$
\begin{aligned}
w_i^*(t, n) &= 0 \\
d_i^*(t, n, \mu, \nu) &= \begin{cases} \text{call if } \mu \in \{1, 2\} \\ 0 \text{ otherwise} \end{cases} \\
e_i^*(t, n, \mu, \nu) &= \begin{cases} 0 \text{ if } \mu \in \{1, 2\} \text{ or } n \geq 2 \\ e_{\max} \text{ otherwise} \end{cases} \\
\phi_i^*(t, \mu, \nu) &= \begin{cases} \{0, 0, 1\} \text{ if } \mu \in \{1, 2\} \\ \{\exp(-\lambda e_{\max} t), \lambda e_{\max} t \exp(-\lambda e_{\max} t), 1 - (\lambda e_{\max} t + 1) \exp(-\lambda e_{\max} t)\} \text{ otherwise.} \end{cases}
\end{aligned}
$$

Proof. Define $V_i(t, n, \mu, \nu)$ as the continuation value for an agent at time t depending on the number of signals they have acquired and the history of announcements of signals by themselves and the other agent. Define $\chi(n)$ as

$$\chi(n) = -\frac{1}{\varepsilon + n\tau} + (1 - \exp(-\lambda e_{\max} T)) \alpha_{n+1} + (1 - \exp(-\lambda e_{\max} T) - \lambda e_{\max} T \exp(-\lambda e_{\max} T)) \alpha_{n+2}.$$

This is the expected value of the decision at the deadline conditional on an agent holding n signals in equilibrium. Note that for $n = 1, 2$ and $T < \tilde{X}$,

$$
\begin{aligned}
\chi(n) - \chi(n-1) &= \exp(-\lambda e_{\max} T) \alpha_n + \lambda e_{\max} T \exp(-\lambda e_{\max} T) \alpha_{n+1} \\
&\quad + (1 - \exp(-\lambda e_{\max} T) - \lambda e_{\max} T \exp(-\lambda e_{\max} T)) \alpha_{n+2} \\
&> \frac{c}{\lambda}.
\end{aligned}
$$

The potential histories in the game can be organized into the following three categories:

$$
\begin{array}{ccl}
(1) & : & (t,0,0,0)\,;(t,1,0,0)\,;(t,2,0,0) \\
(2) & : & (t,0,-i,v)\,;(t,1,-i,v)\,;(t,2,-i,v) \\
(3) & : & (t,1,i,\nu)\,;(t,2,i,\nu)\,.
\end{array}
$$

The first category contains the on-equilibrium histories. The continuation values for the first category are:

$$
\begin{aligned}
V_i\left(t,2,0,0\right) &= \chi\left(2\right) - \delta\left(T - t\right) \\
V_i\left(t,1,0,0\right) &= \left(1 - \exp(-\lambda e_{\max}\left(T - t\right))\right)\chi\left(2\right) + \exp(-\lambda e_{\max}\left(T - t\right))\chi\left(1\right) \\
&\quad -\frac{c}{\lambda}\left(1 - \exp(-\lambda e_{\max}\left(T - t\right))\right) - \delta\left(T - t\right) \\
V_i\left(t,0,0,0\right) &= \exp(-\lambda e_{\max}\left(T - t\right))\chi\left(0\right) + \lambda e_{\max}\left(T - t\right)\exp(-\lambda e_{\max}\left(T - t\right))\chi\left(1\right) \\
&\quad + \left(1 - \exp(-\lambda e_{\max}\left(T - t\right)) - \lambda e_{\max}\left(T - t\right)\exp(-\lambda e_{\max}\left(T - t\right))\right)\chi\left(2\right) \\
&\quad -\delta\left(T - t\right) - \frac{2c}{\lambda}\left(1 - \exp\left[-\lambda e_{\max}\left(T - t\right)\right] - \frac{\lambda e_{\max}\left(T - t\right)}{2}\exp\left[-\lambda e_{\max}\left(T - t\right)\right]\right).
\end{aligned}
$$

The off-equilibrium continuation values for the second category are given by:

$$
\begin{aligned}
V_i\left(t,0,-i,v\right) &= -\frac{1}{\varepsilon + 2\tau} \\
V_i\left(t,1,-i,v\right) &= -\frac{1}{\varepsilon + 3\tau} \\
V_i\left(t,2,-i,v\right) &= -\frac{1}{\varepsilon + 4\tau}.
\end{aligned}
$$

The off-equilibrium continuation values for the third category are given by:

$$
\begin{aligned}
V_i\left(t,1,i,v\right) &= \begin{cases} -\frac{1}{\varepsilon+\tau} + \left(1 - \exp(-\lambda e_{\max}t)\right)\alpha_2 + \left(1 - \left(\lambda e_{\max}t + 1\right)\exp(-\lambda e_{\max}t)\right)\alpha_3 & \text{for } \nu = t \\ -\frac{1}{\varepsilon+3\tau} & \text{for } \nu < t \end{cases} \\
V_i\left(t,2,i,v\right) &= \begin{cases} -\frac{1}{\varepsilon+2\tau} + \left(1 - \exp(-\lambda e_{\max}t)\right)\alpha_3 + \left(1 - \left(\lambda e_{\max}t + 1\right)\exp(-\lambda e_{\max}t)\right)\alpha_4 & \text{for } \nu = t \\ -\frac{1}{\varepsilon+4\tau} & \text{for } \nu < t. \end{cases}
\end{aligned}
$$

Note that $\frac{c}{\lambda} > \alpha_3$ immediately implies that an effort intensity of 0 is optimal when an agent believes that at least two signals have been found. This is the case when an agent themselves has found 2 signals or the agent is at a history where either agents has announced that a signal has been found. In the remaining cases effort is $e_{\max}$ which is optimal provided the continuation values satisfy:

$$
\begin{aligned}
V_i\left(t,2,0,0\right) - V_i\left(t,1,0,0\right) &> \frac{c}{\lambda} \\
V_i\left(t,1,0,0\right) - V_i\left(t,0,0,0\right) &> \frac{c}{\lambda}
\end{aligned}
$$

$\Leftrightarrow$

$$
\begin{aligned}
V_i\left(t,2,0,0\right) - V_i\left(t,1,0,0\right) &= \frac{c}{\lambda} + \exp(-\lambda e_{\max}\left(T-t\right))\left[\chi\left(2\right) - \chi\left(1\right) - \frac{c}{\lambda}\right] \\
&> \frac{c}{\lambda} \\
V_i\left(t,1,0,0\right) - V_i\left(t,0,0,0\right) &= \lambda e_{\max}\left(T-t\right)\exp(-\lambda e_{\max}\left(T-t\right))\left(\chi\left(2\right) - \chi\left(1\right)\right) \\
&\quad + \exp(-\lambda e_{\max}\left(T-t\right))\left(\chi\left(1\right) - \chi\left(0\right)\right) \\
&\quad \frac{c}{\lambda}\left(1 - \exp(-\lambda e_{\max}\left(T-t\right))\right) + c e_{\max}\left(T-t\right)\exp(-\lambda e_{\max}\left(T-t\right)) \\
&= \frac{c}{\lambda} + \lambda e_{\max}\left(T-t\right)\exp(-\lambda e_{\max}\left(T-t\right))\left(\chi\left(2\right) - \chi\left(1\right) - \frac{c}{\lambda}\right) \\
&\quad + \exp(-\lambda e_{\max}\left(T-t\right))\left(\chi\left(1\right) - \chi\left(0\right) - \frac{c}{\lambda}\right) \\
&> \frac{c}{\lambda}.
\end{aligned}
$$

The decision strategy is optimal provided that

$$
\begin{aligned}
V_i\left(t,2,0,0\right) &\geq -\frac{1}{\varepsilon + 2\tau} + \left(1 - \exp(-\lambda e_{\max}t)\right)\alpha_3 + \left(1 - \left(\lambda e_{\max}t + 1\right)\exp(-\lambda e_{\max}t)\right)\alpha_4 \\
V_i\left(t,1,0,0\right) &\geq -\frac{1}{\varepsilon + \tau} + \left(1 - \exp(-\lambda e_{\max}t)\right)\alpha_2 + \left(1 - \left(\lambda e_{\max}t + 1\right)\exp(-\lambda e_{\max}t)\right)\alpha_3 \\
V_i\left(t,0,0,0\right) &\geq -\frac{1}{\varepsilon} + \left(1 - \exp(-\lambda e_{\max}t)\right)\alpha_1 + \left(1 - \left(\lambda e_{\max}t + 1\right)\exp(-\lambda e_{\max}t)\right)\alpha_2
\end{aligned}
$$

Taking the first constraint,

$$
\chi\left(2\right) - \delta\left(T-t\right) \geq -\frac{1}{\varepsilon + 2\tau} + \left(1 - \exp(-\lambda e_{\max}t)\right)\alpha_3 + \left(1 - \left(\lambda e_{\max}t + 1\right)\exp(-\lambda e_{\max}t)\right)\alpha_4
$$

$\Leftrightarrow$

$$
\begin{aligned}
\delta\left(T-t\right) &\leq \left[\left(\lambda e_{\max}t + 1\right)\exp(-\lambda e_{\max}t) - \left(\lambda e_{\max}T + 1\right)\exp(-\lambda e_{\max}T)\right]\alpha_4 \\
&\quad + \exp(-\lambda e_{\max}t)\left(1 - \exp(-\lambda e_{\max}\left(T-t\right))\right)\alpha_3.
\end{aligned}
$$

At $t = T$ the inequality is satisfied. The derivative of the LHS wrt t is $-\delta$ the derivative of the RHS is

$$
-\lambda e_{\max}\left[\exp(-\lambda e_{\max}t)\alpha_3 + \lambda e_{\max}t\exp(-\lambda e_{\max}t)\alpha_4\right].
$$

which for $0 \leq t < \tilde{X}_d$ is strictly less than $-\delta$ hence the inequality is satisfied for $0 \leq t \leq T\left(\leq \tilde{X}\right)$. We may apply the same argument for the other two constraints after noting:

$$
\begin{aligned}
V_i\left(t,1,0,0\right) &\geq \left(1 - \exp(-\lambda e_{\max}T)\right)\alpha_2 + \left(1 - \left(\lambda e_{\max}T + 1\right)\exp(-\lambda e_{\max}T)\right)\alpha_3 - \delta\left(T-t\right) \\
V_i\left(t,0,0,0\right) &\geq \left(1 - \exp(-\lambda e_{\max}T)\right)\alpha_1 + \left(1 - \left(\lambda e_{\max}T + 1\right)\exp(-\lambda e_{\max}T)\right)\alpha_2 - \delta\left(T-t\right).
\end{aligned}
$$

This implies that the announcement strategy is optimal as well,

$$
\begin{aligned}
V_i\left(t,2,0,0\right) &\geq V_i\left(t,2,i,t\right) \\
V_i\left(t,1,0,0\right) &\geq V_i\left(t,1,i,t\right)
\end{aligned}
$$

These conditions are identical to the conditions for the decision strategy earlier.
Finally, for the calling decisions after an announcement, we require:

$$
\begin{array}{rcl}
V_i\left(t,2,-i,v\right) & \geq & V_i\left(s,2,-i,v\right) - \delta\left(s-t\right) \text{ for all } s \in (t,T] \\
V_i\left(t,1,-i,v\right) & \geq & V_i\left(s,1,-i,v\right) - \delta\left(s-t\right) \text{ for all } s \in (t,T] \\
V_i\left(t,0,-i,v\right) & \geq & V_i\left(s,0,-i,v\right) - \delta\left(s-t\right) \text{ for all } s \in (t,T] \\
V_i\left(t,2,i,v\right) & \geq & V_i\left(s,2,i,v\right) - \delta\left(s-t\right) \text{ for all } s \in (t,T] \\
V_i\left(t,1,i,v\right) & \geq & V_i\left(s,1,i,v\right) - \delta\left(s-t\right) \text{ for all } s \in (t,T]
\end{array}
$$

These are all satisfied as $V_i\left(t,\cdot,-i,v\right) = V_i\left(s,\cdot,-i,v\right)$ and $V_i\left(t,\cdot,i,v\right) = V_i\left(s,\cdot,i,v\right)$. ∎

Note that in the proposed equilibrium, the off-equilibrium belief held by a player when her partner announces that she acquired information, is that her partner acquired two signals. This off-equilibrium belief is reasonable, because a player with one signal prefers not to disclose that signal even if she was believed to have one signal. This is true because the best response of the other player would be to call a decision if she held 2 signals, stop putting in effort and delay if she held one signal, and would continue putting in effort until acquiring one signal and delay if she held 0 signals. The benefit for the player with one signal from announcing is that if the other player holds 2 signals then a decision is called immediately. The cost is that the other player would no longer search for a second signal after having acquired a first. It is clear that for sufficiently short deadlines, the probability that the other player has acquired 2 signals is too low for the benefit of announcing a signal to outweigh the cost of reducing the search incentives. We show this more formally below.

The expected payoff from disclosing an acquired signal under this scenario equals

$$
\zeta \equiv \left(1 - (\lambda e_{\max}t + 1)\exp(-\lambda e_{\max}t)\right)\left[-\frac{1}{\varepsilon + 3\tau}\right] + (\lambda e_{\max}t + 1)\exp(-\lambda e_{\max}t)\hat{V},
$$

where

$$
\begin{aligned}
\hat{V} & = & -\frac{1}{\varepsilon + \tau} + \exp(-\lambda e_{\max}(T-t))\left(1 - \frac{\exp(-\lambda e_{\max}T)}{(\lambda e_{\max}t + 1)\exp(-\lambda e_{\max}t)}\right)\alpha_2 \\
& & + (1 - \exp(-\lambda e_{\max}(T-t)))\left[\begin{array}{l}\frac{\exp(-\lambda e_{\max}T)}{(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t)}\alpha_2 \\ + \left(1 - \frac{\exp(-\lambda e_{\max}T)}{(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t)}\right)\alpha_3\end{array}\right] \\
& & -\frac{c}{\lambda}\left(1 - \exp(-\lambda e_{\max}(T-t))\right) - \delta\left(T-t\right)
\end{aligned}
$$

is the payoff in the event that the announcement is not met with a decision being called by the other player. In this case the updated beliefs about the value of information are

$$
\frac{\exp(-\lambda e_{\max}T)}{(\lambda e_{\max}t + 1)\exp(-\lambda e_{\max}t)}\alpha_2 + \left(1 - \frac{\exp(-\lambda e_{\max}T)}{(\lambda e_{\max}t + 1)\exp(-\lambda e_{\max}t)}\right)\alpha_3,
$$

where $\frac{\exp(-\lambda e_{\max}T)}{(\lambda e_{\max}t + 1)\exp(-\lambda e_{\max}t)}$ and $1 - \frac{\exp(-\lambda e_{\max}T)}{(\lambda e_{\max}t + 1)\exp(-\lambda e_{\max}t)}$ are the updated beliefs that the other player will have obtained 0 and 1 signal by the deadline given that they have not called a decision upon hearing the announcement and hence do not hold 2 signals.

Proposition 11 $\exists 0 < T < \tilde{X} : V_i\left(t,1,0,0\right) \geq \zeta.$

Proof. Rearranging $V_i(t, 1, 0, 0) \geq \zeta$ using the expressions above, we obtain

$$\frac{\frac{\lambda e_{\max} t}{\lambda e_{\max} t + 1}\left(1 - \exp\left(-\lambda e_{\max}\left(T - t\right)\right)\right) + \frac{1}{\lambda e_{\max} t + 1}\left(1 - \left(\lambda e_{\max}\left(T - t\right) + 1\right)\exp(-\lambda e_{\max}\left(T - t\right))\right)}{1 - \left(\lambda e_{\max} t + 1\right)\exp(-\lambda e_{\max} t)}$$

$$\geq \frac{\delta\left(T - t\right) + \left(\frac{c}{\lambda} - \alpha_4\right)\left(1 - \exp(-\lambda e_{\max}\left(T - t\right))\right)}{\left(\lambda e_{\max} t + 1\right)\exp(-\lambda e_{\max} t) \times \left[\exp\left(-\lambda e_{\max}\left(T - t\right)\right)\alpha_3 + \left(1 - \exp(-\lambda e_{\max}\left(T - t\right))\right)\alpha_4\right]}$$

This holds with equality for $t = T$ and holds strictly for $t = 0$. It suffices to check that for any $t \in (0, T)$ the following holds,

$$\frac{\lambda e_{\max} t \exp(-\lambda e_{\max} t)}{1 - \left(\lambda e_{\max} t + 1\right)\exp(-\lambda e_{\max} t)} \geq \frac{\delta}{\alpha_3}\frac{T - t}{1 - \exp\left(-\lambda e_{\max}\left(T - t\right)\right)} + \frac{\frac{c}{\lambda} - \alpha_4}{\alpha_3}.$$

Since $\frac{\left(1 - \exp(-\lambda e_{\max}(T - t))\right)}{T - t} < 1$, this condition is implied by

$$\frac{\exp(-\lambda e_{\max} t) - \exp\left(-\lambda e_{\max} T\right)}{T - t}\frac{\lambda e_{\max} t}{1 - \left(\lambda e_{\max} t + 1\right)\exp(-\lambda e_{\max} t)} \geq \frac{\delta}{\alpha_3} + \frac{\frac{c}{\lambda} - \alpha_4}{\alpha_3}. \tag{14}$$

Note that the RHS of (14) is clearly finite. Now consider the first term on the LHS of (14),

$$\frac{\exp(-\lambda e_{\max} t) - \exp\left(-\lambda e_{\max} T\right)}{T - t}.$$

This term is decreasing in t and thus has a finite positive lower bound of $\frac{1 - \exp(-\lambda e_{\max} T)}{T}$ which in the limit $T \to 0$ approaches $\lambda e_{\max}$. Now consider the limit of the second term on the LHS of (14),

$$\lim_{t \to 0}\frac{\lambda e_{\max} t}{\left(1 - \left(\lambda e_{\max} t + 1\right)\exp(-\lambda e_{\max} t)\right)} = \infty$$

Hence $\exists T > 0$ such that (14) is satisfied for all $0 < t \leq T$. ∎

B.5.3 Infinite Deadline

To gain some intuition for what may occur a long way from the deadline we consider an equilibrium of the infinite horizon game. This is done to avoid the complications in a game with a finite horizon, of specifying the changes in behaviour for the subgames as we transition from behavior far away from the deadline to close to the deadline. The infinite horizon case allows one to focus on a setting where there is no effect of a future deadline. We show that the equilibrium strategies are similar to the equilibrium strategies in our earlier model far away from the deadline ($t < T - Y$). We find there is immediate revelation of information by each individual, $w_i^* = n$. Hence, a decision is called whenever the number of signals reaches two, $d_i^* = call$ if $n + w_{-i} \geq 2$. This is similar to the earlier model whereby the agents immediately call a decision upon acquiring information. Individuals also exert less than the maximum effort level $e_i^* = \frac{\delta}{c} < e_{\max}$ as in the earlier model which trades off free riding incentives with incentives to bring forward the decision. We formalize this in the following proposition.

Proposition 12 *A symmetric perfect Bayesian equilibrium of the infinite horizon game is*

$$w_i^* (t,n) = 1 \ \text{if } n \geq 1$$

$$d_i^* (t,n,\mu,\nu) = \begin{cases} call \ if \begin{cases} n \geq 1 \ and \ \mu = -i \\ n = 2 \end{cases} \\ 0 \ otherwise \end{cases}$$

$$e_i^* (t,n,\mu,\nu) = \begin{cases} 0 \ if \begin{cases} n \geq 1 \ and \ \mu = -i \\ n = 2 \end{cases} \\ \frac{\delta}{c} \ otherwise \end{cases}$$

$$\phi_i^* (t,\mu,\nu) = \begin{cases} \{0,1,0\} \ if \ \mu = -i \\ 0 \ otherwise \end{cases}$$

Proof. The continuation values are

$$V_i (t,2,i,\nu) = -\frac{1}{\varepsilon + 2\tau}$$

$$V_i (t,2,-i,\nu) = -\frac{1}{\varepsilon + 3\tau}$$

$$V_i (t,2,0,0) = -\frac{1}{\varepsilon + 2\tau}$$

$$V_i (t,1,i,\nu) = -\frac{1}{\varepsilon + 2\tau} - \frac{c}{\lambda}$$

$$V_i (t,1,-i,\nu) = -\frac{1}{\varepsilon + 2\tau}$$

$$V_i (t,1,0,0) = -\frac{1}{\varepsilon + 2\tau} - \frac{c}{\lambda}$$

$$V_i (t,0,-i,\nu) = -\frac{1}{\varepsilon + 2\tau} - \frac{c}{\lambda}$$

$$V_i (t,0,0,0) = -\frac{1}{\varepsilon + 2\tau} - \frac{2c}{\lambda}.$$

Knowledge of 2 signals results in zero effort as $\alpha_3 < \frac{c}{\lambda}$. This is true when $n = 2$ or $n = 1$ and $\mu = -i$. The non-zero effort strategy is optimal provided that

$$V_i (t,2,i,\nu) - V_i (t,1,i,\nu) = \frac{c}{\lambda}$$

$$V_i (t,2,0,0) - V_i (t,1,0,0) = \frac{c}{\lambda}$$

$$V_i (t,1,0,0) - V_i (t,0,0,0) = \frac{c}{\lambda}$$

$$V_i (t,1,-i,\nu) - V_i (t,0,-i,\nu) = \frac{c}{\lambda},$$

which is straightforward to verify from inspection of the continuation values. The decision strategy not to call is optimal provided

$$V_i (t,1,0,0) \geq -\frac{1}{\varepsilon + \tau}$$

$$V_i (t,1,i,\nu) \geq -\frac{1}{\varepsilon + \tau}$$

$$V_i (t,0,-i,\nu) \geq -\frac{1}{\varepsilon + \tau}$$

$$V_i (t,0,0,0) \geq -\frac{1}{\varepsilon}$$

which is straightforward to verify. For the history $(t, 1, -i, \nu)$, the decision to call is optimal provided that for all $s \geq t$

$$
\begin{aligned}
V_i(t, 1, -i, \nu) \geq{} & \int_t^s \left(-\frac{1}{\varepsilon + 3\tau} - \delta(r - t) \right) \lambda \frac{\delta}{c} \exp\left(-\lambda \frac{\delta}{c}(r - t) \right) dr \\
& + \exp\left(-\lambda \frac{\delta}{c}(s - t) \right) [V_i(s, 1, -i, \nu) - \delta(s - t)] \\
={} & \left[-\frac{1}{\varepsilon + 3\tau} - \frac{c}{\lambda} \right] \left(1 - \exp\left(-\lambda \frac{\delta}{c}(s - t) \right) \right) - \frac{1}{\varepsilon + 2\tau} \exp\left(-\lambda \frac{\delta}{c}(s - t) \right),
\end{aligned}
$$

which follows by $\frac{c}{\lambda} \geq \alpha_3$. An almost identical condition holds for $(t, 2, i, \nu)$ and $(t, 2, -i, \nu)$. A very similar condition is also derived for the history $(t, 2, 0, 0)$:

$$
\begin{aligned}
V_i(t, 2, 0, 0) \geq{} & \int_t^s \left(V_i(r, 2, -i, \nu) - \delta(r - t) \right) \lambda \frac{\delta}{c} \exp\left(-\lambda \frac{\delta}{c}(r - t) \right) dr \\
& + \exp\left(-\lambda \frac{\delta}{c}(s - t) \right) [V_i(s, 2, 0, 0) - \delta(s - t)] \\
={} & \left[-\frac{1}{\varepsilon + 3\tau} - \frac{c}{\lambda} \right] \left(1 - \exp\left(-\lambda \frac{\delta}{c}(s - t) \right) \right) - \frac{1}{\varepsilon + 2\tau} \exp\left(-\lambda \frac{\delta}{c}(s - t) \right),
\end{aligned}
$$

which also follows by $\frac{c}{\lambda} \geq \alpha_3$. The announcement strategy is optimal provided that

$$
\begin{aligned}
V_i(t, 1, i, t) &\geq V_i(t, 1, 0, 0) \\
V_i(t, 2, i, t) &\geq V_i(t, 2, 0, 0),
\end{aligned}
$$

both of which hold with equality. ∎